Multiple Regression Analysis: OLS Asymptotics

Chapter 4 (chapter 5 from the textbook)

Wooldridge: Introductory Econometrics:
A Modern Approach, 5e

Multiple Regression Analysis: OLS Asymptotics

- **So far we focused on properties of OLS that hold for any sample**
- **Properties of OLS that hold for any sample/sample size**
  - Expected values/unbiasedness under MLR.1 – MLR.4
  - Variance formulas under MLR.1 – MLR.5
  - Gauss-Markov Theorem under MLR.1 – MLR.5
  - Exact sampling distributions/tests under MLR.1 – MLR.6
- **Properties of OLS that hold in large samples**
  - Consistency under MLR.1 – MLR.4
  - Asymptotic normality/tests under MLR.1 – MLR.5

Without assuming nor-mality of the error term!

Multiple Regression Analysis: OLS Asymptotics

- **<u>Consistency</u>**

  An estimator θ_n is consistent for a population parameter θ if

  $P\left(|\theta_n - \theta| < \epsilon\right) \rightarrow 1$ for arbitrary $\epsilon > 0$ and $n \rightarrow \infty$.

  Alternative notation: $plim\ \theta_n = \theta$ ← The estimate converges in probability to the true population value

- **Interpretation:**

  Consistency means that the probability that the estimate is arbitrarily close to the true population value can be made arbitrarily high by increasing the sample size

- **Consistency is a minimum requirement for sensible estimators**

Multiple Regression Analysis: OLS Asymptotics

- **Theorem 5.1 (Consistency of OLS)**

$$MLR.1 - MLR.4 \quad \Rightarrow \quad plim\ \widehat{\beta}_j = \beta_j, \quad j = 0, 1, \ldots, k$$

- For OLS to be consistent it is enough that the **explanatory variables are not correlated with the error term** which is less demanding than MLR.4

Multiple Regression Analysis: OLS Asymptotics

- **Asymptotic normality and large sample inference**
  - In practice, the normality assumption MLR.6 is often questionable
  - If MLR.6 does not hold, the results of t- or F-tests may be wrong
  - Fortunately, F- and t-tests still work if the sample size is large enough
  - Also, OLS estimates are normal in large samples even without MLR.6
- **Theorem 5.2 (Asymptotic normality of OLS)**

Under assumptions MLR.1 – MLR.5:

$$\frac{(\widehat{\beta}_j - \beta_j)}{se(\widehat{\beta}_j)} \overset{a}{\sim} N(0,1)$$

In large samples, the standardized estimates are normally distributed

also $plim\ \widehat{\sigma}^2 = \sigma^2$

Multiple Regression Analysis: OLS Asymptotics

- **Asymptotic normality of OLS with heteroscedasticity**

  Under assumptions MLR.1 – MLR.4:

$$\frac{\hat{\beta}_j - \beta_j}{se^*(\hat{\beta}_j)} \overset{a}{\sim} N(0,1)$$

- **Practical consequences**
  - In large samples, the t-distribution is close to the N(0,1) distribution
  - As a consequence, t-tests are valid in large samples without MLR.6
  - The same is true for confidence intervals and F-tests