LISBON SCHOOL OF ECONOMICS & MANAGEMENT
UNIVERSIDADE DE LISBOA

Financial Markets and Instruments

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Duration: 2.5h

Name: **Number:**

GROUP I (30 points)

Answer briefly to the questions of this group (without exceeding the available space).

1. State the assumptions underlying mean-variance theory (MVT) and explain why, even in the presence of risk neutral or risk loving investors, we can focus on the so-called efficient frontier. Finally, discuss equilibrium under MVT, if there would be only risk neutral and/or risk lover investors. [15p]

   Answer:

2 Choose ONE of the following statements and discuss whether they are true or false. ..... [15p]

I. *Most return generating models are based upon unrealistic assumptions, thus, there is no sound ground for applying them in practice.*

II. *An investor worried with safety is indifferent between the optimal portfolios according to Roy, Kataoka or Telsser.*

Comment:.....

GROUP II (20 points)

Explain the relation between the utility function $U(W)$, the risk tolerance function $f(\sigma, \bar{R})$, and the indifference curves of the risk tolerance function. ..... [7.5p]

Show that the second order Taylor approximation of any risk tolerance function is *equivalent* to

$$f\left(\sigma, \bar{R}\right) = \bar{R} - \frac{1}{2} RRA(W_0)\left(\bar{R}^2 + \sigma^2\right).$$

..... [12.5p]

GROUP III

Answer each **Problem** *in separate exam sheets.*

Problem 1 (90 points)

Consider the assumptions of a single factor model (SFM), where for the common factor we have $\bar{R}_m = 15\%$, $\sigma_m = 20\%$. Furthermore, assume there exists a riskless asset that can be used to both lend and borrow with $R_f = 5\%$ and the following information about 6 risky assets.

	$\bar{R}_i$	β_i	σ^2_{ei}
1	25,1%	2	0,002
2	19,8%	1,5	0,003
3	17,0%	1,2	0,004
4	14,8%	1	0,005
5	12,8%	0,8	0,006
6	12,0%	0,7	0,007

1. Consider the single factor used for the SFM model is a good proxy to the market portfolio of CAPM.
   (a) Write down the CAPM equilibrium equation and interpret. [5p]
   (b) Show that only assets 3 and 6 are in equilibrium. [5p]
   (c) Represent the security market line and the 6 assets in the $\left(\beta, \bar{R}^e\right)$ space in and give your buy/sell recommendations. [10p]

2. Suppose Mr. Capm would like to consider only combinations of the two risky assets that are in equilibrium.
   (a) Determinine the MVT inputs for the two assets in equilibrium. [5p]
   (b) What is the geometric shape of the set representing all *efficient* combinations of the two assets. Explain and sketch it in mean-variance space $\left(\sigma, \bar{R}\right)$. [10p]
   (c) If Mr. Capm would consider, in addition, the risk free asset. What would then be:
      (i) the investment opportunity set [5p]
      (ii) the combination of the two risky assets with the highest Sharpe ratio. [10p]
      (ii) the efficient frontier. [2.5p]
   (d) Do you think Mr. Capm should consider also the risk free asset? Why or why not? [5p]

3. Assume now the risky returns are approximately Gaussian.
   (a) Identify all portfolios that have at most 25% probability of negative returns. Derive the mathematical expression and represent it in $\left(\sigma, \bar{R}\right)$ space. [12.5p]
   [For $z \sim N(0,1)$ recall $\mathbb{P}(z \leq -0.6745) = 0.25$.]

(b) Suppose Mr. Capm from Question 2 is a Telser investor. What would you recommend

(i) Considering only combinations of the two risky assets. ..... [7.5p]

(ii) Considering in addition the risk-free asset. Why? ..... [2.5p]

4. Using the cut-off method and assuming shortselling is not allowed, what is your opinion about the fact that Mr. Capm is only considering the two risky assets in equilibrium? I.e., would you recommend considering any other risky asset? Explain your answer. ..... [10p]

Problem 2 (60 points)

In a country *NearByTheSea* the efficient mean-variance frontier is given by

$$\begin{cases} \bar{R}_p = 0.03 + 1.2\sigma_p & \sigma_p \leq 0.10 \\ \sigma_p^2 = 5.56\bar{R}_p^2 - 1.50\bar{R}p + 0.11 & 0.10 < \sigma_p < 0.20 \\ \bar{R}_p = 0.114 + 0.48\sigma_p & 0.20 \leq \sigma_p \leq 0.35 \end{cases}$$

1. Based upon the above information:

(a) Determine the expected returns of the efficient portfolios, $T1$ and $T2$ with 10% and 20% volatility, respectively. ..... [5p]

(b) Knowing the minimum variance portfolio can be seen as the combination of $T1$ and $T2$ where we invest 125% in $T1$, find out the implicit correlation between the returns of portfolios $T1$ and $T2$. ..... [10p]

(c) From the shape of the efficient frontier, what can you conclude about: (i) the existence or not of a riskless asset, (ii) passive and active interest rates, (iii) possible shortselling restrictions, (iv) possible borrowing limits ..... [10p]

(d) Sketch the shape of the investment opportunity set (IOS) set and of the efficient frontier (EF) in the mean-variance $\left(\sigma, \bar{R}\right)$ space and describe how each efficient point could be achieved. ..... [10p]

2. Consider Mr. Quelhas has an utility function $U(W) = -e^{bW}$, with $b < 0$.

(a) Evaluate Mr. Quelhas risk profile, interpreting your conclusions about his absolute and relative risk aversion. ..... [10p]

(b) Take $W_0 = 1$ and use the second order Taylor approximation to the risk tolerance function of Mr. Quelhas to determine for which levels of the parameter b, his optimal volatility is the maximum allowed volatility $\sigma^* = 35\%$. Explain all steps of your solution. ..... [15p]