Advanced Macroeconomics

PhD in Economics

Lisbon School of Economics and Management (ISEG)

Main objectives

The aims of the course are:

- learn some standard models of modern macroeconomics
- obtain a toolbox that enables you to think about macroeconomics in a systematic manner
- acquire research skills
- get used to the idea that models are wrong but extremely helpful - models are vast simplifications of reality since there is no way we can capture absolutely everything

Examples of macro questions

- What determines the level of income in the economy?
- What determines the level of unemployment?
- How does economic policy impact on the economy?
- Optimal policy responses
- What determines long run growth rates?
- What leads to fluctuations in the economy over time?
- What typically happens during booms and recessions?
- Etc.

Notes on models

- Benefit of economic models: clearly state assumptions, follow model to logical conclusion.
- A good economic model abstracts from the unimportant to focus on a particular mechanism.
- However, models are (at best) a stylized notion of reality.
- In particular, the exogenous parameters of one model may very well be the endogenous outcomes of another.
- Economics in general requires careful thought about:
  - How the model assumptions affect the model outcomes ("thinking within the model").
  - What other aspects are missing from the model that may affect the outcomes in reality ("thinking outside the model").

Notes on models

- What does this mean? Roughly speaking, there are a lot of moving pieces.
- To avoid confusion, it is very important to be organized while modeling.
- Three parts of any model:
  - Exogenous model parameters.
  - Endogenous model outcomes.
  - An equilibrium concept.

Notes on models

- The exogenous model parameters are the elements of the model that are taken “as given.”
- You as the economist choose their values.
- They do not respond to other elements of the model.
- But these are the things that you as an economist can change to see how the outcome of the model can change.

Notes on models

- The endogenous model outcomes are the elements of the model that "move around."
- You as the economist do not get to choose their values.
- Instead, they respond to the other elements of the model (including both the exogenous model parameters and other endogenous outcomes).
- These are the parts of the model that you are interested in watching change when you change the exogenous model parameters.

Notes on models

- An equilibrium concept is the set of rules that tells you what the endogenous model outcomes should be for a given set of exogenous model parameters.

- An equilibrium may be one or more conditions.

- Every model should have the following phrase:

  "Given a [insert set of exogenous model parameters here], equilibrium is defined by the [insert endogenous model outcomes here] such that [list equilibrium conditions here]."

Example: supply & demand

- What are the three parts of that model?

- Exogenous model parameters:
  - Supply curve and demand curve.
- Endogenous model outcomes:
  - Price and quantity.
- Equilibrium concept:
  - "For a given supply curve and demand curve, equilibrium is defined by a price and quantity such that demand equals supply."

The Dynamic General Equilibrium Approach to Macroeconomics

- Economic agents are continuously optimizing/re-optimizing subject to their constraints and subject to their information set up. They optimize not only over their current choice variables but also the choices that would be realized in future.
- All agents have rational expectations: thus their ex ante optimal future choices would ex post turn out to be less than optimal if and only if their information set was incomplete and/or there are some random elements in the economy which cannot be anticipated perfectly.
- The optimal choice of all agents are then mediated through the markets to produce an outcome for the macroeconomy.

Introduction (cont.)

- This approach is ‘**dynamic**’ because agents are making choices over variables that relate to both present and future.
- This approach is ‘**equilibrium**’ because the outcome for the macro-economy is the aggregation of individuals’ ‘equilibrium’ behaviour.
- This approach is ‘**general equilibrium**’ because it simultaneously takes into account the optimal behaviour of different types of agents in different markets and ensures that all markets clear.

DGE vs. Traditional Macro

- The need to build macro-models based on internally-consistent, dynamic optimization exercises of rational agents arose once it was realized that ad-hoc micro founations for the aggregative system may not be consistent with one another.
- This begs the following question: Why do we need such optimization based micro-founded framework at all?
- Why cannot we just take the aggregative equations as a representation of the macro-economy and try to estimate various parameters, using aggregative data?
- After all, if we are ultimately interested in knowing how the macroeconomy would respond to various kinds of policy shocks, all that we need to do is to econometrically estimate the parameters of the aggregative system.
- Then from the estimated parameter values or coefficients, we can predict the implcations of various policy changes.

DGE vs. Traditional Macro

- Indeed, this is exactly how macroeconomic analysis was conducted traditionally!
- As we have already seen, traditional macroeconomics was based on some aggregative behavioural relationship (e.g., Keyensian Savings Function - which postulates a relationship between aggregate income and aggregate savings; Phillips Curve - which posits a relationship between umployment rate and inflation rate).
- Often one would construct detailed behavioural equations for the macroeconomy and would try to estimate the parameters of these equations using time series data.
- To be sure some of these equations would be dynamic in nature.
- But **optimization** over time was not considered to be important or even relevant. Indeed, the concept of optimization itself - either by households or firms or even government - was rather alien in the field Macroeconomics.

The Lucas Critique

- The need to build macro models based explicitly on agents' optimization exercises came from the so-called Lucas Critique.
- Lucas (1976) argued that aggregative macro models which are estimated to predict outcomes of economic policy changes are useless simply because the estimated parameters themselves may depend on the existing policies.
- As the policy changes, these coefficients themsleves would change, thereby generating wrong predictions!
- His solution was to build macroecnomic models with clear and specific microeconomic foundations - models that are explicitly based on agents' optimization exercises.

The Lucas Critique

- Such models will enable us to differentiate between **true parameters** - primitives like tastes, technology etc - which are independent of the government policies, and variables that treated as exogenous by the agents but are actually endogenous and are influenced by government policies.
- Moreover such models would take into account agents' expectations about government policies.
- Predictions based on such microfounded models would be more accurate than the aggregative models which club all the true parameters as well as other policy-related parameters together.

A simple example

- Let us see exactly what Lucas critique means in the context of a simple example.
- Consider the Keynesian savings function, specified as an aggregative relationship:

$$S_t = \alpha_1 + \alpha_2 Y_t + \epsilon_t$$

- An aggregative macro model would take the above behavioural relationship as given and would estimate the coefficients α_1 and α_2 from data.

Micro-foundation of the Keynesian Savings Function

- We assume that the economy consists of a finite number (H) of identical households. We can then talk in terms of a 'representative' household.
- Let us define a 2-period utility maximization problem of the representative household as:

$$\underset{\{c_t, c_{t+1}\}}{Max.} \log(c_t) + \beta \log(c_{t+1})$$

subject to,

$$\begin{aligned} \text{(i) } P_t c_t + s_t &= y_t; \\ \text{(ii) } P^e_{t+1} c_{t+1} &= (1 + r^e_{t+1}) s_t + y^e_{t+1}. \end{aligned}$$

- From (i) and (ii) we can eliminate S_t to derive the life-time budget constraint of the household as:

$$P_t c_t + \frac{P^e_{t+1} c_{t+1}}{(1 + r^e_{t+1})} = y_t + \frac{y^e_{t+1}}{(1 + r^e_{t+1})}$$

Micro-foundation of the Keynesian Savings Function

- From the FONCs:

$$\frac{c_{t+1}}{\beta c_t} = (1 + r^e_{t+1}) \left(\frac{P_t}{P^e_{t+1}} \right).$$

Solving we get:

$$P_t c_t = \frac{1}{(1+\beta)} \left[y_t + \frac{y^e_{t+1}}{(1 + r^e_{t+1})} \right]$$

- Thus

$$s_t = \frac{\beta}{(1+\beta)} y_t - \frac{1}{(1+\beta)} \left[\frac{y^e_{t+1}}{(1 + r^e_{t+1})} \right]$$

- Aggregating over all households:

$$S_t = \frac{\beta}{(1+\beta)} Y_t - \frac{1}{(1+\beta)} \left[\frac{Y^e_{t+1}}{(1 + r^e_{t+1})} \right]$$

- Notice that an aggregative model would equate $\frac{\beta}{(1+\beta)}$ to α_2 and $-\frac{1}{(1+\beta)} \left[\frac{Y^e_{t+1}}{(1+r^e_{t+1})} \right]$ to α_1.

Micro-foundation of the Keynesian Savings Function

- While the coefficient α_2 is indeed based on true parameters (primitives) and would therefore be unaffected by policy changes, coefficient α_1 is not.
- Any policy that changes the household's expectation about its future income or future rate of interest rate would affect α_1.
- Thus predicting outcomes of such a policy based on the estimated values of the aggregative equations would be wrong.

The DGE approach

- The Lucas critique and the consequent logical need to develop a unified micro-founded macroeconomic framework which would allow us to accurately predict the macroeconomic outcomes in response to any external shock (policy-driven or otherwise) led to emergence of the modern dynamic general equilibrium approach.
- As before, there are two variants of modern DGE-based approach:
  - One is based on the assumption of perfect markets (the Neoclassical/RBC school). As is expected, this school is critical of any policy intervention, in particular, monetary policy interventions.
  - The other one allows for some market imperfections (the New-Keynesian school). Again, true to their ideological underpinning, this school argues for active policy intervention.

The DGE approach

- However, both frameworks are similar in two fundamental aspects:
  - **Agents optimize over infinte horizon;** and
  - **Agents are forward looking,** i.e., when they optimize over future variable they base their expectations on all available information - including information about (future) government policies. In other words, **agents have rational expectations**.
- We now develop the choice-theoretic frameworks for households and firms under the DGE approach.

The Centralized Economy

Outline

1. Introduction
2. The Basic DGE Closed Economy
3. Golden Rule Solution
4. Optimal Solution
   - The Euler Equation
   - Interpretation
   - Static Equilibrium
   - Dynamics
   - Algebraic Analysis

The Production Economy

Recall a static production economy

$$\mathcal{E} = \{(U^i, \mathbf{e}^i, \theta^{ij}, Y^j) | i \in \mathcal{I},\ j \in \mathcal{J}\}$$

- There are n goods and services.
- Each household $i \in \mathcal{I}$ has continuous, strongly increasing, and strictly quasi-concave utility function $U^i : \mathbb{R}^n_+ \to \mathbb{R}_+$ and is endowed with $\mathbf{e}^i \in \mathbb{R}^n_+$.
- Each competitive firm $j \in \mathcal{J}$ has a production set Y^j that is compact and strongly convex.
- θ^{ij} is the share of household i in firm j.

Really Nice Results from Microeconomic Theory

1. A Walrasian equilibrium exists: there is a price vector $\mathbf{p}^*$ such that

$$\sum_{i \in \mathcal{I}} \mathbf{d}^i(\mathbf{p}^*, m^i(\mathbf{p}^*)) = \sum_{j \in \mathcal{J}} y^j(\mathbf{p}^*) + \sum_{i \in \mathcal{I}} \mathbf{e}^i.$$

   That is, all n markets of goods and services clear.
2. FWTE: The Walrasian equilibrium allocation is Pareto efficient.
3. SWTE: Any desirable Pareto efficient allocation $(\mathbf{x}, \mathbf{y})$ can be achieved as a Walrasian equilibrium allocation after a suitable income transfer program between households.
4. With full information on preferences and technology, communism and capitalism achieve the same outcome in a static economy.

To Make Things Really Simple . . .

- There are two goods, $n = 2$, called capital k and output y.
- The households only consume one good, effectively we can consider them as one big household ($|\mathcal{I}| = 1$)
- One aggregate competitive firm ($|\mathcal{J}| = 1$).
- Of course the ownership share become $\theta^{11} = 1$.
- This is a bit too simple. So instead of static equilibrium, we study this economy over time, $t = 1, 2, \ldots$

The Ramsey Model

- In period t, the endowment is the capital stock k_t. The firm produces output y_t using capital as input:

$$y_t = F(k_t) \tag{2.3}$$

- Output y_t is divided into two parts, consumption c_t and investment i_t:

$$y_t = c_t + i_t. \tag{2.1}$$

  This is called the national income identity, or the resource constraint.

- Investment i_t is saved as capital for next period. In production, the firm consumes only part of the capital stock, δk_t, where δ is called the depreciation rate. Capital stock in period $t+1$ is therefore

$$\Delta k_{t+1} = k_{t+1} - k_t = i_t - \delta k_t. \tag{2.2}$$

Dynamic Resource Constraint

The last three equations gives

$$F(k_t) = c_t + \Delta k_{t+1} + \delta k_t. \quad (2.4)$$

Like the static model, our objective is to maximize utility derived from consumption, not output (we are not communists!) But we have a problem. Should we maximize

1. utility in each period, treating every period as equally important, (golden rule) or,
2. the present value of total utility of all present and future periods, using an appropriate discount factor? (optimal solution)

Golden Rule — The Steady State

The dynamic resource constraint (2.4) can be written as

$$c_t = F(k_t) - k_{t+1} + (1 - \delta)k_t. \tag{2.5}$$

The steady state is attained when all variable are the same in all subsequent periods, i.e., $c_t = c$ and $k_t = k$, $t = 1, 2, \ldots$. Then (2.5) becomes

$$c = F(k) - \delta k. \tag{2.6}$$

The necessary condition for maximization is

$$\frac{\partial c}{\partial k} = F'(k) - \delta = 0, \tag{2.7}$$

which means that marginal product is equal to the depreciation rate when consumption is maximized in the steady state.

Golden Rule Steady State

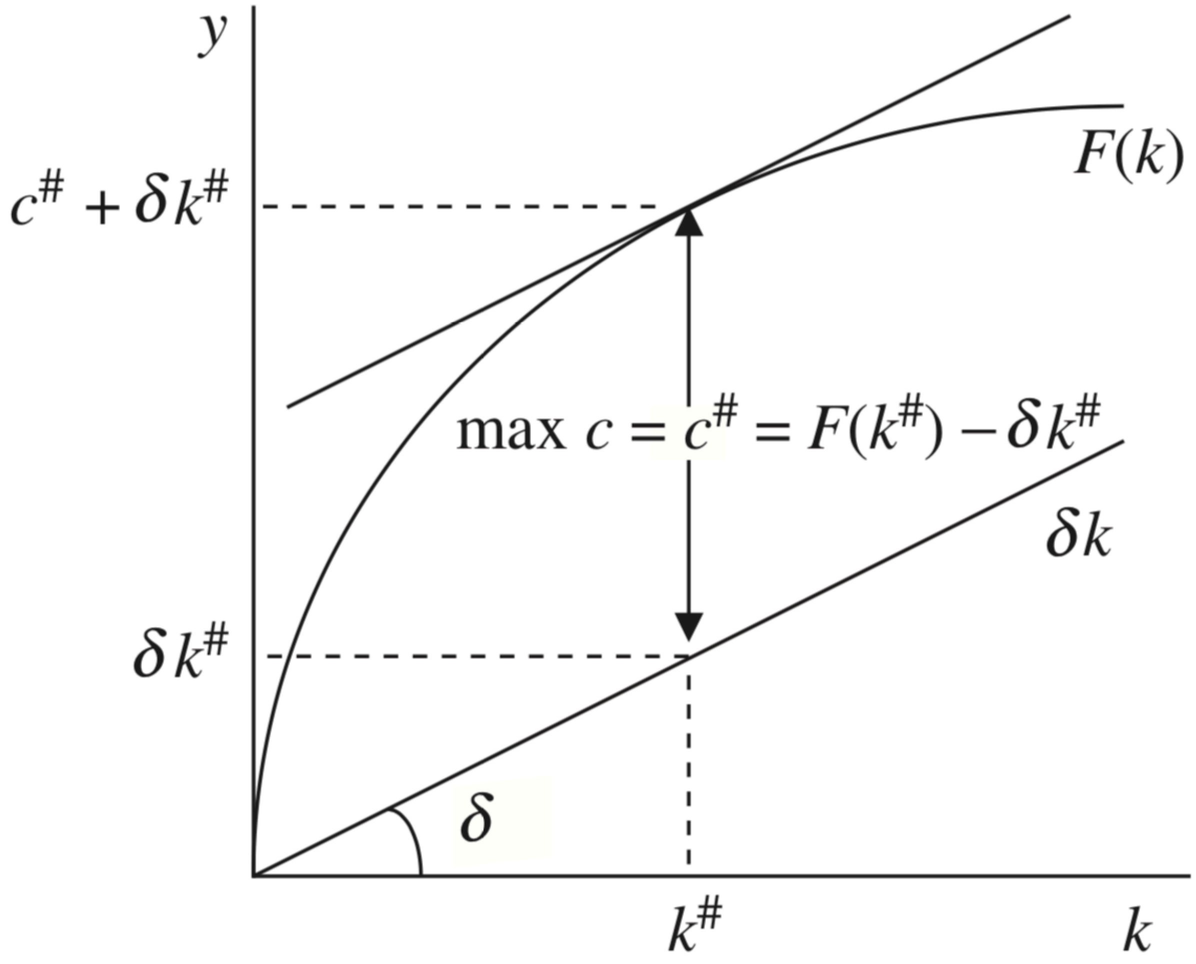

Figure 2.2. Total output, consumption, and replacement investment.

The Optimization Problem

In the optimal solution the present value of current (period t) and future ($t+s$) utility is maximized:

$$\max_{c_{t+s},k_{t+s+1}} \quad \sum_{s=0}^{\infty} \beta^s U(c_{t+s})$$

$$\text{subject to} \quad F(k_{t+s}) = c_{t+s} + k_{t+s+1} - (1-\delta)k_{t+s},$$

where $\beta = 1/(1+\theta)$ and $\theta > 0$ is called the social discount rate. The Lagrangian is

$$\mathcal{L}_t = \sum_{s=0}^{\infty} \Big\{ \beta^s U(c_{t+s}) \\ + \lambda_{t+s}[F(k_{t+s}) - c_{t+s} - k_{t+s+1} + (1-\delta)k_{t+s}] \Big\}, \quad (2.8)$$

where λ_{t+s} is the Lagrange multiplier in period $t+s$.

Necessary Conditions

The first-order conditions are

$$\frac{\partial \mathcal{L}_t}{\partial c_{t+s}} = \beta^s U'(c_{t+s}) - \lambda_{t+s} = 0, \quad s \geq 0, \tag{2.9}$$

$$\frac{\partial \mathcal{L}_t}{\partial k_{t+s}} = \lambda_{t+s}[F'(k_{t+s}) + 1 - \delta] - \lambda_{t+s-1} = 0,$$
$$s \geq 1, \tag{2.10}$$

with the resource constraint

$$F(k_{t+s}) = c_{t+s} + k_{t+s+1} - (1-\delta)k_{t+s}, \quad s \geq 0,$$

and the transversality condition

$$\lim_{s\to\infty} \beta^s U'(c_{t+s})k_{t+s+1} = 0. \tag{2.11}$$

The Euler Equation

Eliminating λ_{t+s} and λ_{t+s-1} in (2.10) using (2.9) gives

$$\beta^s U'(c_{t+s})[F'(k_{t+s}) + 1 - \delta] = \beta^{s-1} U'(c_{t+s-1}), \quad s \geq 0.$$

For $s = 1$ this can be written as

$$\beta \frac{U'(c_{t+1})}{U'(c_t)} \left[F'(k_{t+1}) + 1 - \delta \right] = 1. \tag{2.12}$$

This is called the Euler equation, which is the corner stone of dynamic optimization problems in consumption.

Interpretation of the Euler Equation

The Euler equation reflects the intertemporal substitution of consumption between two consecutive periods. Consider periods t and $t+1$:

$$V_t = U(c_t) + \beta U(c_{t+1}).$$

Using the implicit function theorem, the slope of the indifference curve in the (c_t, c_{t+1}) space is called the marginal rate of time preference:

$$\frac{dc_{t+1}}{dc_t} = -\frac{U'(c_t)}{\beta U'(c_{t+1})}. \tag{2.13}$$

Interpretation continued

The budget constraint in period t and $t+1$ can be written respectively as

$$\begin{aligned} k_{t+1} &= F(k_t) + (1-\delta)k_t - c_t, \\ c_{t+1} &= F(k_{t+1}) - k_{t+2} + (1-\delta)k_{t+1}. \end{aligned}$$

Using the chain rule to differentiate c_{t+1} with respect to c_t, we get

$$\frac{dc_{t+1}}{dc_t} = -[F'(k_{t+1}) + 1 - \delta]. \tag{2.14}$$

This is the slope of the intertemporal production possibility frontier (IPPF). Equating (2.13) and (2.14) gives the Euler equation. That is, at the optimal point (c_t^*, c_{t+1}^*), the indifference curve of the household is tangent to the IPPF.

Graphical Interpretation

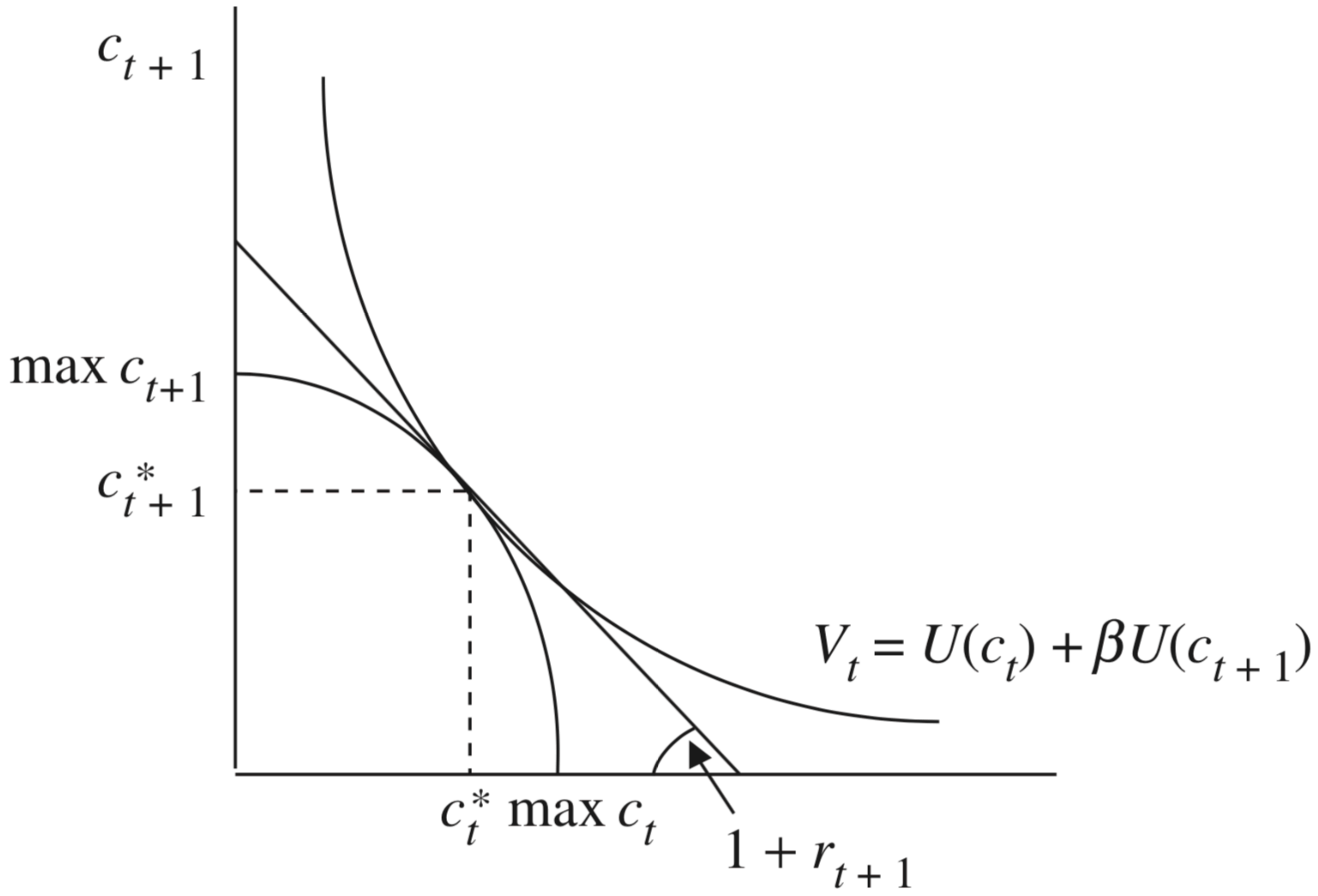

Figure 2.4. A graphical solution based on the IPPF.

Steady-State Solution

In the steady state (long-run), $c_t = c^*$ and $k_t = k^*$ for all t. The Euler equation becomes

$$\beta \frac{U'(c^*)}{U'(c^*)} \left[F'(k^*) + 1 - \delta \right] = 1,$$

or, with $\beta = 1/(1+\theta)$,

$$F'(k^*) = 1/\beta + \delta - 1 = \delta + \theta. \tag{2.21}$$

From the resource constraint we have

$$c^* = F(k^*) - \delta k^*. \tag{2.22}$$

Comparing with the golden rule solution, where $F'(k^{\#}) = \delta$, the long-run capital stock is at a lower level. That is,

$$c^* < c^{\#} \text{ and } k^* < k^{\#}.$$

Comparing Golden Rule and Optimal Solution

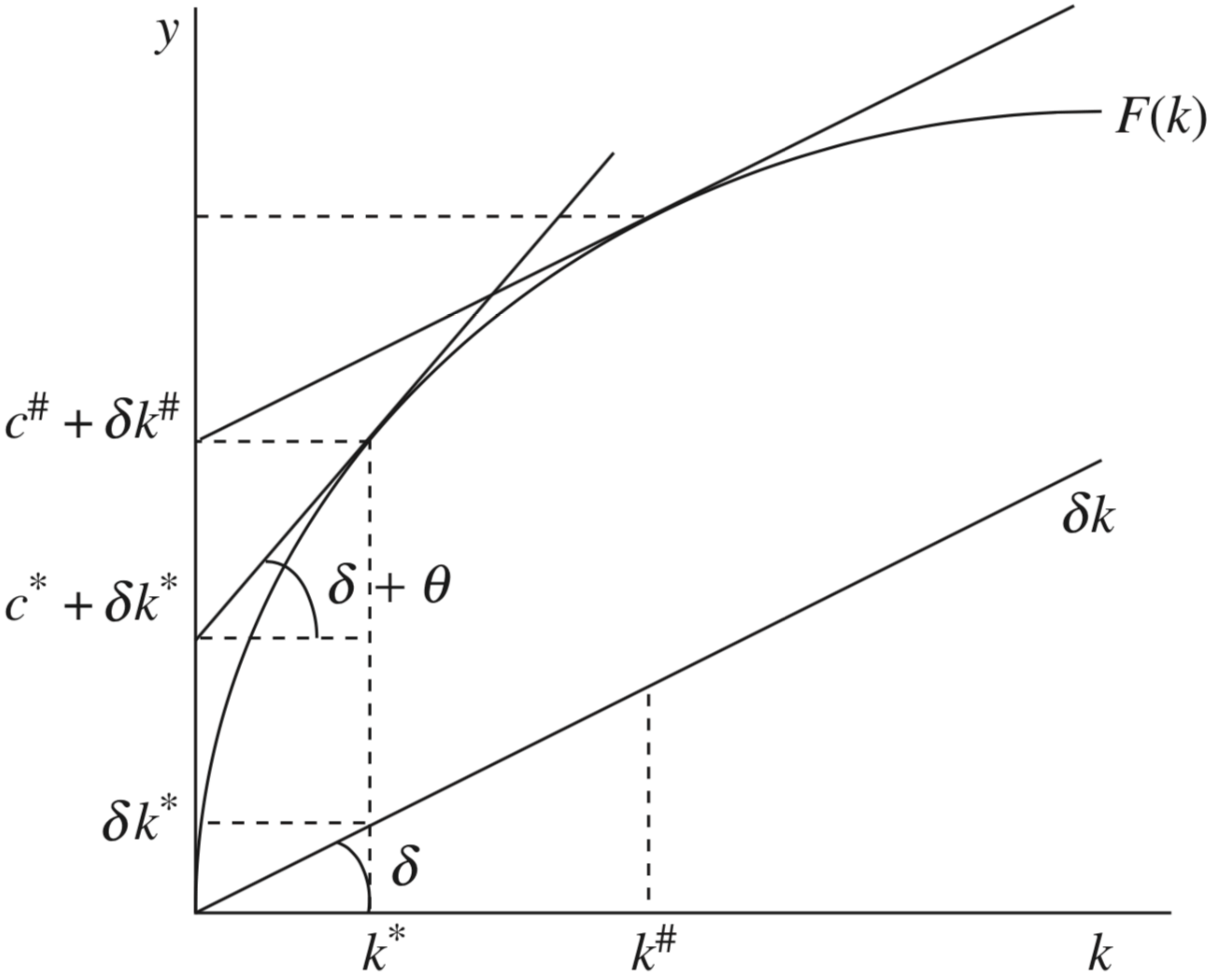

Figure 2.6. Optimal long-run consumption.

Linear Approximation

So far we have established two dynamic relations between two consecutive periods, the Euler equation and the resource constraint:

$$\beta \frac{U'(c_{t+1})}{U'(c_t)} \left[F'(k_{t+1}) + 1 - \delta\right] = 1,$$
$$\Delta k_{t+1} = F(k_t) - \delta k_t - c_t. \tag{2.17}$$

The relation between c_{t+1} and c_t can be better seen by taking a first-order Taylor approximation of $U'(c_{t+1})$ about c_t:

$$U'(c_{t+1}) \simeq U'(c_t) + U''(c_t)\Delta c_{t+1}.$$

The Euler equation becomes, with $U'(c_t)/U''(c_t) \leq 0$,

$$\Delta c_{t+1} = -\frac{U'(c_t)}{U''(c_t)} \left[1 - \frac{1}{\beta[F'(k_{t+1}) + 1 - \delta]}\right]. \tag{2.18}$$

Dynamics of Consumption and Capital

In the steady state, $F'(k_{t+1}) = F'(k^*) = \delta + \theta$ and so

$$\Delta c = -\frac{U'(c^*)}{U''(c^*)}\left[1 - \frac{1}{\beta[\delta + \theta + 1 - \delta]}\right] = 0.$$

Two conclusions:

1. When $k > k^*$, $F'(k) < F'(k^*)$ and by (2.18) $\Delta c < 0$.
2. When $k < k^*$, $F'(k) > F'(k^*)$ and by (2.18) $\Delta c > 0$.

From the resource constraint (2.17),

1. When $c_t > F(k_t) - \delta k_t$, then $\Delta k < 0$
2. When $c_t < F(k_t) - \delta k_t$, then $\Delta k > 0$

Phase Diagrams

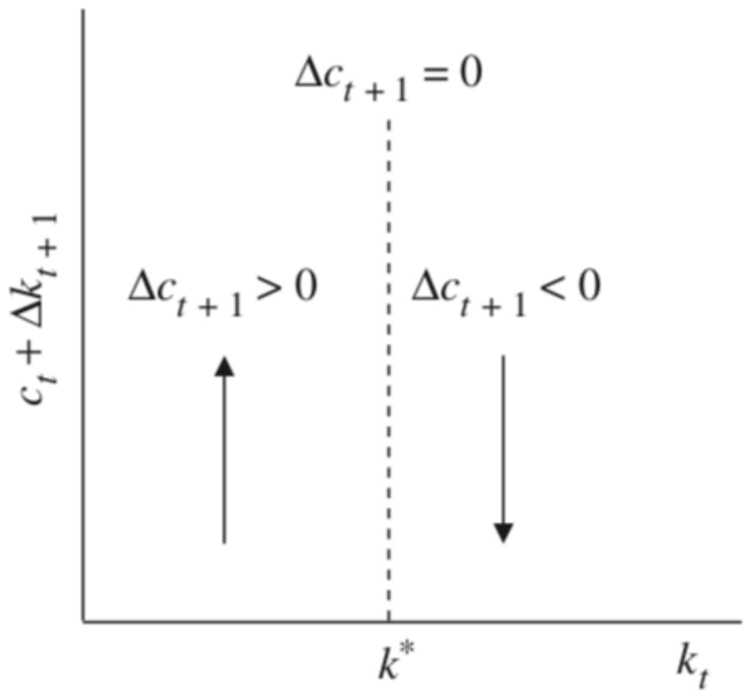

Figure 2.8. Consumption dynamics.

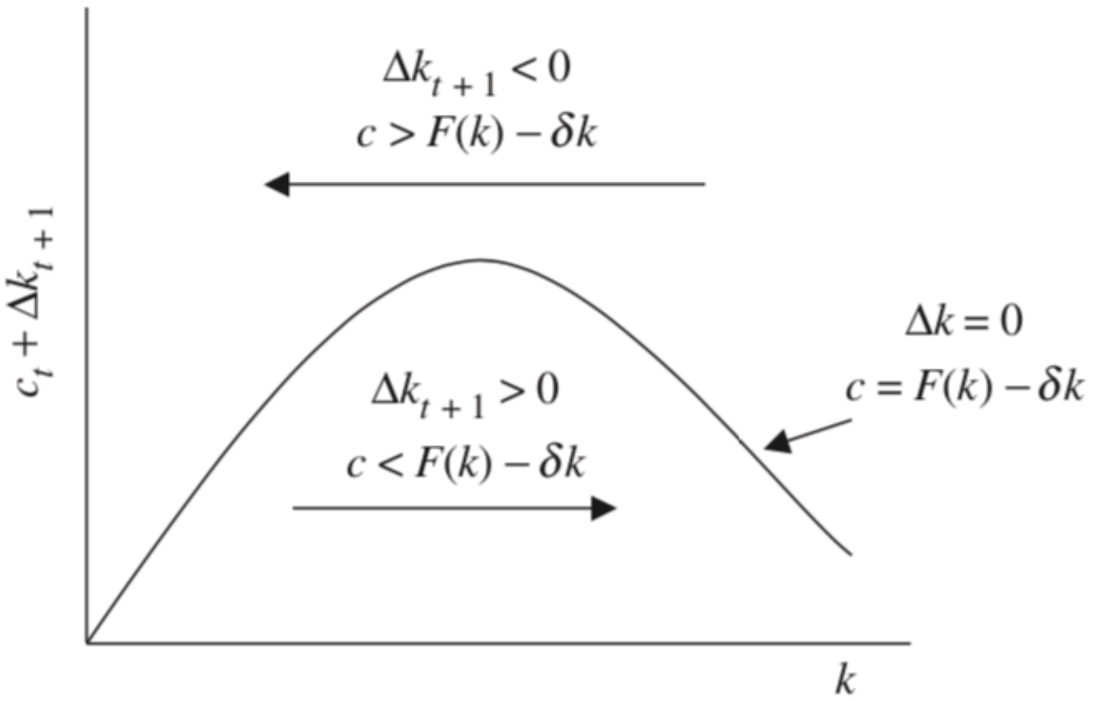

Figure 2.9. Capital dynamics.

The Saddle Path

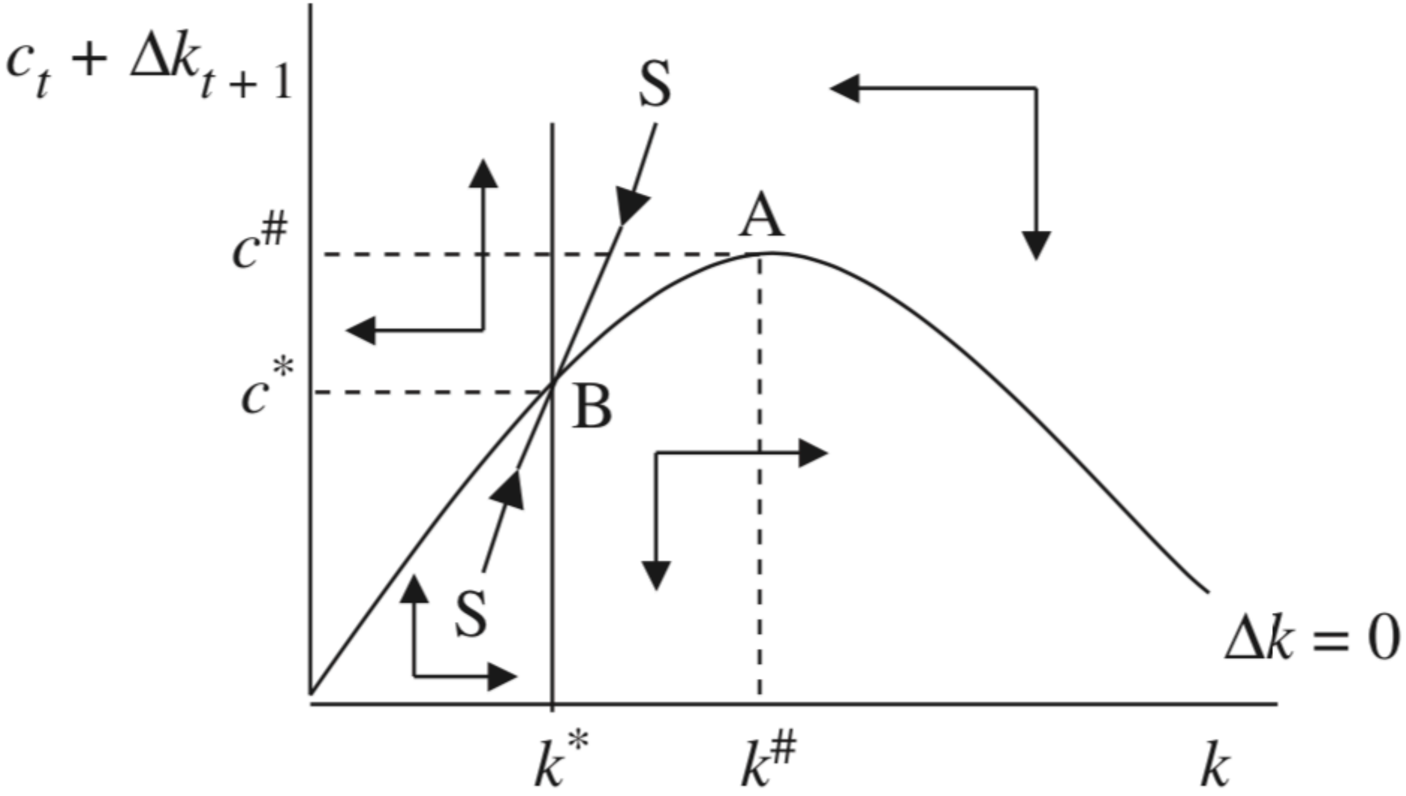

Figure 2.10. Phase diagram.