Exame Recurso de MP — 30/07/2015

[2] $\boxed{1}$

$$\underbrace{(1-B)^2}_{1-2B+B^2} Y_t = (1-\theta_1 B - \theta_2 B^2)\varepsilon_t$$

$$Y_t = 2Y_{t-1} - Y_{t-2} + \varepsilon_t - \theta_1\varepsilon_{t-1} - \theta_2\varepsilon_{t-2}$$

HOLT: $\hat{Y}_t(m) = a(t) + b(t)m$

$$a(t) = \alpha Y_t + (1-\alpha)(a(t-1)+b(t-1))$$
$$b(t) = \beta(a(t)-a(t-1)) + (1-\beta)b(t-1)$$

ARIMA(0,2,2)

$$\hat{Y}_t(1) = 2Y_t - Y_{t-1} - \theta_1\hat{\varepsilon}_t - \theta_2\hat{\varepsilon}_{t-1}$$
$$\hat{Y}_t(2) = 2\hat{Y}_t(1) - Y_t - \theta_2\hat{\varepsilon}_t$$
$$\hat{Y}_t(m) = 2\hat{Y}_t(m-1) - \hat{Y}_t(m-2)\ ,\quad m \geq 3$$

Fazendo $\theta_1 = \alpha+\beta-2$ e $\theta_2 = 1-\alpha$, sai:

[2] $\boxed{2}$ ARIMA(3,1,1): $(1-\phi_1 B - \phi_2 B^2 - \phi_3 B^3)(1-B)Y_t = (1-\theta_1 B)\varepsilon_t$

$$Y_t = (1+\phi_1)Y_{t-1} - (\phi_1-\phi_2)Y_{t-2} - (\phi_2-\phi_3)Y_{t-3} - \phi_3 Y_{t-4} + \varepsilon_t - \theta_1\varepsilon_{t-1}$$
$$\hat{Y}_t(1) = (1+\phi_1)Y_t - (\phi_1-\phi_2)Y_{t-1} - (\phi_2-\phi_3)Y_{t-2} - \phi_3 Y_{t-3} + \theta_1\hat{\varepsilon}_t$$
$$\hat{Y}_t(2) = (1+\phi_1)\hat{Y}_t(1) - (\phi_1-\phi_2)Y_t - (\phi_2-\phi_3)Y_{t-1} - \phi_3 Y_{t-2}$$
$$\hat{Y}_t(3) = (1+\phi_1)\hat{Y}_t(2) - (\phi_1-\phi_2)\hat{Y}_t(1) - (\phi_2-\phi_3)Y_t - \phi_3 Y_{t-1}$$

$\boxed{3}$ $Y_t = Y_{t-1} + \varepsilon_t - 0.1\varepsilon_{t-1}$ $\quad (1-B)Y_t = (1-\theta B)\varepsilon_t \Rightarrow \frac{(1-B)}{1-\theta B}Y_t = \varepsilon_t \Rightarrow \pi(B) = \frac{1-B}{1-\theta B}$

[1] a) ARIMA(0,1,1) Não é estacionário pois $d=1$

É invertível pois $\theta_1 = 0.1 \in [-1,1]$

[1.5] b) ~~$(1-\pi_1 B - \pi_2 B^2 - \pi_3 B^3 - \cdots) = \frac{1-0.1B}{1-B}$~~ $\Rightarrow$ ~~$(1-\pi_1 B - \pi_2 B^2 - \pi_3 B^3 - \cdots)(1-B) = 1-0.1B$~~

~~$1 - B - \pi_1 B + \pi_1 B^2 - \pi_2 B^2 + \pi_2 B^3 - \pi_3 B^3 \cdots = 1-0.1B$~~

~~$1 - (1+\pi_1)B + (\pi_1-\pi_2)B^2 + (\pi_2-\pi_3)B^3 \cdots = 1-0.1B$~~

~~$1+\pi_1 = 0.1$~~

~~$\pi_1 =$~~

$\boxed{\pi_j = (1-\theta)\theta^{j-1}}$, logo $\pi_1 = 1-0.1 = 0.9$; $\pi_2 = (1-0.1)0.1 = 0.09$;

$\pi_3 = (1-0.1)0.1^2 = 0.009$

[1.5] c) $\boxed{\psi_j = (1+\phi)\psi_{j-1} - \phi\psi_{j-2},\ k \geq 2}$ $\quad \psi_0 = 1$; $\psi_1 = 1-0.1 = 0.9$; $\psi_2 = 1-0.1 = 0.9$; $\psi_3 = 1-0.1 = 0.9$

NOTA:

$$\psi(B) = \frac{1-\theta B}{1-B} \Rightarrow (1+\psi_1 B + \psi_2 B^2 + \psi_3 B^3 + \cdots)(1-B) = 1-\theta B$$
$$1 - B + \psi_1 B - \psi_1 B^2 + \psi_2 B^2 - \psi_2 B^3 + \psi_3 B^3 + \cdots = 1-\theta B$$
$$1 - (1-\psi_1)B + (\psi_2-\psi_1)B^2 + (\psi_3-\psi_2)B^3 + \cdots = 1-\theta B$$

$\Leftrightarrow 1-\psi_1 = \theta \Leftrightarrow \psi_1 = 1-\theta = 1-0.1 = 0.9$

$\psi_2 - \psi_1 = 0 \Leftrightarrow \psi_2 = \psi_1 = 0.9$

$\psi_3 - \psi_2 = 0 \Leftrightarrow \psi_3 = \psi_2 = 0.9$ (...)

NOTA:

$$\pi(B) = \frac{1-B}{1-\theta B} \Rightarrow 1 - (0.1+\pi_1)B + (0.1\pi_1 - \pi_2)B^2 + (0.1\pi_2 - \pi_3)B^3 \cdots$$
$$= 1 - B \Leftrightarrow 1 = 1 \wedge 0.1+\pi_1 = 1 \wedge 0.1\pi_1 - \pi_2 = 0 \wedge \ldots$$

4 $E(Y_t)=0$

[2] a) $\text{cov}(Y_t, Y_{t-k}) = \text{cov}(\varepsilon_t + \theta\varepsilon_{t-1} + \theta^2\varepsilon_{t-2} + \cdots + \theta^q\varepsilon_{t-q}, \varepsilon_{t-k} + \theta\varepsilon_{t-k-1}$

$+ \theta^2\varepsilon_{t-k-2} + \cdots + \theta^q\varepsilon_{t-k-q})$

$= \text{cov}(\varepsilon_t + \cdots + \theta^k\varepsilon_{t-k} + \theta^{k+1}\varepsilon_{t-k-1} + \cdots + \theta^q\varepsilon_{t-q}, \ldots)$

$= [\theta^k + \theta^{k+2} + \phi^{k+4} + \cdots + \phi^{k+2(q-k)}]\sigma_\varepsilon^2$

$= (1 + \theta^2 + \theta^4 + \cdots + \theta^{2(q-k)})\sigma_\varepsilon^2\theta^k$

[2] b) $\text{Var}(Y_t) = \text{Var}(\varepsilon_t + \theta\varepsilon_{t-1} + \theta^2\varepsilon_{t-2} + \cdots + \theta^q\varepsilon_{t-q})$

$= (1 + \theta^2 + \theta^4 + \cdots + \theta^{2q})\sigma_\varepsilon^2$

$$\text{CORR}(Y_t, Y_{t-k}) = \frac{(1+\theta^2+\theta^4+\cdots+\theta^{2(q-k)})\theta^k}{1+\theta^2+\theta^4+\cdots+\theta^{2k}}$$

5 $Y_t = 5 + 1.1Y_{t-1} - 0.5Y_{t-2} + \varepsilon_t$ com $\sigma_\varepsilon^2 = 2$ AR(2)

[2] a)

$\hat{Y}_{2007}(1) = 5 + 1.1Y_{2007} - 0.5Y_{2006} = 5 + 1.1(10) - 0.5(11) = 10.5$

$\hat{Y}_{2007}(2) = 5 + 1.1\hat{Y}_{2008} - 0.5Y_{2007} = 5 + 1.1(10.5) - 0.5(10) = 11.55$

[2] b) AR(2): $\psi_0 = 1$; $\psi_1 = \phi_1\psi_0 = \phi_1$, $\psi_j = \phi_1\psi_{j-1} + \phi_2\psi_{j-2}$, $j \geq 2$

logo $\psi_1 = 1.1(1) = 1.1$

[2] c) $\text{Var}(e_t(m)) = \sigma_\varepsilon^2 \sum_{j=0}^{m-1} \psi_j^2$ $\hat{Y}_{2007}(1) \pm 2\sqrt{\sigma_\varepsilon^2}$

$\text{Var}(e_t(1)) = \sigma_\varepsilon^2$ $10.5 \pm 2\sqrt{2} = [7.67, 13.33]$

[2] d) $\hat{Y}_{t+1}(m) = \hat{Y}_t(m+1) + \psi_j(Y_{t+1} - \hat{Y}_t(1))$

$\hat{Y}_{2008}(1) = \hat{Y}_{2007}(2) + \psi_1(Y_{2008} - \hat{Y}_{2007}(1)) = 11.55 + 1.1(12 - 10.5) =$

$= 13.2$

$\hat{Y}_{2008}(1) = \cancel{5} + 1.1Y_{2008} - 0.5Y_{2007} + \varepsilon \ldots$