LISBON SCHOOL OF ECONOMICS & MANAGEMENT
UNIVERSIDADE DE LISBOA

Mathematics Department
Undergraduate Degrees in Economics and Management
Mathematics 2

Normal Period Exam - January 9, 2018

Duration: 1h15

Name:	**Number:**

1. Consider the two variable function defined by

$$f(x, y) = 4x^2 + y^2.$$

a. [1,0 points] Sketch the level curves of f corresponding to the values $k = 1$, $k = 4$ and $k = 0$.

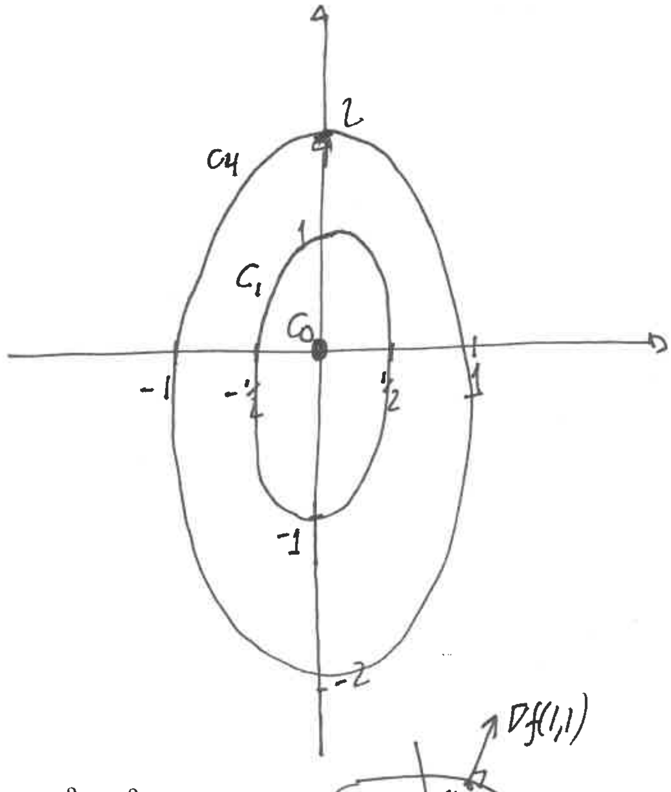
$C_1 = \{(x,y) : 4x^2 + y^2 = 1\}$

$4x^2 + y^2 = 1 \iff \frac{x^2}{\left(\frac{1}{2}\right)^2} + \frac{y^2}{1^2} = 1$

$C_4 = \{(x,y) : 4x^2 + y^2 = 4\}$

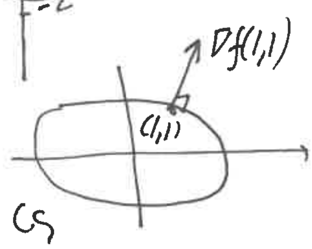
$4x^2 + y^2 = 4 \iff \frac{x^2}{1^2} + \frac{y^2}{2^2} = 1$

$C_0 = \{(x,y) : 4x^2 + y^2 = 0\} = \{(0,0)\}$



b. [1,0 points] Consider the ellipse $\mathcal{E}$ of equation $4x^2 + y^2 = 5$.

Compute all vectors $\vec{u}$ perpendicular to $\mathcal{E}$ at point $(1, 1)$.



$\mathcal{E}$ is the C_5 level curve of f.

We know that $\nabla f(1,1)$ is perpendicular to C_5 at point $(1,1)$

$\nabla f(x,y) = (8x, 2y)$ so $\nabla f(1,1) = (8,2)$

Hence the vectors $\vec{u}$ perpendicular to $\mathcal{E}$ at point $(1,1)$ are the vectors colinear to $\nabla f(1,1)$, namely $\vec{u} = (8k, 2k)$, $k \in \mathbb{R}$