LISBON SCHOOL OF ECONOMICS & MANAGEMENT
UNIVERSIDADE DE LISBOA

Financial Markets and Investments

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Answer directly on the exam sheet. **Duration: 1.5h**

.....

Name: **Number:**

.....

GROUP I (30 points)

1. State the assumptions underlying mean-variance theory (MVT) and explain why, even in the presence of risk neutral or risk loving investors, we can focus on the so-called efficient frontier. Finally, discuss equilibrium under MVT, if there would be only risk neutral and/or risk lover investors. ..... [15p]

   Answer:

2 Choose ONE of the following statements and discuss whether they are true or false. ..... [15p]

I. *Most return generating models are based upon unrealistic assumptions, thus, there is no sound ground for applying them in practice.*

II. *An investor worried with safety is indifferent between the optimal portfolios according to Roy, Kataoka or Telsser.*

Comment: .....

GROUP II (20 points)

Show that if a portfolio stochastically dominates another in a first order sense, then, it is preferred by any investor who prefers more to less, independently of being risk averse, neutral or lover.

Clarify your notation and justify all steps of the proof.

Proof.

GROUP III (50 points)

In a country *NearByTheSea* the efficient mean-variance frontier is given by

$$\begin{cases} \bar{R}_p = 0.03 + 1.2\sigma_p & \sigma_p \leq 0.10 \\ \sigma_p^2 = 5.56\bar{R}_p^2 - 1.50\bar{R}p + 0.11 & 0.10 < \sigma_p < 0.20 \\ \bar{R}_p = 0.114 + 0.48\sigma_p & 0.20 \leq \sigma_p \leq 0.35 \end{cases}$$

1. Based upon the above information:

   (a) Determine the expected returns of the efficient portfolios, $T1$ and $T2$ with 10% and 20% volatility, respectively. ..... [5p]
   Solution:

   (b) Knowing the minimum variance portfolio can be seen as the combination of $T1$ and $T2$ where we invest 125% in $T1$, find out the implicit correlation between the returns of portfolios $T1$ and $T2$..... [5p]
   Solution:

   (c) From the shape of the efficient frontier, what can you conclude about: (i) the existence or not of a riskless asset, (ii) passive and active interest rates, (iii) possible shortselling restrictions, (iv) possible borrowing limits ..... [10p]
   Answer:
   (i)

(ii)

(iii)

(iv)

(d) Sketch the shape of the investment opportunity set (IOS) set and of the efficient frontier (EF) in the mean-variance $\left(\sigma, \bar{R}\right)$ space and describe how each efficient point could be achieved. ..... [7.5p]
Solution:

2. Consider Mr. Quelhas has an utility function $U(W) = -e^{bW}$, with $b < 0$.

   (a) Evaluate Mr. Quelhas risk profile, interpreting your conclusions about his absolute and relative risk aversion. ..... [7.5p]
   Solution:

   (b) Take $W_0 = 1$ and use the second order Taylor approximation to the risk tolerance function of Mr. Quelhas to determine for which levels of the parameter b, his optimal volatility is the maximum allowed volatility $\sigma^* = 35\%$. Explain all steps of your solution. ..... [15p]
   Solution:

. . . .

Extra Answering space (if needed)