# Lecture 15

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Economics II

LISBOA SCHOOL OF ECONOMICS & MANAGEMENT

ISEG

SINCE 1911

# Lecture 15

## Summary:

8.2. Keynesian Model with Government

## Bibliography:

Frank e Bernanke (2011), chapter. 11

Amaral et al. (2007), cap. 5

## Lecture Goals:

After the lecture the student must :

- To understand and use the Keynesian model with government (State).
- To understand how fiscal policy can affect the equilibrium level of the output in the short term.
- To understand the Haavelmo theorem.

# 8.2. The Keynesian model with Government

Remember the model without Government (last lecture):

$$
\begin{cases}
D = C + I \\
C = \bar{C} + c.Y_d \\
Y_d = Y \\
I = \bar{I} \\
Y = D
\end{cases}
$$

## Changes to the equations of the model:

(1) $$D = C + I \boxed{+ G}$$

- The state also has purchasing intentions (expense) in final goods and services, at constant base year prices, for public consumption.

(3) $$Y_d = Y \boxed{- T} \boxed{+ TR}$$

- Direct taxes ($T$) reduce the disposable income of the households.
- The State transfers to households (*TR*) increase the disposable income of the households.

(6) $$I = \boxed{I^{\text{Priv}}} + \boxed{I^{\text{Publ}}}$$ NEW

- It represents the spending intentions for investment at constant base year prices:
  - The private agents (households and families) have intentions to invest – private investment ($I^{\text{Priv}}$).
  - The State also has intentions to invest - public investment ($I^{\text{Publ}}$).
- Is a definition equation.
- Thus, the equation (4) is modified to represent only the private investment intentions:

(4) $$I^{\text{Priv}} = \overline{I^{\text{Priv}}}$$

(7) $$G = \bar{G}$$ NEW

- It represents the government expenditure <u>intentions</u> in consumer goods at constant base year prices.
- It is a <u>behavioral</u> equation.
- Do not depend on other variables in the model, so those intentions are explained by exogenous factors which area exogenous to the model.

(8) $$TR = \overline{TR}$$ NEW

- It represents the State's spending <u>intentions</u> on transfers to households at constant base year prices.
- It is a <u>behavioral</u> equation.
- It is also an exogenous variable.

(9) $$I^{\text{Publ}} = \overline{I^{\text{Publ}}}$$ NEW

- It represents the state's spending intentions on capital goods at constant base year prices.
- It is a behavioral equation.
- Do not depend on other variables in the model, so they are explained by exogenous factors to the model.

(10) $$T = \bar{T} + t.Y$$ NEW

- It represents the intentions of state tax revenue, at constant base year prices.
- It is a behavioral equation.

The following exogenous variables are controlled by the state:

- public consumption ($G$);
- public investment ($I^{\text{Publ}}$);
- transfers to households ($TR$);
- autonomous taxes ($\overline{T}$);
- marginal tax rate ($t$).

Thus, these five variables can be used as <u>instruments of economic policy.</u>

In this case (Keynesian Model with Government), the model in its <u>structural formula</u> is given by:

$$
\begin{cases}
D = C + I + G \\
C = \bar{C} + c.Y_d \\
Y_d = Y - T + TR \\
I^{\text{Priv}} = \overline{I^{\text{Priv}}} \\
Y = D \\
I = I^{\text{Priv}} + I^{\text{Publ}} \\
G = \bar{G} \\
TR = \overline{TR} \\
I^{\text{Publ}} = \overline{I^{\text{Publ}}} \\
T = \bar{T} + t.Y
\end{cases}
$$

Solving by substitution:

(1)+... $D = C + I + G \Leftrightarrow$

(2)+... $\Leftrightarrow D = \left(\bar{C} + c.Y_d\right) + I + G \Leftrightarrow$

(3)+... $\Leftrightarrow D = \left[\bar{C} + c.\left(Y - T + TR\right)\right] + I + G \Leftrightarrow$

(6)+... $\Leftrightarrow D = \left[\bar{C} + c.\left(Y - T + TR\right)\right] + \left(I^{\text{Priv}} + I^{\text{Publ}}\right) + G \Leftrightarrow$

(4)+... $\Leftrightarrow D = \left[\bar{C} + c.\left(Y - T + TR\right)\right] + \left(\overline{I^{\text{Priv}}} + I^{\text{Publ}}\right) + G \Leftrightarrow$

(7)+... $\Leftrightarrow D = \left[\bar{C} + c.\left(Y - T + TR\right)\right] + \left(\overline{I^{\text{Priv}}} + I^{\text{Publ}}\right) + \bar{G} \Leftrightarrow$

$$(8)+\ldots \Leftrightarrow D=\left[\bar{C}+c.\left(Y-T+\overline{TR}\right)\right]+\left(\overline{I^{\mathrm{Priv}}+I^{\mathrm{Publ}}}\right)+\bar{G}\Leftrightarrow$$

$$(9)+\ldots \Leftrightarrow D=\left[\bar{C}+c.\left(Y-T+\overline{TR}\right)\right]+\left(\overline{I^{\mathrm{Priv}}}+\overline{I^{\mathrm{Publ}}}\right)+\bar{G}\Leftrightarrow$$

$$(10)+\ldots \Leftrightarrow D=\left\{\bar{C}+c.\left[Y-\left(\bar{T}+t.Y\right)+\overline{TR}\right]\right\}+$$

$$+\left(\overline{I^{\mathrm{Priv}}}+\overline{I^{\mathrm{Publ}}}\right)+\bar{G}\Leftrightarrow$$

$$(5)+\ldots \Leftrightarrow Y=\bar{C}+c.Y-c.t.Y-c.\bar{T}+c.\overline{TR}+$$

$$+\overline{I^{\mathrm{Priv}}}+\overline{I^{\mathrm{Publ}}}+\bar{G}\Leftrightarrow$$

$$\Leftrightarrow Y - c.Y + c.t.Y = \bar{C} + \overline{I^{Priv}} + \overline{I^{Publ}} + \bar{G} + c.\left(\overline{TR} - \bar{T}\right) \Leftrightarrow$$

$$\Leftrightarrow \left[1 - c.\left(1 - t\right)\right].Y = \bar{C} + \overline{I^{Priv}} + \overline{I^{Publ}} + \bar{G} + c.\left(\overline{TR} - \bar{T}\right) \Leftrightarrow$$

$$\Leftrightarrow \boxed{Y = \frac{\bar{C} + \overline{I^{Priv}} + \overline{I^{Publ}} + \bar{G} + c.\left(\overline{TR} - \bar{T}\right)}{1 - c.\left(1 - t\right)}}$$

Reduced form to equilibrium product.

The effect of a small change in public consumption on the product of equilibrium is given by

$$\frac{\partial Y}{\partial \overline{G}} = \frac{1}{1 - c.(1 - t)} > 1$$

- because $0 < c.(1 - t) < 1$.

The impact of an increase in public consumption on GDP of equilibrium is higher than the increase in public consumption.

- There is a multiplier effect for public consumption.
- The economic policy authorities, under certain circumstances, may increase the level of economic activity by increasing public spending.

In what circumstances is this increase in public spending be justified?

- When there is an excess of production capacity.
- When taxes or public debt can raise.

## Some interesting properties:

1. The multiplier of public consumption is equal to the public investment multiplier, the private investment multiplier or the autonomous consumption multiplier:

$$\frac{\partial Y}{\partial \overline{G}} = \frac{\partial Y}{\partial \overline{I^{\text{Publ}}}} = \frac{\partial Y}{\partial \overline{I^{\text{Priv}}}} = \frac{\partial Y}{\partial \overline{C}} = \frac{1}{1-c.(1-t)}$$

2. These multipliers

- ➢ Increase with the marginal propensity to consume
- ➢ Decrease with the marginal tax rate

3. The multiplier of the transfers is lower than the multiplier of the public consumption:

$$\frac{\partial Y}{\partial \overline{\overline{TR}}} = \frac{c}{1-c.(1-t)} < \frac{1}{1-c.(1-t)}$$

4. The multiplier of autonomous taxes is negative and equal to the symmetrical of the multiplier of transfers :

$$\frac{\partial Y}{\partial \overline{\overline{T}}} = -\frac{\partial Y}{\partial \overline{\overline{TR}}} = -\frac{c}{1-c.(1-t)}$$

- This is due to the fact that the tax are "negative transfer"

But a change in fiscal policy instruments causes changes in the budget balance.

- These changes have consequences on the stock of public debt.
- The budget balance is an endogenous variable because it depends on:
  - the fiscal policy variables ;
  - the output of equilibrium.

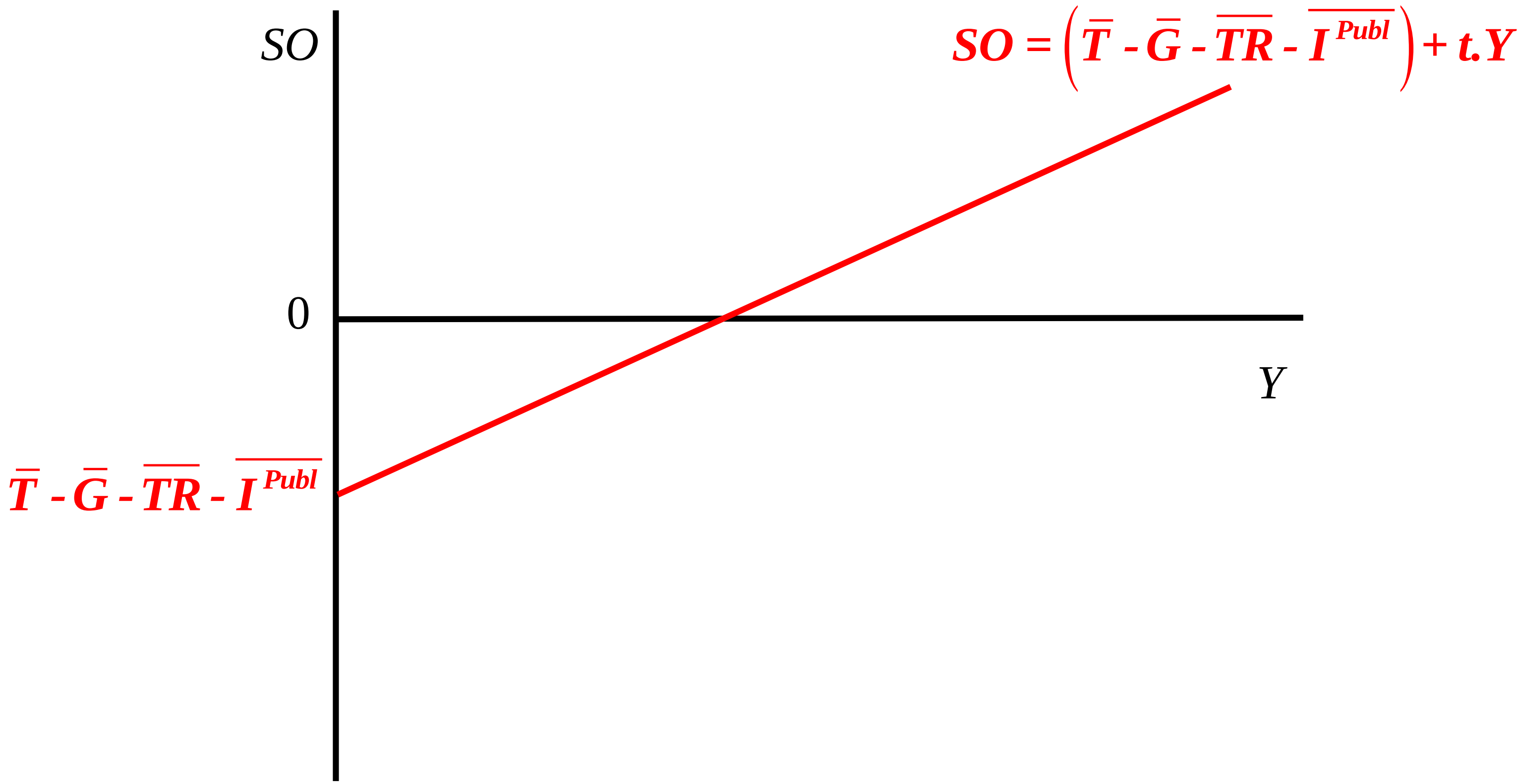
*SO* = BUDGET BALANCE (=*BB*)

$$SO = T - \left( G + TR + I^{\mathrm{Publ}} \right)$$

$$SO = \left( \bar{T} + t.Y \right) - \left( \bar{G} + \overline{TR} + \overline{I^{\mathrm{Publ}}} \right)$$

# Graphic representation of the budget balance:

*SO* = BUDGET BALANCE (=*BB*)



## Zero budget balance - Haavelmo theorem:

- Suppose that if you want a budget always balanced.
  - Public debt remains constant.
  - This is the long term goal of Euro Zone countries.
- In our model, we can write:

*SO* = BUDGET BALANCE (=*BB*)

NEW equation!

$$\boxed{SO = 0} \Leftrightarrow T = G + TR + I^{\text{Publ}}$$

- Consequently, households' disposable income is given by:

$$Y_d = Y - T + TR =$$

$$= Y - \left(G + TR + I^{\text{Publ}}\right) + TR =$$

$$Y_d = Y - G - I^{\text{Publ}}$$

- As a result, private consumption intentions are given by:

$$C = \bar{C} + c.Y_d = \bar{C} + c.\left(Y - G - I^{\text{Publ}}\right)$$

- Thus, the product of equilibrium is given by:

$$Y = C + I + G \Leftrightarrow$$

$$\Leftrightarrow Y = \bar{C} + c.\left(Y - G - I^{\text{Publ}}\right) + \left(I^{\text{Publ}} + I^{\text{Priv}}\right) + G \Leftrightarrow$$

$$\Leftrightarrow (1-c).Y = \bar{C} + \overline{I^{\text{Priv}}} + (1-c).\left(\bar{G} + \overline{I^{\text{Publ}}}\right) \Leftrightarrow$$

$$\Leftrightarrow \boxed{Y = \frac{\bar{C} + \overline{I^{\text{Priv}}}}{1-c} + \left(\bar{G} + \overline{I^{\text{Publ}}}\right)}$$

Reduced form for equilibrium output with balanced budget.

## In the model with a balanced budget:

- The multiplier of public consumption is equal to 1.
- The multiplier od the public investment is also equal to 1.
- The product increases exactly by the same amount as consumption or public investment.

$$\frac{\partial Y}{\partial \bar{G}} = \frac{\partial Y}{\partial \overline{I^{\text{Publ}}}} = 1$$

## Why?

- If the public consumption or the public investment increase, ...
- ... to have balanced (in equilibrium) budget balance which remains balanced ($\Delta SO = 0$), consequently:

*SO* = BUDGET BALANCE (=*BB*)

$$\Delta SO = 0 \Leftrightarrow \Delta T - \Delta TR = \Delta G + \Delta I^{\text{Publ}}$$

- The change in disposable income is given by :

$$\Delta Y_d = \Delta Y - \left(\Delta T - \Delta TR\right) = \Delta Y - \left(\Delta G + \Delta I^{\text{Publ}}\right)$$

- On the other hand, the change in the balance of product (if the autonomous private consumption and the autonomous private investment remain constant):

$$\Delta Y = c.\Delta Y_d + \left(\Delta G + \Delta I^{\text{Publ}}\right)$$

- So that the budget remains balanced (it mean in equilibrium), an increase in public consumption or public investment will have to be offset by:
  - an equal increase in taxes or...
  - ... an equal decrease in transfers.
- In this way the disposable income of households is unchanged:

$$\Delta Y_d = \left[ c.\Delta Y_d + \left( \Delta G + \Delta I^{\text{Publ}} \right) \right] - \left( \Delta G + \Delta I^{\text{Publ}} \right) \Leftrightarrow$$

$$\Leftrightarrow \Delta Y_d = c.\Delta Y_d \Leftrightarrow$$

$$\Leftrightarrow \boxed{\Delta Y_d = 0} \Rightarrow \boxed{\Delta Y = \Delta G + \Delta I^{\text{Publ}}}$$