Financial Markets and Instruments

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Duration: 2.5h

....

Name: **Number:**

....

GROUP I (30 points)

Answer directly on the exam sheet (without exceeding the available space).

1. Explain Lintner's definition of portfolio and its connection with shortselling restrictions. Consider an arbitrary number of $n > 3$ risky assets and a risk-free asset that can be used for both lending and borrowing. Sketch in the plan $(\sigma, \bar{R})$ the investment opportunity set under 3 different scenarios: (i) shortselling fully allowed, (ii) limited shortselling *a la*, Lintner, and (iii) shortselling forbidden. .... [15p]

   Answer:

2 Choose <u>ONE</u> of the following statements and discuss whether they are true or false. .....[15p]

    I. *To an investor who does not verify the Von-Neuman-Morgensten axioms, one should recommend safe portfolios according to criteria such as Roy, Kataoka or Telser.*

    II. *If some analysts believe in a two-factor APT equilibrium model and others in the classical CAPM equilibrium model, they will never agree about equilibrium returns.*

Comment:......

Answer directly on the exam sheet (without exceeding the available space).

1. Characterize the risk profile of a *log–investor* with $U(W) = a + b\log(W)$, for $b > 0$. ....[10]

2. Show that a portfolio that maximizes the geometric expected return is the optimal portfolio for a log–investor (for simplicity you may consider a discrete distribution of returns). .... [10p]

Answers:

Answer each **Problem** *of this group in SEPARATE exam sheets.*

Problem 1 (75 points)

The efficient frontier in the market under analysis is given by

$$\bar{R}_p = 3\% + SR_T \sigma_p \ ,$$

where SR_T is the highest possible attainable Sharpe Ratio.

In addition, we know the portfolios under consideration are based upon 16 risky assets and that the only combination of *just risky assets* that is efficient happens to be the *homogeneous* portfolio with expected return of 12% and a volatility of 15%.

1. What can you conclude about: (i) the existence or not a a riskless asset, (ii) de possibility of borrowing to invest in risky assets, (iii) the composition of the tangent portfolio, (iv) the value of SR_T. Explain. .... [10p]

2. Show that, for an average volatility of risky assets of 17.7%, the market implied average correlation is 0.7. .... [7.5p]

3. Mr. Iseg has a risk profile well described by the indifference curves $\bar{R}_p = \sigma_p^2 + 0.3\sigma_p + K$, with $K \in \mathbb{R}$.

   (a) Show the optimal investment volatility for Mr. Iseg is 15%........................... [7.5p]
   (b) How should Mr. Iseg invest 250 000 euros, to reach that desired level of risk? .... [5p]
   (c) What would need to be the certain return, R_C, that would make Mr.Iseg indifferent to invest or not in his optimal portfolio? Explain your computations. .... [10p]

4. In terms of (i) the market's efficient frontier and (ii) the optimal allocation for Mr. Iseg, what would change if:

   (a) Shortselling is forbidden. .... [5p]
   (b) It is not possible to take a loan for investment in risky assets. .... [5p]

5. Assume this market is in a standard CAPM equilibrium.

   (a) Write down the capital market line (CML). Explain................................ [5p]
   (b) What can you conclude about the *market portfolio* ? Explain...................... [7.5p]
   (c) Determine the securities market line (SML) equation. .... [7.5p]
   (d) What is the average beta of the 16 risky assets existent in this market? Why?........ [5p]

Problem 2 (75 points)

Consider two Gaussian risky assets with $\bar{R}_1 = 12\%, \bar{R}_2 = 6\%, \sigma_1 = 20\%, \sigma_2 = 15\%$ and $\rho = +0.5$. Shortselling is allowed without bound but it is not possible to get a loan to invest in risky assets. Still, there exist a riskless rate $R_f = 3\%$ for deposits.

1. Sketch the investment opportunity set (IOS) and the efficient frontier (EF) in the mean-variance plan. Explain. . . . . [10p]

2. Find out the minimum variance portfolio, its expected return and volatility. . . . . [5p]

3. Derive the equations of the efficient frontier. . . . . [15p]

4. Consider that Mr. Exact wants a portfolio E with $\bar{R}_E = 8\%$ and $\sigma_E = 18\%$.

   (a) What can you conclude about the efficiency of E? Explain. . . . . [5p]

   (b) Find out the composition of portfolio E. Motivate all steps. . . . . [10p]

   (c) Suggest an alternative to portfolio E that Mr. Exact would always accept, no matter this risk profile. Explain. . . . . [7.5p]

5. Suppose now a new financial institution, Safety Bank, appears in this market. The Safety Bank is willing to give credit to investments in financial markets at a 3% interest rate, provided the probability of not getting paid (capital plus interest) is not higher than 10%.

   (a) Write down the Safety Bank credit condition and represent it in mean-variance space. [recall that for $z \sim N(0,1)$ $Pr(z \leq -1.2816) \leq 10\%$] . . . . [10p]

   (b) Find the Telser portfolio satisfying the Safety Bank restriction. . . . . [7.5p]

   (c) What type of investors will use the credit services of Safety Bank? Explain. . . . . [5p]