LISBON SCHOOL OF ECONOMICS & MANAGEMENT
UNIVERSIDADE DE LISBOA

Quantitative Finance – 1st yr/Economics-Finance-Management, 1st sem, Exam 2 01.02.2019, 9:00

1st Part (70/200 marks) Total Time: 2h

1st Part: 70 Marks. All answers shall be given in the space available. During the examination no comments or questions should be asked. Write your name and number on every sheet on the place available. No mobile phones, or any device with *bluetooth* or *wifi*, are allowed at any time.

Name:____**Number:**_________

***In the following groups of questions, every right answer has 2.5 marks each, wrong answers have −2.5 each (2.5 penalty mark).** Each group of questions will have a mark between **0** (minimum) and **10** (maximum).*

Write True (**T**) or False (**F**), *with an "**X**" in the appropriate entry.*

1. Consider Simple and Compound Interest calculation:	**T**	**F**
Both in Simple and Compound Interest, an anual nominal interest rate, positive and compounded quarterly, can never be equivalent to the corresponding effective rate.		X
For an interest rate $i = 10\%$, a principal applied under simple interest takes about +37,5% more time to double the accumulated capital than the same application if done under coumpound interest.	✓	
For interest rates higher than 100%, Simple Interest takes always more time to double the accumulated capital when compared to Compound Interest situationn.		X
Let two non-negative interest rates referred to the same period, such as $i_1 > i_2$. Then, the corresponding (anual) nominal interest rates also verify $i_1{}^{(m)} > i_2{}^{(m)}$.	✓	

2. Consider annuities, immediate, differed, perpetuities. Always, $i > 0$:	**T**	**F**
Let $a_{\overline{\infty}\mid i} = 20$. Then we have $a_{\overline{n}\mid i} / \ddot{a}_{\overline{n}\mid i} = 1.05$.		X
An annuity is defined as a set of periodic constant payments, in equally spaced periods, and where a constant interest rate is applied.		X
If we set that $a_{\overline{\infty}\mid i} = 20$ and $s_{\overline{\infty}\mid i} = 30$, we can state that we are facing a perpetuity where a constant interest rate of 5% is considered.		X
Notation $\ddot{a}_{\overline{n}\mid i}$ denotes an annuity due with n constant payments all equal to *R*, for a constant interest rate *i*.		X

3. Consider the following financial operations:	**T**	**F**
In a constant and periodic repayment loan, both the principal and interest payments are non-increasing along time.		X
A bond loan, completey subscribed, changes the loan amount if units are issued above the par. That is, the new loan amount is equal to the global nominal value plus the issuance premia received.		X
The *Leasing* operation is basically a bank loan with an extra optional payment known as the residual value.		X
A non public company is allowed to issue shares if the issuance is public and pays interest.		X

4. Consider the following situations:	**T**	**F**
A five year €100,000 loan is redempted four times with constant amortization payments of €25,000 each, with an annual interest rate of 5%. Interest payment in the last year totals €1250.00.	✓	
For rate $i > 0$, we have that $s_{\overline{n}\mid i} > n$.	✓	
Consider a bond loan issued above the par where redemption is paid under the par. Investor's yield rate is lower than the coupon rate (annual).	✓	
Let an annuity and the corresponding interest rate $i \geq 0$. We have that $1/a_{\overline{n}\mid i} - 1/s_{\overline{n}\mid i} = i$.	✓	

In the next group of questions, tick ✓ or write X in the box next to the answer you consider to be correct (only one is). In each group, a correct answer has 5 marks and a wrong answer gets -1.25 marks (penalty 1.25).

5. Consider simple interest and a monthly interest rate of 1.2%. Dr. Zen got a loan of €57,000.00. Principal and interest were paid altogether once at the end summing €63.839,54. What is the term of the loan? Approximately: $57\,000(1+0.012x) = 63\,839{,}54 \rightarrow x \simeq 10$ months

a) 1 year, 2 months and 3 days ☐; b) 9 months and 10 days ☐;

c) 9 months and 15 days ☐; d) None of the others ☑.

6. Dr. Zen made a four year application of amount €5,000.00, under compound interest. It accumulated at maturity an amount of €6,281.78. Determine the quarterly interest rate that would produce the same acummulated amount with the same term (approximately).

a) 1.43% ☑; b) 1.60% ☐; c) 1.46% ☐; d) None of the others ☐.

$5000(1+i)^{16} = 6281.78 \rightarrow i \simeq 1.4365 \rightarrow i \simeq 1.43\%$

7. The company where Dr. Zen works issued a bond loan. Issuance was done at the par value, redemption also at the par (no other expenses considered). A bond is sold in the market at the nominal value at the middle of a coupon payment period. The **yield rate to the selling investor**, who had bought at the issuance date, is (relatively to the coupon rate):

a) Higher, than the coupon rate ☐; b) Equal ☐; c) Lower ☑; d) Information is missing ☐.

8. Consider compound interest. Choose the quarterly rate that is equivalent to the rate corresponding to the annual nominal rate of 12%, compounded semi-annually (approximate to the €0.01):

a) 2,96% ☑; **b)** 3% ☐; **c)** 3,14% ☐; **d)** 5,83% ☐.

9. Consider the following amortization schedule of a loan (Dr. Zen's) with annual constant payments (principal plus interest) to be redempted in eight years (in €):

Year	OB_{k-1}	I_k	P_k	R_k	Accum P_k	OB_k
3	2,604,704.00	260,470.00	227,766.00	488,236.00	623,062.00	2,376,938.00

Principal Amortization is (approximately):

a) €511 211.00 ☐; **b)** €488 236.00 ☐; **c)** €443 851.00€ ☐; **d)** None of the others ☑.

$m_8 < T_8 \rightarrow$ a) and b) are not answers. Neither is c) since $443851(1+i) \neq T_8$

10. Dr. Zen has to receive today 10 monthly amounts, increasing in geometric progression, increasing at a monthly rate of 1%. First payment is being received within a month from today, with a value of €10. For a monthly effective rate of $i_M = 1\%$, the present value of the set of amounts is given by (in €):

a) $^{10(0.1)}/_{1.01}$ ☐; b) ∞ ☐; c) $^{10^2}/_{1.01}$ ☑; d) None of the others ☐.

$$P.V. = \frac{10}{1.01} + 10\,\frac{1.01}{1.01^2} + \cdots + 10\,\frac{1.01^9}{1.01^{10}} = 10\left(\frac{10}{1.01}\right) = \frac{10^2}{1.01}$$

Name:____No.:_________

2nd Part (130/200 marks)

In this group write your calculations in the space below the question and write the final answer in the box provided. Do not forget to present all formulae and intermediate calculations needed.

1. (50 marks)

"Zen's PLC" issued a bond loan under the following terms:

- Issuance date: 01/01/2016;
- Face value: €10.00;
- Nº of bonds, above the par: 120,000;
- Issue value: €9.80;
- Maturity: 3 years;
- Coupon annual interest rate, variable, compounded semi-annually: 1st year 6%; Following years: 6.2%;
- 1st redemption, 1 year after issuance;
- Coupon paid semi-annually, 1st payment on 01/07/2016;
- Redemption: Principal constant payments, annually;
- Redemption premium: €0.20 per bond in the 1st year and €0.30 in the following.

a) Compute the value of the loan.

loan : 120 000 (10,00) = 1 200 000

R:

b) Fill up the Amortization Schedule, for the **1st year and half**, only:

Time	Initial Balance	Interest	No. of Bonds redempted	Principal	Premium	Payment	Outstanding Balance
0							1 200 000
1	1 200 000	36 000	—	—	—	36 000	1 200 000
2	1 200 000	36 000	40 000	400 000	8000	444 000	800 000
3	800 000	24 800	—	—	—	24 800	800 000
4	800 000	24 800	40 000	400 000	12 000	436 800	400 000
5	400 000	12 400	—	—	—	12 400	400 000
6	400 000	12 400	40 000	400 000	12 000	424 400	0

c) Dr. Zen's school mate, Dr. Zorba, bought 100 bonds at the issuance date and sold them exactly three months after the payment of the 3rd coupon. He got an yield rate of 11% on the investment, **write the equation** that allows to calculate how much money Dr. Zorba received for the complete sell.

Pay at issuance : 100 (9.80) = 980.00 , Yield rate : $r = 0.11$ and $r^* = (1.11)^{1/2} - 1 \simeq 5.36\%$

Coupons 1 and 2 : 100(10)(0.03) = 30 , Coupon 3 = 100(10)(0,031) = 31 , S : Sell value

(980) +30 +30 +31 S

0 1 2 3 3½ 4 Sem.

$$\boxed{S + 30\, s_{\overline{2}|r^*}(1+r^*)^{1.25} + 31(1+r^*)^{0.25} = 980(1+r)^{1.75}}$$

2. (30 marks)

Dr. Zen is acquiring an *SUV* vehicle through a leasing contract. Contract value is €30,000. Dr. Zen received the following proposal from *LeasingZappa, Ltd*:

- Nominal annual interest rate, compounded quarterly: 8%;
- Values to be paid:
  - Initial payment, in the contract signing date: 10% of the purchase value;
  - 10 quarterly constant installments: The first is to be paid 6 months after the contract signing date;
  - Residual value: 15% of purchase value, option to be exercised (and paid) to be paid one month after the last installment, exactly.

a) Calculate the amount of each quarterly installment.

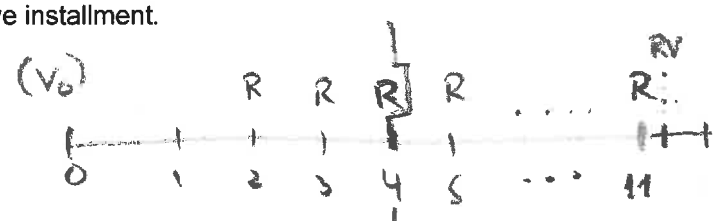
$V_0 = 30\,000 \quad , \quad i^{(4)} = 8\% \rightarrow i_4 = 2\%$

$30\,000 = 3000 + R_{\,1|}a_{\overline{10|}} + 4500\,(1.02)^{-11\,1/3}$

$R \simeq 2657{,}67$

R: € 2657.67

b) Calcule the amount yet to be paid one year after the contract date, immediately after the payment of the respective installment.



After 1 year: 7 instalm. are remaining:

$2657{,}67\; a_{\overline{7|}2\%} \cong 17{,}200.40$

Residual value is optional.

R: € 17,200.40

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3. (50 pontos)

O Dr. Zen agreed to sell his second/older vehicle to his friend Dr. Zaid for €12,000.00. The agreement determines that Dr. Zen will receive today a down payment of €3,500.00, plus ten equal installments at the end of each quarter. Consider an effective annual rate of 10.9%.

a) The first of the ten periodic payments is due within three months. Calculate the value of each installment.

$i_A = 10.9\% \iff i_T = (1.109)^{+1/4} - 1 = 2.62\,202\%$, $a_{\overline{10|}\,i_T} \simeq 8.69795$

$12000 - 3500 = R\, a_{\overline{10|}\,i_T} \iff R = \frac{8500}{a_{\overline{10|}\,i_T}} \simeq 977.24$ €

R: $\sim$ € 977.24

b) The €3,500.00 down payment can be suppressed if the periodic payments are due at the beginning of each quarter. Calculate the amount of each installment under these new conditions (no change in the interest rate).

$12000 = R^* \ddot{a}_{\overline{10|}\,i_T} = R\, a_{\overline{10|}} (1.0262202)$

$R^* \simeq 1344.39$ €

R: $\sim$ € 1344.39

c) Get back to the initial situation in item a). Dr. Zaid, due to unexpected financial troubles, concluded that is not going to be able to pay the last installment. Dr. Zen, who is a kind young man, understood the situation and forgave the last payment. In this case, what was the value of the discount (to the initial sell agreement) that Dr. Zen conceided to his friend, referred today (date of the agreement)?

Value of the Discount: $977.24\,(1.0262202)^{-10} \simeq 754.52$ €

R: $\sim$ € 754.52