

Duration: 60 Minutes

Name: \_\_\_\_\_

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|-----------|----|----|-------|
| Question: | 1  | 2  | Total |
| Points:   | 30 | 70 | 100   |

**Justify** all your answers. You are required to show your work on each problem on this test. **Organize your work.** Work scattered all over the page will receive very little credit. A correct answer in a multiple choice question is worth 10 points; an incorrect one is worth  $-2.5$  points. **You are allowed to use only statistical tables, a calculator and a formula sheet** (2 pages maximum).

1. Consider the following linear regression model:

$$y_i = \beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2} + \beta_3 x_{i3} + \beta_4 x_{i4} + u_i, \quad i = 1, \dots, 500 \quad (1)$$

Assume that the sample  $\{(x_{i1}, x_{i2}, x_{i3}, x_{i4}, y_i) : i = 1, \dots, 500\}$  is randomly drawn from the population and that the condition  $E(u_i | x_{i1}, x_{i2}, x_{i3}, x_{i4}) = 0$  is verified for all  $i$ .

- (10) (a) Choose the **correct** statement:
- If  $x_{i1} = x_{i2}$  then the OLS estimates for  $\beta_j, j = 1, \dots, 4$  in equation (1) exist and  $\hat{\beta}_1 = \hat{\beta}_2$ .
  - If  $x_{i3} + x_{i4} = 1$  then the OLS estimator of  $\beta_j, j = 1, \dots, 4$  does not exist because the model is linear in the parameters.
  - If the OLS estimate  $\hat{\beta}_0 = 0$ , it implies that the sample average of  $y$  is equal to zero.
  - None of the above.
- (10) (b) Suppose that  $R^2 = 0.873$ . This means that ...
- ... the sample variation in  $y$  is 87.3% higher than sample variation of  $u$ .
  - ... 87.3% of the sample average in  $y$  is explained by  $x_{i1}, x_{i2}, x_{i3}$  and  $x_{i4}$ .
  - ... 87.3% of the sample variation in  $y$  is explained by  $x_{i1}, x_{i2}, x_{i3}$  and  $x_{i4}$ .
  - None of the above.
- (10) (c) Suppose that a researcher is interested in testing the hypothesis  $H_0 : \beta_1 = 2\beta_4$ . The OLS estimates of  $\hat{\beta}_1, \hat{\beta}_4, \text{se}(\hat{\beta}_1)$  and  $\text{se}(\hat{\beta}_4)$  are also given. In addition, it is known that  $\widehat{\text{Cov}}(\hat{\beta}_1, \hat{\beta}_4) = 0$ . The observed value of the test statistic is given by:
- $t_{obs} = \frac{\hat{\beta}_1 - 2\hat{\beta}_4}{\sqrt{[\text{se}(\hat{\beta}_1)]^2 - 2[\text{se}(\hat{\beta}_4)]^2}}$
  - $t_{obs} = \frac{\hat{\beta}_1 - 2\hat{\beta}_4}{\sqrt{[\text{se}(\hat{\beta}_1)]^2 + 4[\text{se}(\hat{\beta}_4)]^2}}$
  - $t_{obs} = \frac{\hat{\beta}_1 - 2\hat{\beta}_4}{\text{se}(\hat{\beta}_1) - 2\text{se}(\hat{\beta}_4)}$
  - $t_{obs} = \frac{\hat{\beta}_1 - 2\hat{\beta}_4}{\text{se}(\hat{\beta}_1) + 4\text{se}(\hat{\beta}_4)}$

2. The portuguese parliament approved the National budget for 2019 in November 2018. With this approval, the law that allows the portuguese banks to concede loans to the portuguese students, to pay their tuition and related University expenses, can be implemented. There are however some banks that want to concede these loans according to a given set of characteristics of the student. The following model was estimated:

$$\begin{aligned} ploan_i = & \beta_0 + \beta_1 age_i + \beta_2 avgrade_i + \beta_3 avgrade_i^2 + \beta_4 \log(hours_i) \\ & + \beta_5 meduc_i + \beta_6 feduc_i + \beta_7 sibs_i + e_i \end{aligned} \quad i = 1, \dots, 722 \quad (2)$$

The variables have the following meaning:

- ***ploan*** is the probability of conceding a loan to a student;
- ***age*** is the age of a student;
- ***avgrade*** is the current overall average grade of the student;
- ***hours*** is the total amount of hours that the student spent studying during the last academic year;
- ***meduc*** is the mother's education of the student;
- ***feduc*** is the father's education of the student;
- ***sibs*** is the number of siblings of the student.

The estimated output of this equation can be found in **Annex Part I**. Other results that, may be useful to solve the following questions, can be found in **Annex Part I**, as well.

- (15) (a) Interpret the estimated coefficients  $\hat{\beta}_4$  and  $\hat{\beta}_7$ , and test their individual significance, as well.

- (10) (b) Write the partial effect of *avgrade* over *plan* and interpret it.

- (15) (c) Consider a new variable *female*, a dummy variable equal to 1 if the student is female (and equal to 0 otherwise). Are there notable differences between the estimates for women and men, with regard to equation (2)? Justify using an appropriate test.

- (15) (d) Test the hypothesis that a unitary change in both parents education, results in a change in the probability of conceding a loan equal to one.

- (15) (e) Equation 4 in Annex Part I aims to perform a test. Identify and formalize this test. What can you conclude?