LISBOA SCHOOL OF ECONOMICS & MANAGEMENT

# Financial Econometrics 2019

## Practice Exam — Just for practice: not an example nor a model!

NAME: _____ ID: _____

- Answer the questions in the space provided supplying enough information to prove you performed the necessary computations.
- Throughout this test, both $\epsilon_t$ and $a_t$ represent a white noise process.

**1**. Classify the following sentences as true or false.

**i**. The process $Z_t$, defined by $Z_t = 0.5Z_{t-1} + a_t - a_{t-2}$, has an AR unit root.
☐ True ☐ False

**ii**. An autoregressive process is never stationary but can be non-invertible.
☐ True ☐ False

**iii**. An ARMA model may be able to describe adequately the dynamics of a given time series with fewer parameters than an AR model.
☐ True ☐ False

**iv**. The assumption that financial markets' returns are normally distributed is strongly sustained by empirical evidence.
☐ True ☐ False

**v**. A Jarque-Bera test should be used for deciding about the existence of a trend in a time series.
☐ True ☐ False

**2**. Consider the following process $X_t = 2 + 0.5X_{t-1} - 0.3X_{t-2} + 0.3\epsilon_{t-1} + \epsilon_t$ where $\epsilon_t \sim N(0,1)$.

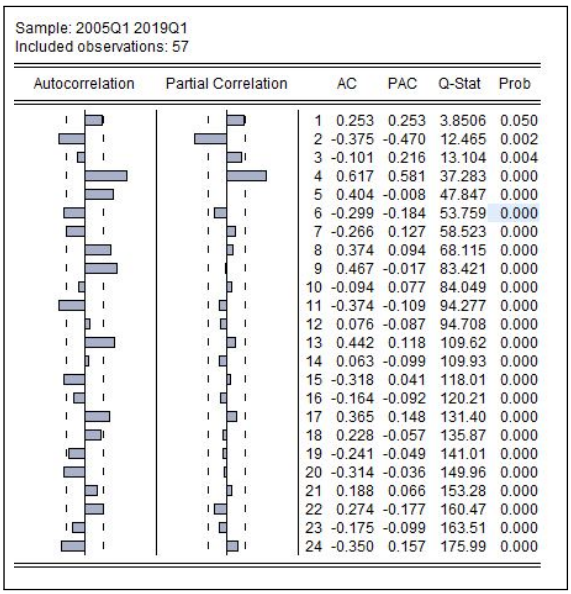
(a) Is the process stationary? Justify your answer.

(b) Is the process invertible? Justify your answer.

(c) Compute the unconditional expected value of the process.

**3**. Without using the operators $B$ nor $\nabla$, write in equation form the process $Y_t$ that follows an $ARIMA(1,1,1)(1,0,0)_4$ in terms of the observations $Y_t$, $Y_{t-1}\ldots$, the noise $a_t, a_{t-1}\ldots$, and the corresponding parameters $\phi_i, \Phi_i, \Theta_i, \theta_i$, as applicable.

**4**. The next figure presents SACF/SPACF of a quarterly time series. Which time series model do you think could be appropriate for this time series? Justify your option.


Sample: 2005Q1 2019Q1
Included observations: 57

| | Autocorrelation | Partial Correlation | | AC | PAC | Q-Stat | Prob |
|---|---|---|---|---|---|---|---|
| 1 | | | 1 | 0.253 | 0.253 | 3.8506 | 0.050 |
| 2 | | | 2 | -0.375 | -0.470 | 12.465 | 0.002 |
| 3 | | | 3 | -0.101 | 0.216 | 13.104 | 0.004 |
| 4 | | | 4 | 0.617 | 0.581 | 37.283 | 0.000 |
| 5 | | | 5 | 0.404 | -0.008 | 47.847 | 0.000 |
| 6 | | | 6 | -0.299 | -0.184 | 53.759 | 0.000 |
| 7 | | | 7 | -0.266 | 0.127 | 58.523 | 0.000 |
| 8 | | | 8 | 0.374 | 0.094 | 68.115 | 0.000 |
| 9 | | | 9 | 0.467 | -0.017 | 83.421 | 0.000 |
| 10 | | | 10 | -0.094 | 0.077 | 84.049 | 0.000 |
| 11 | | | 11 | -0.374 | -0.109 | 94.277 | 0.000 |
| 12 | | | 12 | 0.076 | -0.087 | 94.708 | 0.000 |
| 13 | | | 13 | 0.442 | 0.118 | 109.62 | 0.000 |
| 14 | | | 14 | 0.063 | -0.099 | 109.93 | 0.000 |
| 15 | | | 15 | -0.318 | 0.041 | 118.01 | 0.000 |
| 16 | | | 16 | -0.164 | -0.092 | 120.21 | 0.000 |
| 17 | | | 17 | 0.365 | 0.148 | 131.40 | 0.000 |
| 18 | | | 18 | 0.228 | -0.057 | 135.87 | 0.000 |
| 19 | | | 19 | -0.241 | -0.049 | 141.01 | 0.000 |
| 20 | | | 20 | -0.314 | -0.036 | 149.96 | 0.000 |
| 21 | | | 21 | 0.188 | 0.066 | 153.28 | 0.000 |
| 22 | | | 22 | 0.274 | -0.177 | 160.47 | 0.000 |
| 23 | | | 23 | -0.175 | -0.099 | 163.51 | 0.000 |
| 24 | | | 24 | -0.350 | 0.157 | 175.99 | 0.000 |

Figure 1: Correlogram of $y_t$

**5**. Is a white noise process stationary to the second order (i.e., «wide sense stationary» or «weakly stationary» or «covariance stationary»)?

**6**. Consider the process $y_t = 0.5y_{t-2} + \epsilon_t + 0.1\epsilon_{t-1}$

(a) Write the Wold representation form of $y_t$.

(b) What is the numerical value of $\gamma_0 = \text{Var}(Y_t)$?

(c) Compute the numerical value $\rho_3$ of the $y_t$ process.

**7**. Consider the process $y_t = \mu + y_{t-1} + x_t$, in which $x_t \sim \text{iid}(0, \sigma^2)$. Show that the series $\nabla y_t$ is stationary and compute its mean and autocovariance function.

**8**. Briefly distinguish *ex-ante* from *ex-post* forecasting and explain the goal of the latter.

**9**. What is the goal of measures such as the AIC, AICc, BIC , and SIC?

**10**. We have observed 110 anual values of the CPI, $z_t$. Complete the spaces in order to explain the rational behind the following steps.

(a) Firstly, we took logs and obtaines $w_t = \log(z_t)$, in order to obtain a series which is _____ in _____.

(b) Then, we took the first differences $x_t = (1-B)w_t$ trying to obtain a _____ process.

(c) Then, we subtracted the sample mean $\bar{x} = 0.7$ to these $x_t$, in order to obtain a process with _____.

(d) Finally, we adjusted an AR(2) to the series $(x_t - 0.7)$, which is equivalent to assume that $w_t$ is an ARIMA(_,_,_).

(e) In order to verify whether the residuals are _____, we performed the Ljung-Box test.

(f) We have obtained $Q = n(n+2)\sum_{k=1}^{15}(109-k)^{-1}\hat{\rho}_k^2 = 11.243 < 23.362 = \chi^2_{0.05}(13)$, where $n =$ _____.

(g) This means we haven't rejected $H_0 : \rho_1 = \rho_2 = \ldots = \rho_{15} =$ ___, for the residuals.

(h) That means we concluded that the model AR(2) is _____ for the data $(x_t - 0.7)$.

**11**. The next figures illustrate the time series plot, SACF/SPACF and part of the EViews output from statistical tests applied to the series Yields_10y.

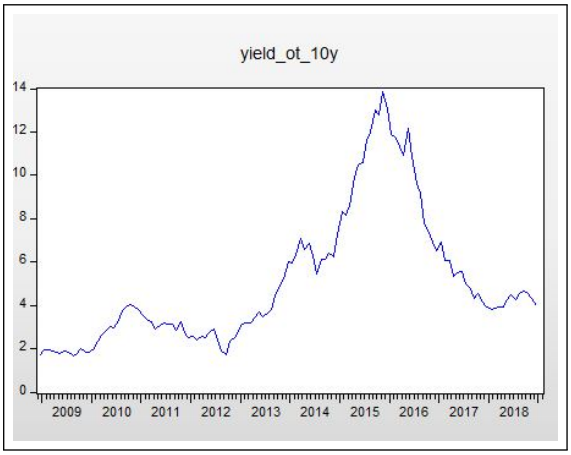



Figure 2: Yields_10y plot

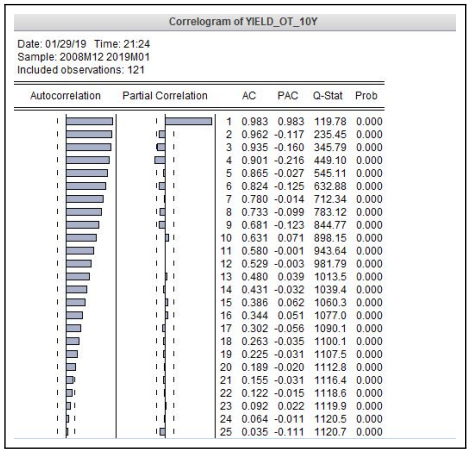



Figure 3: Correlogram

Null Hypothesis: YIELDS_OT_10Y has a unit root
Exogenous: Constant
Lag Length: 3 (Automatic - based on SIC, maxlag=12)

| | | t-Statistic | Prob.* |
|---|---|---|---|
| Augmented Dickey-Fuller test statistic | | -1.794055 | 0.3819 |
| Test critical values: | 1% level | -3.487046 | |
| | 5% level | -2.886290 | |
| | 10% level | -2.580046 | |

*MacKinnon (1996) one-sided p-values.

Augmented Dickey-Fuller Test Equation
Dependent Variable: D(YIELDS_OT_10Y)
Method: Least Squares
Date: 01/29/19 Time: 21:39
Sample (adjusted): 5 121
Included observations: 117 after adjustments

| Variable | Coefficient | Std. Error | t-Statistic | Prob. |
|---|---|---|---|---|
| YIELDS_OT_10Y(-1) | -0.024369 | 0.013583 | -1.794055 | 0.0755 |
| D(YIELDS_OT_10Y(-1)) | 0.097548 | 0.089977 | 1.084143 | 0.2806 |
| D(YIELDS_OT_10Y(-2)) | 0.184164 | 0.089141 | 2.065990 | 0.0411 |
| D(YIELDS_OT_10Y(-3)) | 0.279110 | 0.090435 | 3.086316 | 0.0026 |
| C | 0.131151 | 0.081490 | 1.609406 | 0.1103 |

| | | | |
|---|---|---|---|
| R-squared | 0.168032 | Mean dependent var | 0.018120 |
| Adjusted R-squared | 0.138319 | S.D. dependent var | 0.485999 |
| S.E. of regression | 0.451137 | Akaike info criterion | 1.287704 |
| Sum squared resid | 22.79475 | Schwarz criterion | 1.405745 |
| Log likelihood | -70.33066 | Hannan-Quinn criter. | 1.335627 |
| F-statistic | 5.655130 | Durbin-Watson stat | 1.976090 |
| Prob(F-statistic) | 0.000349 | | |

Figure 4: Yields_10y unit root test

(a) Just by inspection of Figure 2, do you think a random walk with a constant drift could be appropriate for the series? Why or why not?

(b) On Figure 3, is the statistic $Q_{24}$ indicating significance of the autocorrelations? Which one(s)? Answer it in one line.

(c) Do you think that Yields_OT_10y is stationary? Justify your answer using the information provided in Figures 2, 3 and 4. In case you make use of a statistical testing procedure formalize the test: write the estimated equation, null and alternative hypotheses, test statistics, significance level, critical value/region, result and conclusion of the test.

**12**. The following figures present the outputs of an estimated model.

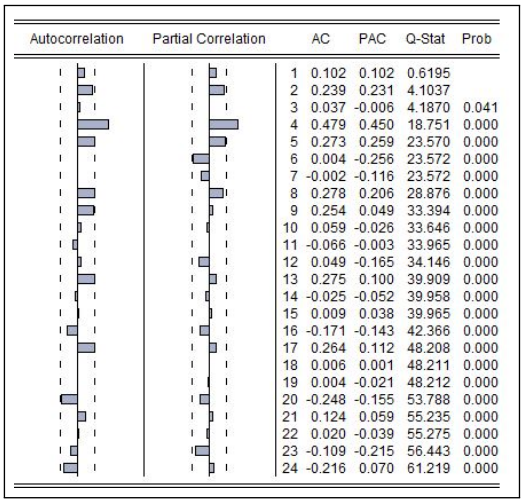
Dependent Variable: Y
Method: ARMA Maximum Likelihood (OPG - BHHH)
Sample: 2005Q1 2019Q1
Included observations: 57
Convergence achieved after 9 iterations
Coefficient covariance computed using outer product of gradients

| Variable | Coefficient | Std. Error | t-Statistic | Prob. |
|---|---|---|---|---|
| C | -0.003392 | 0.140974 | -0.024058 | 0.9809 |
| AR(1) | 0.391105 | 0.110596 | 3.536337 | 0.0009 |
| AR(2) | -0.492100 | 0.126729 | -3.883084 | 0.0003 |

| | | | |
|---|---|---|---|
| R-squared | 0.292294 | Mean dependent var | 0.005276 |
| Adjusted R-squared | 0.252235 | S.D. dependent var | 1.372350 |
| S.E. of regression | 1.186718 | Akaike info criterion | 3.258830 |
| Sum squared resid | 74.63990 | Schwarz criterion | 3.402202 |
| Log likelihood | -88.87666 | Hannan-Quinn criter. | 3.314549 |
| F-statistic | 7.296604 | Durbin-Watson stat | 1.768559 |
| Prob(F-statistic) | 0.000348 | | |

| | | |
|---|---|---|
| Inverted AR Roots | .20-.67i | .20+.67i |

Figure 5: Estimated model




| | AC | PAC | Q-Stat | Prob |
|---|---|---|---|---|
| 1 | 0.102 | 0.102 | 0.6195 | |
| 2 | 0.239 | 0.231 | 4.1037 | |
| 3 | 0.037 | -0.006 | 4.1870 | 0.041 |
| 4 | 0.479 | 0.450 | 18.751 | 0.000 |
| 5 | 0.273 | 0.259 | 23.570 | 0.000 |
| 6 | 0.004 | -0.256 | 23.572 | 0.000 |
| 7 | -0.002 | -0.116 | 23.572 | 0.000 |
| 8 | 0.278 | 0.206 | 28.876 | 0.000 |
| 9 | 0.254 | 0.049 | 33.394 | 0.000 |
| 10 | 0.059 | -0.026 | 33.646 | 0.000 |
| 11 | -0.066 | -0.003 | 33.965 | 0.000 |
| 12 | 0.049 | -0.165 | 34.146 | 0.000 |
| 13 | 0.275 | 0.100 | 39.909 | 0.000 |
| 14 | -0.025 | -0.052 | 39.958 | 0.000 |
| 15 | 0.009 | 0.038 | 39.965 | 0.000 |
| 16 | -0.171 | -0.143 | 42.366 | 0.000 |
| 17 | 0.264 | 0.112 | 48.208 | 0.000 |
| 18 | 0.006 | 0.001 | 48.211 | 0.000 |
| 19 | 0.004 | -0.021 | 48.212 | 0.000 |
| 20 | -0.248 | -0.155 | 53.788 | 0.000 |
| 21 | 0.124 | 0.059 | 55.235 | 0.000 |
| 22 | 0.020 | -0.039 | 55.275 | 0.000 |
| 23 | -0.109 | -0.215 | 56.443 | 0.000 |
| 24 | -0.216 | 0.070 | 61.219 | 0.000 |

Figure 6: Residuals correlogram

(a) Write explicitly the standard mathematical form of the estimated equation with parameter estimates. Use approximations to three decimal places.

(b) With the available information, do you think that this model is reasonably well specified? Justify your answer.

(c) Given that the last three observations $z_T = 0.1$ , $z_{T-1} = -0.5$ and $z_{T-2} = 0.5$ obtain the one step-ahead forecast $\hat{z}_{T+1|T}$. Use approximations to three decimal places.

**13**. Explain what is the volatility clustering effect present on financial time series.