# Serial Correlation and Heteroscedasticity in Time Series Regressions

## Chapter 10 (Ch. 12 of the textbook)

Wooldridge: Introductory Econometrics:
A Modern Approach, 5e

# Analyzing Time Series: Serial Correl. and Heterosced.

- **Properties of OLS with serially correlated errors**
  - **OLS still unbiased and consistent** if errors are serially correlated
  - Correctness of R-squared also does not depend on serial correlation
  - **OLS standard errors and tests will be invalid**
  - **OLS will not be efficient** anymore if there is serial correlation
- **Serial correlation and the presence of lagged dependent variables**
  - Is OLS inconsistent if there are ser. corr. and lagged dep. variables?
  - No: Including enough lags so that TS.3‵ holds guarantees consistency
  - Including too few lags will cause an omitted variable problem and serial correlation because some lagged dep. var. end up in the error term

# Analyzing Time Series: Serial Correl. and Heterosced.

- **<u>Testing for serial correlation</u>**
- **Testing for AR(1) serial correlation with strictly exog. regressors**

$$y_t = \beta_0 + \beta_1 x_{t1} + \cdots + \beta_k x_{tk} + u_t$$

AR(1) model for serial correlation (with an i.i.d. series $e_t$)

$$u_t = \rho u_{t-1} + e_t$$

Replace true unobserved errors by estimated residuals

$$\widehat{u}_t = \rho \widehat{u}_{t-1} + error$$

# Analyzing Time Series: Serial Correl. and Heterosced.

- **<u>Test procedure</u>**
- OLS of $y_t = \beta_0 + \beta_1 x_{t1} + \cdots + \beta_k x_{tk} + u_t \longrightarrow \hat{u}_t$
- Auxiliary test regression

  OLS of $\hat{u}_t = \rho \hat{u}_{t-1} + error \longrightarrow \hat{\rho}, se(\hat{\rho})$
- Test

  $H_0 : \rho = 0$ $\boxed{\textbf{t-test}}$
- **Example: Static Phillips curve (see above)**

$$\hat{u}_t = inf_t - (1.42 + 0.468 unem_t) \quad n = 49$$

$$\underset{(0.116)}{\hat{\hat{u}}_t = 0.573 \hat{u}_{t-1}} \quad n = 48 \longrightarrow t_{obs} = \frac{0.573}{0.116} = 4.94$$

Reject null hypothesis of no serial correlation

# Analyzing Time Series: Serial Correl. and Heterosced.

- **Testing for AR(1) serial correlation with general regressors**
  - The t-test for autocorrelation can be easily generalized to allow for the possibility that the explanatory variables are not strictly exogenous:

$$\hat{u}_t = \alpha_0 + \alpha_1 x_{t1} + \cdots + \alpha_k x_{tk} + \boxed{\rho}\,\hat{u}_{t-1} + error$$

The test now allows for the possibility that the strict exogeneity assumption is violated.

Test for $H_0 : \rho = 0$

  - The test may be carried out in a heteroscedasticity robust way

# Analyzing Time Series: Serial Correl. and Heterosced.

- **General Breusch-Godfrey test for AR(q) serial correlation**
  - OLS of $y_t = \beta_0 + \beta_1 x_{t1} + \cdots + \beta_k x_{tk} + u_t \longrightarrow \hat{u}_t$
  - OLS of

$$\hat{u}_t = \alpha_0 + \alpha_1 x_{t1} + \ldots + \alpha_k x_{tk} + \rho_1 \hat{u}_{t-1} + \rho_2 \hat{u}_{t-2} + \ldots + \rho_q \hat{u}_{t-q} + v \longrightarrow R^2_{\hat{u}}$$

  - Test

$$H_0 : \rho_1 = \rho_2 = \ldots = \rho_q = 0 \qquad H_1 : H_0 \text{ false}$$

$LM = nR^2_{\hat{u}} \overset{\cdot}{\sim} \chi^2(q)$ **Reject** $H_0$ if $LM_{obs} > \chi^2_\alpha$ with $\chi^2_\alpha : P\left(\chi^2(q) > \chi^2_\alpha\right) = \alpha$

  - If $H_0$ is rejected there is evidence of autocorrelation of order q

# Analyzing Time Series: Serial Correl. and Heterosced.

- **Heteroscedasticity in time series regressions**
  - Heteroscedasticity usually receives less attention than serial correlation
  - Heteroscedasticity-robust standard errors also work for time series
  - Heteroscedasticity is automatically corrected for if one uses the serial correlation-robust formulas for standard errors and test statistics
- **Testing for heteroscedasticity**
  - The usual heteroscedasticity tests assume absence of serial correlation
  - Before testing for heteroscedasticity one should therefore test for serial correlation first, using a heteroscedasticity-robust test if necessary
  - After ser. corr. has been corrected for, test for heteroscedasticity