Lecture notes, exercises, and study suggestions

Class 1 - February 20

1. Make yourself familiar with some time series application such as ITSM, EViews, or R.
2. Get some of the Wei's book data sets at the author's website. Plot some of the data series and check whether they present a trend, seasonality, cycles and noise.
3. Choose one data set that presents seasonality of period s and apply a moving-average filter of amplitude s to the same set.
4. Chose one data set without seasonality and apply a moving-average filter of order 2, than one of order 5, than one of order 10, and plot the filtered series superimposed with the four resulting filtered series. You may simply use Excel to do it.
5. Recall that $B^i X_t = X_{t-i}$ and $\nabla = (1 - B)$. Express the following in terms of X_i:
   - BX_{t-2}
   - $\nabla^2 X_t$
   - $BX_t - B^{-2} X_t$
   - $(1 - B^s) X_t$
6. Read carefully Wei's book definition of stochastic processes and make sure you understand that time series can be seen as cases of stochastic processes in which we are interested in the sequence of X_t variables or observations as functions of t.

Class 2 - March 6

1. Make sure you understand the concepts of *strict stationarity* and *second-order stationarity.*
2. Review the properties of γ_k and ρ_k. Understand the concept and computation of ϕ_{kk}.

3. Do the following exercises from Wei's book: 2.1; 2.3(c); 2.7(a), (b), and (c) with Excel; 2.7(d) and (e) with an appropriate computer program.

4. By using the appropriate system of equations, compute ϕ_{22} for a process such that $\gamma_0 = 4, \gamma_1 = 2, \gamma_2 = 1$.

5. Do the same for a process such that $\gamma_0 = 4, \gamma_1 = 2, \gamma_2 = 1/2$

Class 3 - March 13

1. Make sure you understand the concepts of white noise, iid and iin processes
   - Give one example of a process $a_t \sim \text{wn}(0, \sigma^2)$ which is not iid
   - Give one example of a process $\epsilon_t \sim$ iid which is not $\text{wn}(0, \sigma^2)$

2. Make sure you understand what is an *ergodic in mean* process and that a sufficient condition for this property is that $\rho_k \to 0$.

3. Review Bartlett's formula and understand that you can construct bands for testing wether $\rho_k = 0$ or $\phi_{kk} = 0$ on the form $\pm 1.96/\sqrt{n}$

4. Review MA(∞) representation of a process, also called Wold representation

5. Use the *autocovariance generating function* to deduce the ACVF and the ACF of the process $Z_t = a_t + \frac{1}{2}a_{t-1} + a_{t-2}$ with $a_t \sim \text{iin}(0, 1)$.

6. Consider a stationary process with the autocovariance function $\gamma_k = \phi^k$, with $\phi < 1$. Find the variance of $\hat{\gamma}_0$ using Bartlett's approximation.

Classes 4, 5, and 6 - March 20, 27, and 29

1. Make sure you are able to derive theoretically the ACF and the ACVF of AR(1), AR(2), MA(1), and MA(2) processes, both by using the autocovariance generating function and by computing the expected values $\text{E}(Z_t Z_{t+k})$

2. Make sure you understand why $\rho_k = 0$ for $k > q$ for MA(q) processes

3. Consider the ARMA process Z_t that follows the equation:

$$Z_t - \frac{1}{4}Z_{t-1} = a_t - \frac{5}{6}a_{t-1} - \frac{1}{6}a_{t-2}$$

   and write it in inverted form, i.e., as an AR(∞) process. Show the numeric values of ψ_j for $j = 1, 2, 3, 4$

4. Solve exercises 3.1, 3.2, 3.6, 3.8, 3.12, and 3.15. You can use an appropriate software to check the results.

5. By using an appropriate software, work out the exercise 3.17.

6. Make sure you understand the differences between a trend-stationary process, i.e., a process you can transform to a stationary one by removing a deterministic trend, and a difference stationary process, i.e., a process you can transform to a stationary one by differencing it.

7. Make sure you understand what is a random walk process, with and without drifts.

8. Make sure you understand the recursive idea behind exponential smoothing (equations 4.2.10 and the following).

9. Solve exercise 4.1 and 4.5.

10. Referring to the previous exercise make sure you understand the following concepts:

    - under-differencing, i.e, not differencing a process when you need to do so for it to become stationary, therefore observing an
    - AR unit root – non-stationarity (non-causality)
    - over-differencing, i.e, differencing a process when you don't need to do so for it to become stationary and so introducing a
    - MA unit root – non-invertibility

    Verify that overdifferencing increases the variance

11. By using an appropriate software, work out the exercise 4.3.

Classes 7, 8, 9, and 10 - April 3, 10, 24, and 26

1. Make sure you understand the Box-Cox transformation: what is its purpose, when it should be applied, the special case $\lambda = 0$, and the meaning of differenced logs for a financial or economic time series.

2. Understand the difference between stationarity and nonstationarity in seasonality—SARIMA vs. SARMA—and try out a few examples with observed time series.

3. Know how to write a SARIMA model as an ARIMA model with restrictions on the parameters. Try out a practical example with ITSM.

4. Review the whole structure of the so-called Box-Jenkins Identification Method.

5. Make sure you understand the need to use criteria such as AIC and SIC and try out some examples. Verify that the best R^2 fit (or minimum residual variance) is not necessarily the best model to chose.

6. Review the concepts of *ex-ante* and *ex-post* forecasting.

7. Work out the book exercises 4.4, 5.1, 5.4 (a) and (b).

8. By using an appropriate software, work out the exercises 6.2 and 6.3 (Data sets WW19, ww20, WW21, WW22, and WW23) and 7.6.

9. Solve exercise 8.1.

10. By using an appropriate software, work out exercises 8.6 and 8.7 (Data sets WW25 and WW26).

PLEASE CHECK:

1. NEW **CALENDAR FOR GROUP PRESENTATIONS**

2. NEW **VOLATILITY TOPICS SLIDES**