

Duration: 60 Minutes

Name: _____

Question:	1	2	Total
Points:	30	70	100

Justify all your answers except for multiple choice questions. **Organize your work.** Work scattered all over the page will receive very little credit. A correct answer in a multiple choice question is worth 10 points; an incorrect one is worth -2.5 points. **You are allowed to** use only statistical tables, a calculator and a formula sheet (2 pages maximum).

1. Consider the following linear regression model:

$$y_i = \beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2} + \beta_3 x_{i3} + \beta_4 x_{i4} + u_i, \quad i = 1, \dots, 500 \quad (1)$$

Suppose that the sample $\{(x_{i1}, x_{i2}, x_{i3}, x_{i4}, y_i) : i = 1, \dots, 500\}$ is randomly drawn from the population and the condition $E(u_i | x_{i1}, x_{i2}, x_{i3}, x_{i4}) = 0$ is verified.

(10) (a) In a preliminary analysis, two auxiliary equations were estimated:

$$\hat{x}_{i2} = 3.465 + 1.211x_{i1} - 0.077x_{i3} - 0.055x_{i4}, \quad R^2 = 0.988 \quad (2)$$

$$\hat{x}_{i2} = 5.780 + 1.455x_{i1}, \quad R^2 = 0.971 \quad (3)$$

Which of the following statements is **TRUE**:

- One cannot obtain OLS estimates for β_j , $j = 0, 1, 2, 3, 4$, in equation (1), because x_{i2} and x_{i1} are almost perfectly correlated.
- The OLS standard error for $\hat{\beta}_2$ in equation (1) should be very high.
- The variables x_{i3} and x_{i4} are relevant to explain y_i .
- None of the above.

(10) (b) Suppose that x_{i4}^2 is added to the model (with coefficient β_5). If $H_0 : \beta_4 = 0$ and $H_0 : \beta_5 = 0$ are not rejected, then:

- The null hypothesis $H_0 : \beta_4 = \beta_5 = 0$ will not be rejected as well.
- The variable x_{i4} has no effect on y_i .
- The squared term, x_{i4}^2 , must be removed from the model's specification.
- None of the above.

- (10) (c) If the conditional variance of the error is given by $\text{Var}(u_i | x_{i1}, x_{i2}, x_{i3}, x_{i4}) = 10 + \delta x_{i4}^2$, where δ is an unknown parameter, then:
- The OLS estimator of the coefficients β_j , $j = 0, 1, \dots, 4$, in equation (1) will be inconsistent for any value of $\delta > 0$.
 - The error, u , is homoskedastic for $\delta = 0$.
 - If $\delta > 0$, the usual OLS t statistics follow approximately a standard Normal distribution.
 - None of the above.

2. An external evaluation committee is analyzing the quality of the courses offered in the Economics and Management programs of each University in Lisbon. The goal is to give accreditation to these programs. With regard to the Econometrics courses (both portuguese speaking and english speaking), the following equation was estimated.

$$\text{grade}_i = \beta_0 + \beta_1 \text{dist}_i + \beta_2 \text{dist}_i^2 + \beta_3 \log(\text{tuition}_i) + \beta_4 \text{study}_i + \beta_5 \text{work}_i + u_i \quad (4)$$

with $i = 1, \dots, 554$. The variables have the following meaning:

- **grade** is the final grade in Econometrics (in a scale from 0 to 20) of the student;
- **dist** is the distance between the house of the student and the University;
- **tuition** is the tuition fee paid by the student;
- **study** is the total daily amount of time (in hours) studying;
- **work** is equal to 1 if the student has a job (equal to 0 otherwise).

The estimated output of this equation is in the **Annex**. Other results that may be necessary to solve the following questions are included in the **Annex** as well.

- (15) (a) Interpret the estimated coefficients $\hat{\beta}_3$ and $\hat{\beta}_5$, and test their individual significance, as well.

- (10) (b) Write the partial effect of *dist* over *grade* and interpret it.

- (15) (c) Is equation 2 in the Annex better to explain *grade* than equation 1 in the Annex? Justify, using the result of one test of hypothesis.

- (15) (d) Formalize and test the hypothesis that there is no difference in the final grade of a student with a job that studies two more hours than a student with no job, holding all the other factors the same.

- (15) (e) Is there evidence that the model in Equation 1 in the Annex is not correctly specified? Use the result of a statistical test to conclude. Indicate the observed value of the test statistic and the statistical conclusion.