Performance of Combined Double Seasonal Univariate Time Series Models for Forecasting Water Demand

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Abstract: This paper examines the daily water demand forecasting performance of double seasonal univariate time series models (Holt-Winters, ARIMA, and GARCH) based on multistep ahead forecast mean squared errors. A within-week seasonal cycle and a within-year seasonal cycle are accommodated in the various model specifications to capture both seasonalities. The study investigates whether combining forecasts from different methods could improve forecast accuracy. The results suggest that the combined forecasts perform quite well, especially for short-term forecasting. On the other hand, the individual forecasts from Holt-Winters exponential smoothing and GARCH models can improve forecast accuracy on specific days of the week.

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Introduction

Water demand forecasting is of great economic and environmental importance. Many factors can influence directly or indirectly water consumption. These include rainfall, temperature, demography, land use, pricing, and regulation. Weather conditions have been widely used as inputs of multivariate statistical models for hydrological time series modeling and forecasting.

Maidment and Miaou (1986), Fildes et al. (1997), Zhou et al. (2000), Jain et al. (2001), and Bougadis et al. (2005) adopted regression and time series models for water demand forecasting by using climate effects as explanatory variables for their models. Wong et al. (2007) used a nonparametric approach based on the transfer function model to forecast a time series of riverflow. Such methods are useful for assessing water demand under some stability conditions. However, their ability to project demand into the future may be limited as a result of weather condition variability and changes in consumer behavior and technology. During the last few decades, some researchers have also employed a variety of black-box methods for hydrological forecasting, including artificial neural networks models (Coulibaly and Baldwin 2005; Chau 2006; Jain and Kumar 2007; Cheng et al. 2008) and support vector regression (Sivapragasam et al. 2001; Yu et al. 2006; Wu et al. 2008).

Water demand is highly dominated by daily, weekly, and yearly seasonal cycles. The univariate time series models based on the historical data series can be quite useful for short-term demand forecasting as we accommodate the various periodic and seasonal cycles in the model specifications and forecasts. To avoid their sensibility to changes in weather conditions and other seasonal patterns, we may combine forecasts derived from the most accurate forecasting methods for different forecast origins and horizons. Combining forecasts can reduce errors by averaging of individual forecasts (Clemen 1989; Armstrong 2001) and is particularly useful when we are uncertain about which forecasting method is better for future prediction. Some relevant works on combined forecasts of univariate time series models were made by Makridakis and Winkler (1983), Sanders and Ritzman (1989), Lobo (1992), and Makridakis et al. (1993).

In this paper, I examined the water demand forecasting performance of double seasonal univariate time series models based on multistep ahead forecast mean squared errors (MSEs). I investigated whether combining forecasts from different methods could improve forecast accuracy. Our interest in this problem arose from the time series competition organized by Spanish IEEE Computational Intelligence Society at the SICO 2007 Conference.

The remainder of the paper is organized as follows. The second section gives a brief description of the data set used in this study. The third section presents the methodology used in time series modeling and forecasting. The fourth section reports the empirical findings. Concluding remarks are provided in the last section.

Data

I analyzed the daily water consumption series in Spain from January 1, 2001 to June 30, 2006 (2006 observations). I dropped February 29 in the leap year 2004 to maintain 365 days in each year. This series is plotted in Fig. 1. The data set was obtained from the Spanish IEEE Computational Intelligence Society (http://www.congresocedi.es/2007/).

I used the first 1976 observations from January 1, 2001 to May 31, 2006 as training sample for model estimation, and the remaining 30 observations from June 1, 2006 to June 30, 2006 as post-sample for forecast evaluation. The series exhibits periodic behavior with a within-week seasonal cycle of seven periods and a within-year cycle of 365 periods. The observed increases (de-

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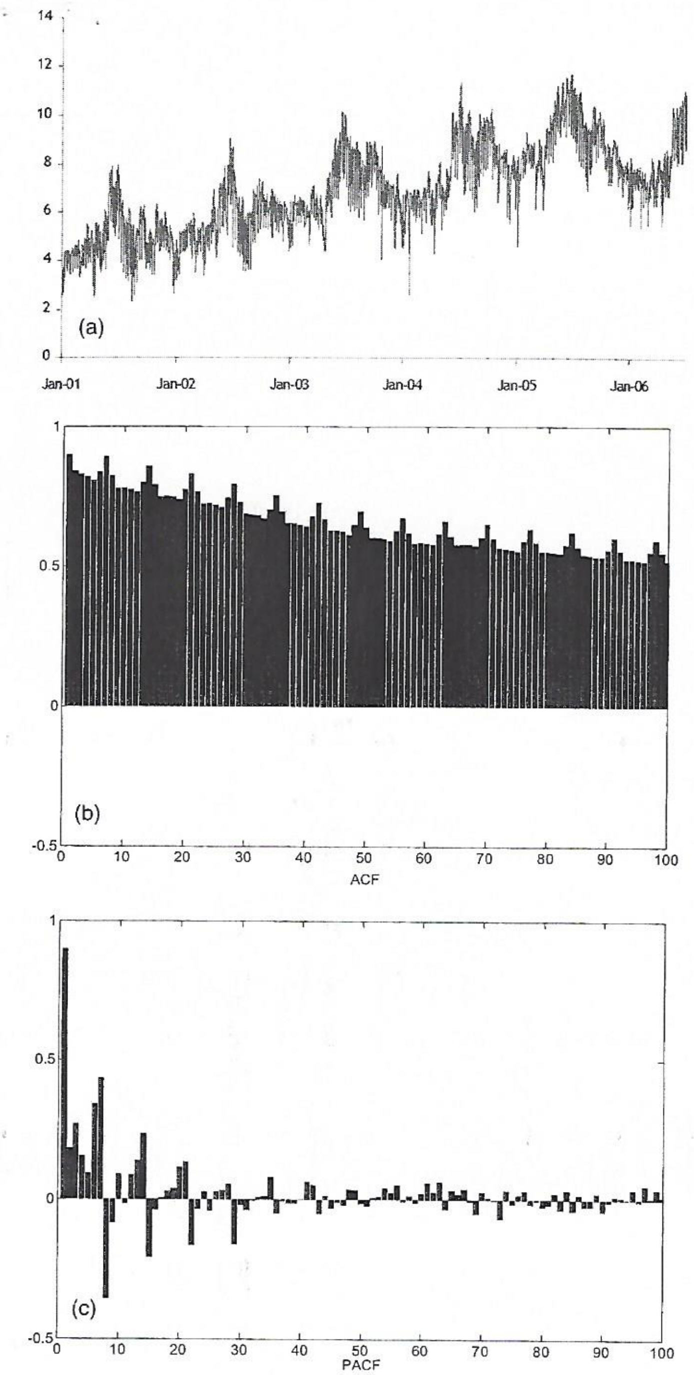

Fig. 1. Daily water demand in Spain for the period January 1, 2001 to June 30, 2006; ACF and PACF

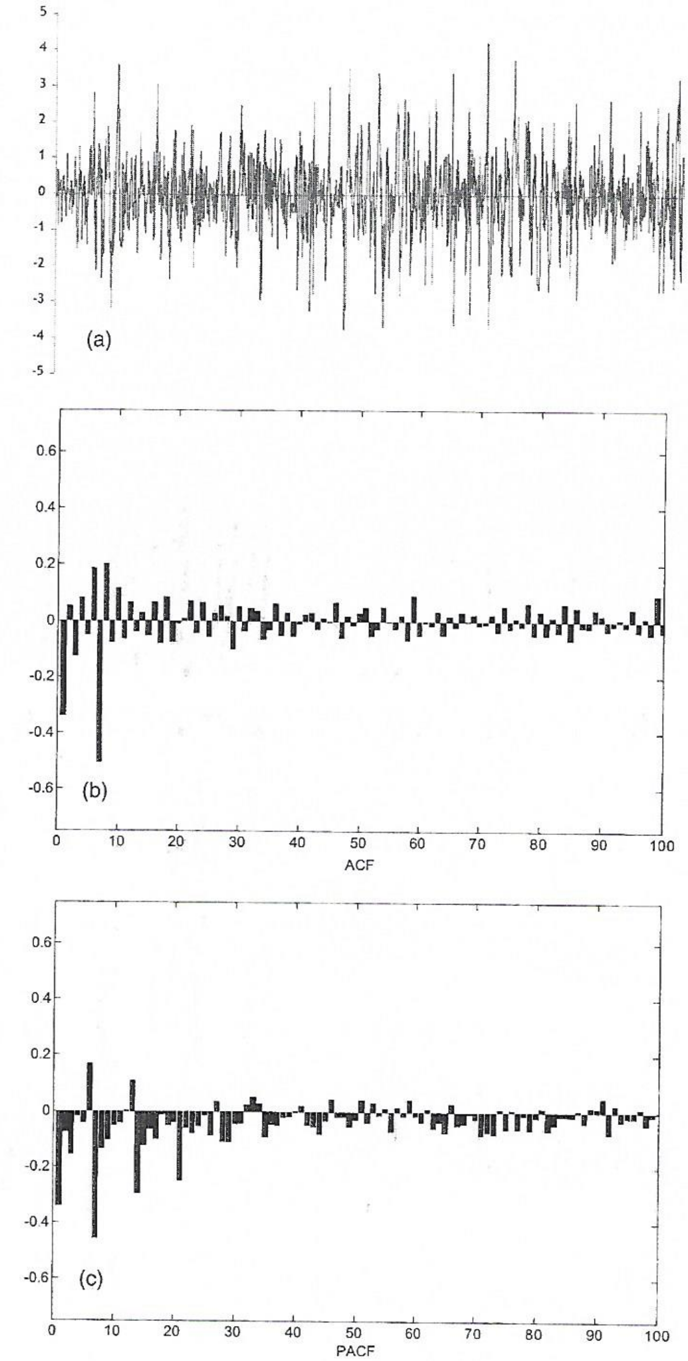

Fig. 2. Water demand series after yearly seasonal differencing and weekly seasonal differencing; ACF and PACF

creases) in demand in the summer (winter) days seem to be caused by good (bad) weather. The analysis of weekly seasonality shows a consumption drop in demand on Saturdays and Sundays as a result of the shutdown of industry.

Fig. 1 shows also the sample autocorrelations and the sample partial autocorrelations for the training sample. The autocorrelation function (ACF) decays very slowly at regular lags and at multiples of Seasonal Periods 7 and 365. The partial autocorrelation function (PACF) has a large spike at Lag 1 and cutoff to 0 after Lag 2. This suggests both a weekly seasonal difference $(1-B^7)$ and a yearly seasonal difference $(1-B^{365})$ to achieve stationarity. Fig. 2 presents the double seasonal differenced series $(1-B^7)(1-B^{365})Y_t$ and their estimated ACF and PACF.

Methodology

Forecast Evaluation

I denoted the actual observation for time period t by Y_t and the forecasted value for the same period by F_t. The MSE statistic for the postsample period $t=m+1,m+2,\ldots,n$ is defined as follows:

$$\text{MSE}=\frac{1}{n-m}\sum_{t=m+1}^{n}(Y_t-F_t)^2 \tag{1}$$

This statistic is used to evaluate the out-of-sample forecast accuracy using a training sample of observations of size $m<n$ (where n is the sample size) to estimate the model, and then computing

recursively the one-step ahead forecasts for time periods $m+1, m+2, \ldots$, by increasing the training sample by one. For k-step ahead forecasts, we begin at the start of the training sample and we compute the forecast errors for time periods $t=m+k$, $m+k+1, \ldots$, using the same recursive procedure.

RW

The naive version of the random walk (RW) model is defined as

$$F_{t+1} = Y_t \tag{2}$$

This purely deterministic method uses the most recent observation as a forecast, and is used as a basis for evaluating of time series models described below.

Exponential Smoothing

Exponential smoothing is a simple but very useful technique of adaptive time series forecasting. Standard seasonal methods of exponential smoothing includes the Holt-Winters additive trend, multiplicative trend, damped additive trend, and damped multiplicative trend [see Gardner, Jr. (2006)]. I implemented the double seasonal versions of the Holt-Winters exponential smoothing (Taylor 2003) to take into account the two seasonal cycle periods in the daily water consumption: a within-week cycle of 7 days and a within-year cycle of 365 days. In an application to one-half hourly electricity demand, Taylor (2003) used a within-day seasonal cycle of 48 half hours and a within-week seasonal cycle of 336 half hours.

The double seasonal additive methods outperformed the double seasonal multiplicative methods. Within the double seasonal additive methods, the additive trend was found to be the best for one-step ahead forecasting.

The forecasts for Taylor's exponential smoothing for double seasonal additive method with additive trend are determined by the following expressions:

$$L_t = \alpha(Y_t - S_{t-7} - D_{t-365}) + (1-\alpha)(L_{t-1} + T_{t-1}) \tag{3}$$

$$T_t = \beta(L_t - L_{t-1}) + (1-\beta)T_{t-1} \tag{4}$$

$$S_t = \gamma(Y_t - L_t - D_{t-365}) + (1-\gamma)S_{t-7} \tag{5}$$

$$D_t = \delta(Y_t - L_t - S_{t-7}) + (1-\delta)D_{t-365} \tag{6}$$

$$F_{t+h} = L_t + T_t \times h + S_{t+h-7} + D_{t+h-365} + \phi^h[Y_t - (L_{t-1} - T_{t-1} - S_{t-7} - D_{t-365})] \tag{7}$$

where L_t=smoothed level of the series; T_t=smoothed additive trend; S_t=smoothed seasonal index for weekly period ($s_1=7$); D_t=smoothed seasonal index for yearly period ($s_2=365$); α and β=smoothing parameters for the level and trend; γ and δ=seasonal smoothing parameters; ϕ=adjustment for first-order autocorrelation; and F_{t+h}=forecast for h periods ahead, with $h \leq 7$. We initialize the values for the level, trend and seasonal periods as follows:

$$L_7 = \frac{1}{7}\sum_{t=1}^{7} Y_t, \quad L_{365} = \frac{1}{365}\sum_{t=1}^{365} Y_t$$

$$T_7 = \frac{1}{7^2}\left(\sum_{t=8}^{14} Y_t - \sum_{t=1}^{7} Y_t\right), \quad T_{365} = \frac{1}{365^2}\left(\sum_{t=366}^{730} Y_t - \sum_{t=1}^{365} Y_t\right)$$

$$S_1 = Y_1 - L_7, \; S_2 = Y_2 - L_7, \ldots, \; S_7 = Y_7 - L_7$$

$$D_1 = Y_1 - L_{365}, \; D_2 = Y_2 - L_{365}, \ldots, \; D_{365} = Y_{365} - L_{365}$$

The smoothing parameters α, β, γ, δ, and ϕ are chosen by minimizing the MSE statistic for one-step-ahead in-sample forecasting using a linear optimization algorithm.

ARIMA Model

I implemented a double seasonal multiplicative autoregressive integrated moving average (ARIMA) model [see Box et al. (1994)] of the form

$$\phi_p(B)\Phi_{P_1}(B^{s_1})\Pi_{P_2}(B^{s_2})(1-B)^d(1-B^{s_1})^{D_1}(1-B^{s_2})^{D_2}(Y_t - c) = \theta_q(B)\Theta_{Q_1}(B^{s_1})\Psi_{Q_2}(B^{s_2})\varepsilon_t \tag{8}$$

where c=constant term; B=lag operator such that $B^kY_t = Y_{t-k}$; $\phi_p(B)$ and $\theta_q(B)$=regular autoregressive and moving average polynomials of orders p and q; $\Phi_{P_1}(B^{s_1})$, $\Pi_{P_2}(B^{s_2})$, $\Theta_{Q_1}(B^{s_1})$, and $\Psi_{Q_2}(B^{s_2})$=seasonal autoregressive and moving average polynomials of orders P_1, P_2, Q_1, and Q_2, respectively; s_1 and s_1=seasonal periods; d, D_1, and D_2=orders of integration; and ε_t=white noise process with 0 mean and constant variance. The roots of the polynomials $\phi_p(B)=0$, $\theta_q(B)=0$, $\Phi_{P_1}(B^{s_1})=0$, $\Pi_{P_2}(B^{s_2})=0$, $\Theta_{Q_1}(B^{s_1})=0$, and $\Psi_{Q_2}(B^{s_2})=0$ should lie outside the unit circle. This model is often denoted as ARIMA$(p, d, q) \times (P_1, D_1, Q_1)_{s_1} \times (P_2, D_2, Q_2)_{s_2}$.

I examined the sample autocorrelations and the partial autocorrelations of the differenced series to identify the integer values p, q, P_1, Q_1, P_2, and Q_2. After identifying a tentative ARIMA model, we estimate the parameters by Marquardt nonlinear least-squares algorithm [for details, see Davison and MacKinnon (1993)]. I checked the adequacy of the model by using suitable fitted residuals tests. I used the Schwarz Bayesian Criterion (SBC) for model selection.

GARCH Model

In many practical applications to time series modeling and forecasting, the assumption of nonconstant variance may be not reliable. The models with nonconstant variance are referred to as conditional heteroscedasticity or volatility models. To deal with the problem of heteroscedasticity in the errors, Engle (1982) and Bollerslev (1986) proposed the autoregressive conditional heteroskedasticity (ARCH) and the generalized ARCH (or GARCH) to model and forecast the conditional variance (or volatility). The GARCH(p, q) model assumes the form

$$\sigma_t^2 = \omega + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^2 + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 \tag{9}$$

where p=order of the GARCH terms and q=order of the ARCH terms. The necessary conditions for the model (9) to be variance and covariance stationary are: $\omega > 0$; $\beta_j \geq 0$, $j=1,\ldots,p$; $\alpha_i \geq 0$, $i=1,\ldots,q$; and $\Sigma_{j=1}^{p}\beta_j + \Sigma_{i=1}^{q}\alpha_i < 1$. The last summation quantifies the shock persistence to volatility. A higher persistence indicates that periods of high (slow) volatility in the process will last longer. In most economical and financial applications, the simple GARCH(1,1) model has been found to provide a good representation of a wide variety of volatility processes as discussed by Bollerslev et al. (1992).

Table 1. Seasonal ARIMA Model Estimates for Water Demand Series

Model: ARIMA(4, 0, 9)$\times$(2, 1, 3)$_7\times$(0, 1, 1)$_{365}$				Residual ACF		Residual PACF	
Parameter	Lag	Estimate	Standard error	Lag	Estimate	Lag	Estimate
c		−0.004	0.007	1	0.004	1	0.004
ϕ_1	1	0.592	0.025	2	0.009	2	0.009
ϕ_2	2	0.134	0.027	3	−0.020	3	−0.020
ϕ_4	4	0.061	0.023	4	0.001	4	0.001
θ_9	9	−0.053	0.024	5	−0.026	5	−0.025
Φ_1	7	−0.757	0.023	6	0.015	6	0.015
Φ_2	14	−0.561	0.029	7	−0.010	7	−0.010
Θ_3	21	−0.366	0.032				
Ψ_1	365	−0.644	0.023				
R^2 adjusted=0.662; $Q(20)$=18.31 (0.11)							

Note: $Q(20)$=Ljung-Box statistic for serial correlation in the residuals up to order 20; p value in parentheses.

In order to capture seasonal and cyclical components in the volatility dynamics, I implemented a seasonal-periodic GARCH model of the form

$$\sigma_t^2 = \omega + \beta_1\sigma_{t-1}^2 + \alpha_1\varepsilon_{t-1}^2 + \alpha_7\varepsilon_{t-7}^2 + \alpha_{365}\varepsilon_{t-365}^2 + \sum_{k=1}^{M}\left[\lambda_k\cos\left(\frac{2\pi kS_t}{7}\right) + \varphi_k\sin\left(\frac{2\pi kS_t}{7}\right) + \eta_k\cos\left(\frac{2\pi kD_t}{365}\right) + \nu_k\sin\left(\frac{2\pi kD_t}{365}\right) + \lambda_k'\varepsilon_{t-7}^2\cos\left(\frac{2\pi kS_t}{7}\right) + \varphi_k'\varepsilon_{t-7}^2\sin\left(\frac{2\pi kS_t}{7}\right) + \eta_k'\varepsilon_{t-365}^2\cos\left(\frac{2\pi kD_t}{365}\right) + \nu_k'\varepsilon_{t-365}^2\sin\left(\frac{2\pi kD_t}{365}\right)\right] \quad (10)$$

where S_t and D_t=repeating step functions with the days numerated from 1 to 7 within each week, and from 1 to 365 within each year, respectively. A similar approach was used by Campbell and Diebold (2005) to model conditional variance in daily average temperature data, and by Taylor (2006) to forecast electricity consumption. In the empirical study, I set $M=3$ for the Fourier series, which the SBC criterion indicates is more than sufficient to capture cyclical dynamics. I estimated the model by the method of maximum likelihood, assuming a generalized error distribution (GED) for the innovations series [see Nelson (1991)].

Combining Forecasts

I examined whether combining forecasts from the various univariate methods for different forecast origins and horizons could provide more accurate forecasts than the individual methods being combined. The forecasts can be combined by using simple and optimal weights.

I considered all possible combinations of the forecast methods Holt-Winters (HW), ARIMA (A), and GARCH (G), and I computed the simple (unweighted) average of the forecasts for 1–7 days ahead

$$F_t^S = \frac{F_t^{(HW)} + F_t^{(A)} + F_t^{(G)}}{3} \quad (11)$$

where $F_t^{(\cdot)}$=forecasted value of method (•) in time period t. This approach is simple and useful when we have no evidence about which forecasting method is more accurate. I dropped the RW (the worst method tested by the MSE statistic, as we will see in the next section) of the combination.

I considered two approaches for computing optimal weights. First, I computed the optimal combination of the forecasts using weights by the inverse of the MSE of each of the individual methods [see Makridakis and Winkler (1983)] as follows:

$$F_t^{MSE} = \frac{(M-\mathrm{MSE}^{(HW)})F_t^{(HW)} + (M-\mathrm{MSE}^{(A)})F_t^{(A)} + (M-\mathrm{MSE}^{(G)})F_t^{(G)}}{2M} \quad (12)$$

where $\mathrm{MSE}^{(\cdot)}$=forecast MSE of method (•) as defined in Eq. (1) and $M=\mathrm{MSE}^{(HW)}+\mathrm{MSE}^{(A)}+\mathrm{MSE}^{(G)}$=sum of the postsample forecast MSE of the three methods. Second, I computed optimal combination of the postsample forecasts using weights by the inverse of each of the forecast squared errors (SE) of each of the individual methods as follows:

$$F_t^{SE} = \frac{(S_t-\mathrm{SE}_t^{(HW)})F_t^{(HW)} + (S_t-\mathrm{SE}_t^{(A)})F_t^{(A)} + (S_t-\mathrm{SE}_t^{(G)})F_t^{(G)}}{2S_t} \quad (13)$$

where $\mathrm{SE}_t^{(\cdot)}=(Y_t-F_t^{(\cdot)})^2$=forecast SE of method (•) and $\mathrm{SE}_t^{(\cdot)}=\mathrm{SE}_t^{(HW)}+\mathrm{SE}_t^{(A)}+\mathrm{SE}_t^{(G)}$=sum of the postsample forecast SEs of the three methods for each time period t. If the performance of the individual methods changes during the forecasting period, then combining forecasts using inverse SE weights can result in more accurate forecasts than the method that uses inverse MSE weights.

Empirical Study

Estimation Results

The implementation of the double seasonal Holt-Winters method to the water demand series Y_t gives the values $\alpha=0.000$, $\beta=0.755$, $\gamma=0.303$, $\delta=0.294$, and $\phi=0.607$. After evaluating different ARIMA formulations, I applied the following multiplicative double seasonal ARIMA model:

$$(1-\phi_1B-\phi_2B^2-\phi_4B^4)(1-\Phi_1B^7-\Phi_2B^{14})(1-B^7)(1-B^{365})(Y_t - c) = (1-\theta_9B^9)(1-\Theta_3B^{21})(1-\Psi_1B^{365})\varepsilon_t$$

This model can be represented as ARIMA$(4,0,9)\times(2,1,3)_7\times(0,1,1)_{365}$, with $\phi_3=0$, $\theta_1=\theta_2=\ldots=\theta_8=0$, and $\Theta_1=\Theta_2=0$. The estimated results and diagnostic checks are shown in Table 1. All the parameter estimates are significant at the 5% significance level. The residual ACF and PACF exhibit no patterns up to the

Table 2. Seasonal-Periodic GARCH Model Estimates for Water Demand Series

Model: ARIMA(4, 0, 9)×(2, 1, 3)$_7$×(0, 1, 1)$_{365}$–GARCH(1, 1)×(0, 1)$_{365}$

Mean equation				Residual ACF		Residual PACF	
Parameter	Lag	Estimate	Standard error	Lag	Estimate	Lag	Estimate
c		−0.011	0.008	1	−0.007	1	0.007
ϕ_1	1	0.502	0.029	2	0.023	2	0.023
ϕ_2	2	0.137	0.030	3	−0.028	3	−0.028
ϕ_4	4	0.075	0.024	4	−0.026	4	−0.026
θ_9	9	−0.064	0.023	5	−0.042	5	−0.040
Φ_1	7	−0.747	0.023	6	0.026	6	0.027
Φ_2	14	−0.534	0.028	7	−0.006	7	−0.006
Θ_3	21	−0.346	0.031				
Ψ_1	365	−0.640	0.025				
Variance equation				**Squared residual ACF**		**Squared residual PACF**	
Parameter	Lag	Estimate	Standard error	Lag	Estimate	Lag	Estimate
ω		0.107	0.028	1	0.012	1	0.012
α_1	1	0.103	0.037	2	−0.030	2	−0.031
β_1	1	0.483	0.108	3	0.028	3	0.029
α_{365}	365	0.109	0.032	4	0.018	4	0.016
φ_1		0.026	0.011	5	0.008	5	0.009
φ_3'	365	0.062	0.035	6	−0.023	6	−0.023
GED		1.361	0.055	7	0.015	7	0.015

R^2 adjusted=0.657; $Q(20)=19.20$ (0.08); $Q^2(20)=13.61$ (0.33)

Note: $Q(20)$ [$Q^2(20)$]=Ljung-Box statistic for serial correlation in the residuals (squared residuals) up to order 20; p value in parentheses.

order of 7. The Ljung-Box statistic, $Q=18.31$, based on 20 residual autocorrelations is not significant at the conventional levels. These results suggest that the model is appropriate for modeling the water demand series.

I then fitted a significant parameter ARIMA-GARCH model of the form

$$(1-\phi_1 B-\phi_2 B^2-\phi_4 B^4)(1-\Phi_1 B^7-\Phi_2 B^{14})(1-B^7)(1-B^{365})(Y_t - c)=(1-\theta_9 B^9)(1-\Theta_3 B^{21})(1-\Psi_1 B^{365})\varepsilon_t$$

and

$$\sigma_t^2=\omega+\beta_1\sigma_{t-1}^2+\alpha_1\varepsilon_{t-1}^2+\alpha_{365}\varepsilon_{t-365}^2+\varphi_1\sin\left(\frac{2\pi D_t}{365}\right)+\varphi_3'\varepsilon_{t-365}^2\sin\left(\frac{6\pi D_t}{365}\right).$$

The model estimates and diagnostic checks are given in Table 2. The Ljung-Box test statistics show evidence of no serial correlation in the residuals (mean equation) and no serial correlation in the squared residuals (variance equation) up to order 20. Thus, I conclude that this model is also adequate for the data.

Forecast Evaluation Results

The performance of the estimated univariate methods was evaluated by computing MSE statistics for multistep forecasts from 1 to 7 days ahead. Table 3 and Fig. 3 give the forecasts results for the postsample period from June 1, 2006 to June 30, 2006. An initial interpretation of the results suggests that the ability to forecast water demand did not decrease as the forecast horizon increased, except from 1 to 2 days ahead.

The ARIMA and GARCH models appear to have the same forecast performance especially for short-term forecasts (1–2 days ahead). For 1–4 day ahead forecasts, the ARIMA and GARCH

Table 3. MSE for Multistep-Ahead Forecasts for Postsample Period

					Simple combination				Optimal combination	
Horizon	RW	HW	A	G	HW-A	HW-G	A-G	HW-A-G	MSE	SE
One-step	0.96	0.38	0.35	0.35	0.35	0.35	0.35	0.35	0.35	0.33
Two-step	1.55	0.51	0.45	0.45	0.46	0.45	0.45	0.45	0.45	0.41
Three-step	1.82	0.49	0.47	0.45	0.45	0.45	0.45	0.45	0.45	0.42
Four-step	2.09	0.48	0.45	0.46	0.46	0.46	0.46	0.46	0.46	0.44
Five-step	2.23	0.43	0.44	0.46	0.43	0.43	0.45	0.44	0.44	0.42
Six-step	1.91	0.42	0.45	0.47	0.43	0.43	0.46	0.44	0.44	0.42
Seven-step	1.33	0.40	0.44	0.46	0.41	0.42	0.45	0.43	0.42	0.41
Average	1.70	0.44	0.44	0.44	0.43	0.43	0.44	0.43	0.43	0.41

Note: RW=random walk; HW=Holt-Winters; A=ARIMA model; and G=GARCH model.

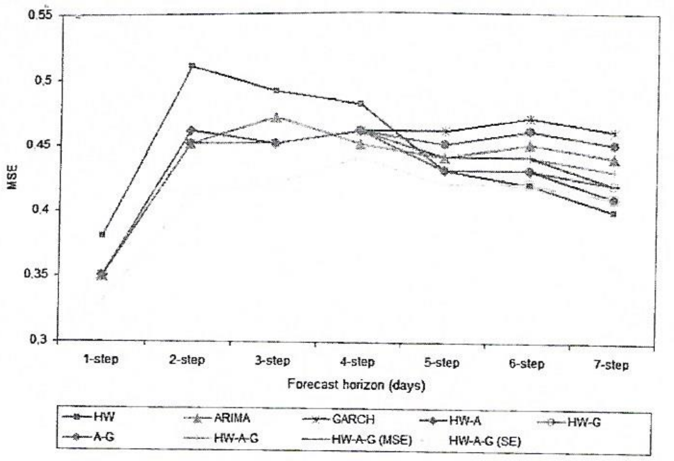

Fig. 3. Comparison of multistep ahead forecasts for postsample period

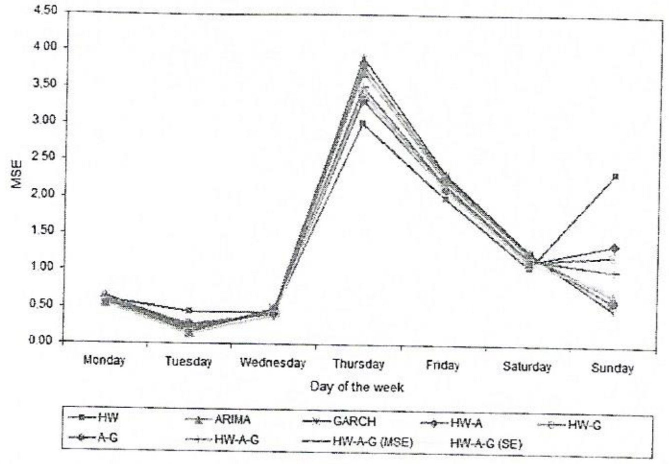

Fig. 4. Comparison of multistep ahead averaged forecasts for each day of the week

models performed better than the Holt-Winters method. In contrast, the Holt-Winters outperformed the ARIMA and GARCH models in long horizons. The RW model ranked last for any of the forecast horizons considered.

The optimal combination of Holt-Winters, ARIMA, and GARCH weighted by inverse SEs is more accurate than the various simple combinations, except for seven-step ahead forecasting in which the Holt-Winters outperformed the optimal combined forecasting. For 1 day ahead, the average MSE for the individual forecasting methods (HW, ARIMA, and GARCH) was 0.36 while it was 0.33 for the optimal combined forecasts—a error reduction of 8.33%. For 2- and 3-day ahead forecasts, combining reduced the MSE by 12.77 and 10.64%, respectively.

Table 4 and Fig. 4 give the forecast results for each of the 7

Table 4. MSE for Multistep-Ahead Forecasts for Each Day of the Week

						Simple combination				Optimal combination	
Horizon	Day	RW	HW	A	G	HW-A	HW-G	A-G	HW-A-G	MSE	SE
One-step	Mon	16.18	2.33	1.18	1.25	1.71	1.75	1.21	1.55	1.54	1.34
	Tue	0.28	0.53	0.20	0.19	0.34	0.34	0.19	0.29	0.28	0.21
	Wed	0.18	0.14	0.25	0.26	0.19	0.20	0.26	0.21	0.22	0.20
	Thu	3.15	4.19	5.26	5.40	4.71	4.78	5.33	4.93	4.94	4.84
	Fri	0.47	0.37	0.54	0.54	0.45	0.45	0.54	0.48	0.48	0.35
	Sat	3.00	0.23	0.64	0.58	0.39	0.37	0.61	0.45	0.46	0.40
	Sun	1.20	1.26	0.40	0.33	0.70	0.61	0.36	0.53	0.53	0.41
Four-step	Mon	3.86	0.42	0.43	0.54	0.42	0.48	0.48	0.46	0.46	0.44
	Tue	2.66	0.15	0.16	0.17	0.15	0.15	0.16	0.16	0.16	0.12
	Wed	8.39	0.48	0.69	0.77	0.58	0.62	0.73	0.64	0.64	0.59
	Thu	11.27	3.63	3.79	4.14	3.71	3.88	3.96	3.85	3.85	3.73
	Fri	1.83	1.78	1.88	1.94	1.83	1.86	1.91	1.87	1.87	1.84
	Sat	4.14	1.29	1.21	1.26	1.25	1.28	1.24	1.25	1.25	1.25
	Sun	10.23	3.23	1.10	0.81	2.03	1.82	0.95	1.56	1.55	1.18
Seven-step	Mon	0.30	0.19	0.24	0.38	0.21	0.28	0.30	0.26	0.26	0.25
	Tue	0.15	0.07	0.06	0.08	0.06	0.06	0.06	0.06	0.06	0.04
	Wed	1.09	0.27	0.39	0.29	0.33	0.28	0.34	0.31	0.31	0.29
	Thu	13.60	2.54	3.33	3.42	2.92	2.96	3.38	3.08	3.07	2.99
	Fri	7.91	2.14	2.25	2.38	2.19	2.26	2.32	2.26	2.25	2.17
	Sat	4.19	1.43	1.48	1.59	1.46	1.51	1.54	1.50	1.50	1.49
	Sun	0.70	1.14	0.29	0.22	0.63	0.51	0.26	0.42	0.44	0.31
Average	Mon	4.79	0.61	0.55	0.65	0.57	0.62	0.59	0.59	0.59	0.53
	Tue	4.13	0.44	0.16	0.17	0.25	0.26	0.16	0.21	0.21	0.14
	Wed	4.89	0.43	0.46	0.48	0.41	0.41	0.47	0.42	0.42	0.39
	Thu	7.52	3.01	3.71	3.91	3.33	3.43	3.80	3.51	3.51	3.41
	Fri	5.21	1.99	2.25	2.32	2.12	2.15	2.28	2.18	2.18	2.11
	Sat	4.02	1.06	1.24	1.26	1.13	1.14	1.25	1.17	1.17	1.13
	Sun	6.10	2.35	0.68	0.50	1.38	1.22	0.59	1.02	1.02	0.75

Note: RW=random walk; HW=Holt-Winters; A=ARIMA model; and G=GARCH model.

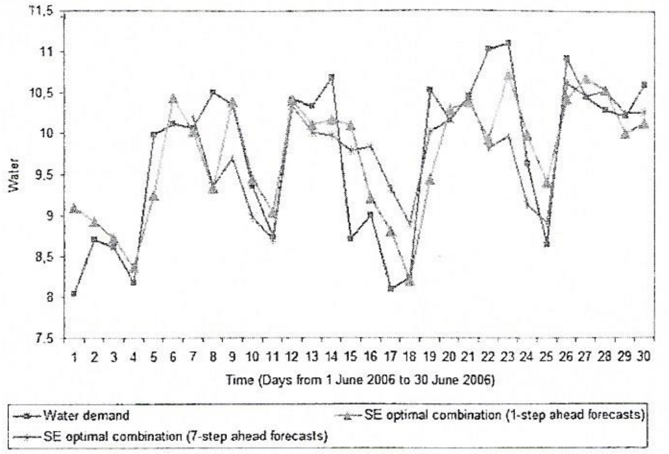

Fig. 5. One-step and seven-step ahead forecasts of water demand made using the SE optimal combination HW-ARIMA-GARCH

days of the week in the same period. The results suggest that the Thursdays exhibit irregular demand patterns in the postsample period used in this study. From the data, we found that the water consumption decreased 10.37% on the first Thursday of the postsample period (June 1, 2006), whereas it increased 4.22 and 18.44% on the following Thursdays (June 8, 2006 and June 15, 2006, respectively). Possible reasons for this unusual pattern are weather changes and any restrictions on water demand.

In terms of the day of the week effect on forecasting performance, the SE optimal combination HW-A-G appears to be most useful for Monday, Tuesday, and Wednesday forecasts—combining reduced the MSE of multistep ahead averaged forecasts by 12.1, 45.45, and 14.60%, respectively, when compared with the average of the individual methods. The Holt-Winters appears to be the most appropriate method for Thursday, Friday, and Saturday forecasts and the GARCH model appears to be the best method for Sunday forecasts. Fig. 5 shows the one-step ahead and seven-step ahead forecasts of water demand in the evaluation forecasting period June 1, 2006 to June 30, 2006, made using the SE optimal combined method.

Conclusions

In this article, I compared the forecast accuracy of individual and combined univariate time series models for multistep ahead daily water demand forecasting in Spain. I implemented double seasonal versions of the Holt-Winters, ARIMA, and GARCH models to accommodate the two seasonal effects (within-week cycle of 7 days and within-year cycle of 365 days) on the variability of the data.

The results suggest that the optimal combined forecasts can be quite useful especially for short-term forecasting. However, the forecasting performance of this approach is not consistent over the 7 days of the week. The Holt-Winters method and the GARCH model can be used independently to improve forecast accuracy of water demand on Thursdays to Saturdays and Sundays, respectively.

In future research, it would be interesting to investigate whether combining individual forecasts derived from different univariate and multivariate methods for hydrological forecasting (incorporating factors such as temperature, rainfall, land use, or others) or different data sets (training and test sets) or both can help to improve accuracy.

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