Financial Forecasting

M.Sc. in Finance

List of Exercises

(based on the Lecture Notes of Prof. António Costa (ISEG) and Prof. Nuno Sobreira (ISEG) and on the textbook Gloria Gonzalez-Rivera, *Forecasting for Economics and Business*, Pearson, 2013)

Exercises 5, 6a) (chapter 3 Textbook)

Exercises 7 a) b), 8a) and 9 (chapter 3 Textbook)

1. Consider the following stochastic processes where $\varepsilon_t \sim WN(0, \sigma_\varepsilon^2)$, $\beta_1, \beta_2 \neq 0$:
   i. $X_t = \alpha + \varepsilon_t$
   ii. $X_t = \beta_0 + \beta_1 t + \beta_2 t^2 + \varepsilon_t$
   iii. $X_t = \alpha + X_{t-1} + \varepsilon_t$ with X_0 fixed

a. Identify the processes that are stationary.

b. For the stationary processes verify that $\rho_k = Corr(X_t, X_{t-k}) \to 0$ with $k \to \infty$

c. For the nonstationary processes, propose a transformation that makes the process stationary.

Exercise 3 (chapter 6 Textbook)

2. Consider the MA(1) process $y_t = \varepsilon_t - 0.12\ \varepsilon_{t-1}$ where $\varepsilon_t \sim WN(0, \sigma_\varepsilon^2)$
   a. Find the ACF of the process.
   b. Is the model stationary? Justify your answer.
   c. Is the model invertible? Justify your answer.
   d. Characterize the behavior of the Partial Autocorrelation Function of the process.

3. Consider the MA(2) process $y_t = 14 + \varepsilon_t - 0.1\ \varepsilon_{t-1} + 0.21\varepsilon_{t-2}$ where $\varepsilon_t \sim WN(0, \sigma_\varepsilon^2)$
   a. Find the ACF of the process.
   b. Is the model stationary? Justify your answer.
   c. Is the model invertible? Justify your answer.
   d. Characterize the behavior of the Partial Autocorrelation Function of the process.

4. Suppose that you have time series data of a given country's inflation denoted as y_t. With these data the following model was estimated:

Dependent Variable: INFL
Method: Least Squares
Date: 11/08/12 Time: 16:08
Sample: 1971M01 2011M12
Included observations: 492
Convergence achieved after 35 iterations
MA Backcast: 1970M10 1970M12

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	1.000127	0.007137	140.1296	0.0000
MA(1)	0.372642	0.034864	10.68835	0.0000
MA(2)	-0.287517	0.036356	-7.908265	0.0000
MA(3)	-0.642579	0.035015	-18.35160	0.0000

R-squared	0.568424	Mean dependent var	1.002714
Adjusted R-squared	0.565771	S.D. dependent var	0.536875
S.E. of regression	0.353779	Akaike info criterion	0.767809
Sum squared resid	61.07791	Schwarz criterion	0.801943
Log likelihood	-184.8810	Hannan-Quinn criter.	0.781212
F-statistic	214.2467	Durbin-Watson stat	1.389038
Prob(F-statistic)	0.000000		

Inverted MA Roots	.85	-.61+.62i	-.61-.62i

a. Obtain the general theoretical expression for $E[y_t]$ and $Var(y_t)$
b. Using the Eviews output provide an estimate for $E[y_t]$ and $Var(y_t)$

Exercises 1, 5 and 6 (chapter 7 Textbook)

5. Consider the AR(2) process $y_t = 2 + 0.8y_{t-1} - 0.1y_{t-2} + \varepsilon_t$ where $\varepsilon_t \sim \mathrm{WN}(0, \sigma_\varepsilon^2)$
   a. Is the process stationary?
   b. Compute the unconditional mean of the process.
   c. Determine the PACF and describe the ACF of the process.

6. Suppose that you have time series data of a given country's inflation denoted as y_t. With these data the following model was estimated:

Dependent Variable: INFL
Method: Least Squares
Date: 11/07/12 Time: 14:02
Sample (adjusted): 1971M03 2011M12
Included observations: 490 after adjustments
Convergence achieved after 3 iterations

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.999994	0.016895	59.18885	0.0000
AR(1)	1.006304	0.029335	34.30388	0.0000
AR(2)	-0.769362	0.029310	-26.24904	0.0000

R-squared	0.718467	Mean dependent var	1.000375
Adjusted R-squared	0.717311	S.D. dependent var	0.536716
S.E. of regression	0.285364	Akaike info criterion	0.336001
Sum squared resid	39.65768	Schwarz criterion	0.361681
Log likelihood	-79.32032	Hannan-Quinn criter.	0.346087
F-statistic	621.4079	Durbin-Watson stat	2.779603
Prob(F-statistic)	0.000000		

Inverted AR Roots	.50+.72i	.50-.72i

a. Obtain the general theoretical expression for $E[y_t]$.

b. Using the Eviews output provide an estimate for $E[y_t]$.

c. Suppose that the inflation rate at November 2011 and December 2011 were 1 and 1.2 respectively. Obtain the optimal forecast estimate for the inflation rate according to this model for:

   i. January 2012

   ii. February 2012

d. Provide estimates for the forecast uncertainty for:

   i. January 2012

   ii. February 2012

11. Write the equation that defines a process *ARMA*(0,1)(0,1)12 with parameters $\theta_{12} = 0.8$ and $\theta_1 = 0.6$ and find the ACF of the process.

12. Write the equation that defines a process *ARMA*(1,0)(1,0)4 with parameters $\phi_4 = 0.8$ and $\phi_1 = 0.6$ and find the PACF of the process.

13. Suppose that you want to analyze a given time series data with the correlogram of Figure 1 . According to this information, what is the best model for this time series? Justify your answer.

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	-0.444	-0.444	22.515	0.000
		2	0.028	-0.211	22.608	0.000
		3	0.015	-0.083	22.634	0.000
		4	-0.097	-0.155	23.728	0.000
		5	0.094	-0.027	24.779	0.000
		6	-0.108	-0.118	26.176	0.000
		7	0.043	-0.075	26.400	0.000
		8	0.036	-0.003	26.561	0.001
		9	0.040	0.089	26.755	0.002
		10	-0.036	0.027	26.913	0.003
		11	0.118	0.183	28.651	0.003
		12	-0.404	-0.367	49.305	0.000
		13	0.259	-0.111	57.870	0.000
		14	-0.037	-0.049	58.049	0.000
		15	-0.035	-0.037	58.213	0.000
		16	0.087	-0.023	59.213	0.000
		17	-0.120	-0.077	61.134	0.000
		18	0.252	0.157	69.727	0.000
		19	-0.176	0.039	73.950	0.000
		20	-0.047	-0.071	74.249	0.000
		21	0.056	0.021	74.690	0.000
		22	-0.056	-0.046	75.134	0.000
		23	0.053	0.018	75.530	0.000
		24	0.039	-0.095	75.753	0.000
		25	-0.104	-0.109	77.318	0.000
		26	0.083	-0.020	78.342	0.000
		27	-0.045	-0.090	78.644	0.000
		28	-0.015	-0.029	78.676	0.000
		29	0.087	0.055	79.838	0.000
		30	-0.210	-0.027	86.688	0.000

Figure 1

14. Are the following processes stationary/causal? Are the following processes invertible? Justify your answers. Consider that $\varepsilon_t \sim WN(0, \sigma_\varepsilon^2)$.
    a. $y_t = \varepsilon_t + 0.8\,\varepsilon_{t-1} - \varepsilon_{t-2}$
    b. $y_t = 0.6\,y_{t-1} + 0.4 y_{t-2} + \varepsilon_t$
    c. $y_t = (1 - 0.7L + 0.3L^2)\,\varepsilon_t$

15. Consider the process ARMA(1,1) with $\phi = 0.8$ and $\theta = 0.5$ and with mean equal to 10.
    a. Formulate the equation that defines the process
    b. Find the ACF of the process.

16. For the following processes identify the orders of the autoregressive and moving average part and write the ARMA representation without the lag operator:

i. $Y_t = (1 - 0.5L)\varepsilon_t$

ii. $(1 + 0.8L)Y_t = (1 - 1.2L)\varepsilon_t$

iii. $(1 - 0.7L + 0.4L^2)Y_t = (1 - 1.2L)\varepsilon_t$

iv. $(1 + 0.8L)Y_t = (1 - 0.7L + 0.4L^2 + L^3)\varepsilon_t$

17. Consider the following models where $\varepsilon_t \sim WN(0, \sigma_\varepsilon^2)$:

v. $Y_t = Y_{t-1} + \varepsilon_t - 1.5\varepsilon_{t-1}$

vi. $Y_t = 0.8Y_{t-1} + \varepsilon_t - 0.5\varepsilon_{t-1}$

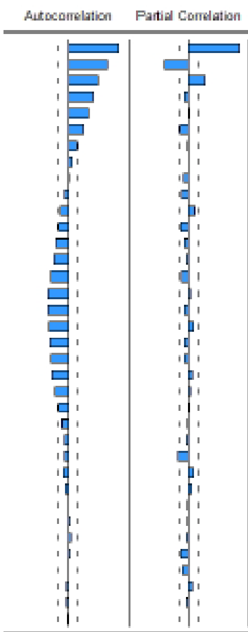
vii. $Y_t = 1.1Y_{t-1} + 0.8Y_{t-1} + \varepsilon_t - 1.7\varepsilon_{t-1} + 0.72\varepsilon_{t-2}$

viii. $Y_t = 0.6Y_{t-1} + \varepsilon_t - 1.2\varepsilon_{t-1} + 0.2\varepsilon_{t-2}$

a) Verify if Y_t is stationary and invertible.

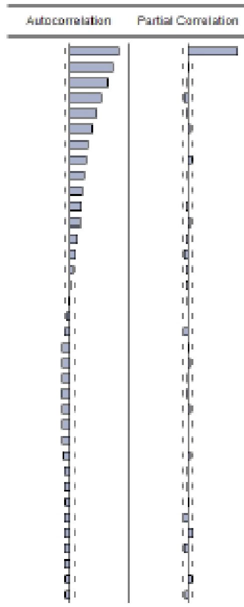
b) Characterize the behavior of the ACF and PACF.

18. In the following figure you may find the ACF and PACF of four time series.



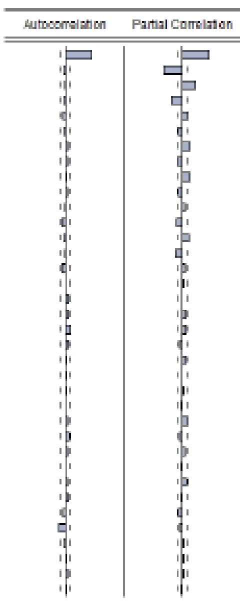
	AC	PAC	Q-Stat	Prob
1	0.877	0.877	103.76	0.000
2	0.673	-0.413	165.34	0.000
3	0.524	0.277	203.04	0.000
4	0.430	-0.075	228.61	0.000
5	0.357	0.019	246.34	0.000
6	0.264	-0.162	256.14	0.000
7	0.159	-0.011	259.72	0.000
8	0.075	-0.002	260.53	0.000
9	0.010	-0.086	260.54	0.000
10	-0.065	-0.124	261.15	0.000
11	-0.121	0.112	263.30	0.000
12	-0.161	-0.133	267.13	0.000
13	-0.207	-0.071	273.53	0.000
14	-0.252	-0.032	283.06	0.000
15	-0.303	-0.127	296.95	0.000
16	-0.340	0.033	314.58	0.000
17	-0.350	-0.076	333.44	0.000
18	-0.332	0.095	350.53	0.000
19	-0.305	-0.072	365.06	0.000
20	-0.291	-0.073	378.48	0.000
21	-0.274	0.080	390.41	0.000
22	-0.230	0.032	398.95	0.000
23	-0.162	0.022	403.23	0.000
24	-0.099	-0.017	404.85	0.000
25	-0.062	-0.041	405.48	0.000
26	-0.071	-0.190	406.31	0.000
27	-0.084	0.088	407.51	0.000
28	-0.055	0.042	408.03	0.000
29	-0.008	-0.012	408.04	0.000
30	0.033	-0.019	408.22	0.000
31	0.051	-0.026	408.68	0.000
32	0.033	-0.133	408.87	0.000
33	-0.008	-0.102	408.88	0.000
34	-0.031	0.075	409.05	0.000
35	-0.033	-0.029	409.25	0.000
36	-0.023	0.002	409.35	0.000

(a) Time series x_t



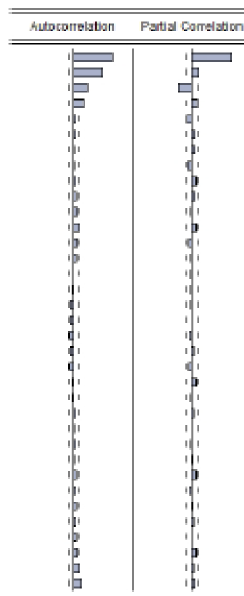
	AC	PAC	Q-Stat	Prob
1	0.869	0.869	379.63	0.000
2	0.760	0.023	671.12	0.000
3	0.661	-0.017	892.02	0.000
4	0.558	-0.071	1049.6	0.000
5	0.460	-0.046	1156.8	0.000
6	0.385	0.033	1232.2	0.000
7	0.322	0.006	1285.1	0.000
8	0.285	0.068	1326.5	0.000
9	0.250	-0.009	1358.5	0.000
10	0.221	-0.001	1383.5	0.000
11	0.190	-0.031	1402.0	0.000
12	0.174	0.041	1417.5	0.000
13	0.147	-0.040	1428.6	0.000
14	0.105	-0.077	1434.3	0.000
15	0.063	-0.039	1436.3	0.000
16	0.019	-0.043	1436.5	0.000
17	-0.021	-0.011	1436.7	0.000
18	-0.051	0.001	1438.1	0.000
19	-0.095	-0.089	1442.8	0.000
20	-0.119	0.017	1450.2	0.000
21	-0.126	0.032	1458.5	0.000
22	-0.134	-0.018	1467.9	0.000
23	-0.143	-0.026	1478.6	0.000
24	-0.134	0.036	1488.1	0.000
25	-0.125	0.006	1496.4	0.000
26	-0.116	-0.003	1503.5	0.000
27	-0.102	0.030	1509.0	0.000
28	-0.093	-0.011	1513.7	0.000
29	-0.080	-0.015	1518.0	0.000
30	-0.079	0.018	1521.3	0.000
31	-0.094	-0.091	1526.0	0.000
32	-0.085	0.087	1529.8	0.000
33	-0.092	-0.072	1534.3	0.000
34	-0.096	-0.007	1539.3	0.000
35	-0.078	0.079	1542.5	0.000
36	-0.072	-0.054	1545.3	0.000

(b) Time series y_t



	AC	PAC	Q-Stat	Prob
1	0.462	0.462	107.40	0.000
2	-0.040	-0.322	108.19	0.000
3	-0.017	0.221	108.33	0.000
4	-0.028	-0.204	108.72	0.000
5	-0.058	0.095	110.45	0.000
6	-0.031	-0.080	110.92	0.000
7	0.041	0.125	111.77	0.000
8	0.033	-0.096	112.33	0.000
9	0.023	0.116	112.61	0.000
10	0.026	-0.080	113.03	0.000
11	-0.018	0.026	113.20	0.000
12	-0.074	-0.103	116.00	0.000
13	-0.031	0.104	116.49	0.000
14	-0.016	-0.137	116.62	0.000
15	-0.070	0.039	119.18	0.000
16	-0.002	0.016	119.18	0.000
17	0.045	-0.002	120.22	0.000
18	0.045	0.061	121.28	0.000
19	0.089	0.061	125.38	0.000
20	0.054	-0.045	126.93	0.000
21	0.011	0.063	127.00	0.000
22	0.022	-0.005	127.25	0.000
23	0.018	0.011	127.41	0.000
24	-0.007	-0.019	127.44	0.000
25	0.031	0.095	127.96	0.000
26	0.071	-0.035	130.60	0.000
27	0.034	0.029	131.20	0.000
28	-0.002	-0.023	131.20	0.000
29	0.028	0.074	131.62	0.000
30	0.055	-0.018	133.25	0.000
31	-0.054	-0.078	134.84	0.000
32	-0.119	-0.033	142.38	0.000
33	-0.044	0.020	143.41	0.000
34	0.009	0.016	143.45	0.000
35	0.027	0.013	143.84	0.000
36	0.023	-0.017	144.12	0.000

(c) Time series w_t



	AC	PAC	Q-Stat	Prob
1	0.678	0.678	231.06	0.000
2	0.516	0.105	365.18	0.000
3	0.258	-0.235	398.87	0.000
4	0.178	0.091	414.82	0.000
5	0.039	-0.091	415.59	0.000
6	0.035	0.057	416.22	0.000
7	0.020	0.054	416.42	0.000
8	0.012	-0.073	416.50	0.000
9	0.029	0.072	416.91	0.000
10	0.061	0.055	418.82	0.000
11	0.072	-0.021	421.49	0.000
12	0.099	0.072	426.62	0.000
13	0.073	-0.062	429.27	0.000
14	0.054	-0.018	430.79	0.000
15	0.003	-0.015	430.79	0.000
16	-0.013	-0.018	430.87	0.000
17	-0.052	-0.023	432.27	0.000
18	-0.053	-0.001	433.71	0.000
19	-0.076	-0.044	436.70	0.000
20	-0.052	0.035	438.12	0.000
21	-0.072	-0.056	440.84	0.000
22	-0.016	0.073	440.98	0.000
23	-0.017	-0.014	441.13	0.000
24	0.034	0.025	441.73	0.000
25	0.024	0.005	442.03	0.000
26	0.041	-0.013	442.92	0.000
27	0.021	0.012	443.15	0.000
28	0.056	0.078	444.82	0.000
29	0.037	-0.042	445.54	0.000
30	0.047	0.022	446.73	0.000
31	0.038	0.026	447.48	0.000
32	0.054	0.007	449.07	0.000
33	0.069	0.075	451.64	0.000
34	0.104	0.029	457.51	0.000
35	0.141	0.059	468.18	0.000

(d) Time series z_t

According to the previous figures identify an appropriate ARMA model for each series. Justify.

19. Consider the following estimation outputs for a fitted model on a prince index (IPI).

   b. Write the estimated model in equation form.

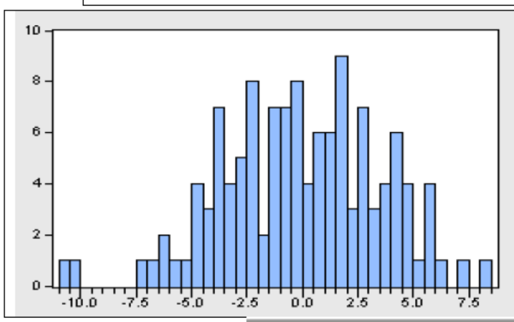
   c. Comment on the residuals distribution.

   d. Is the proposed model acceptable?

Dependent Variable: D(IPI,1,12)
Sample (adjusted): 2001M02 2011M04
Included observations: 123 after adjustments
MA Backcast: 2000M01 2001M01

Variable	Coefficient	Std. Error	t-Statistic	Prob.
MA(1)	-0.740267	0.060517	-12.23246	0.0000
SMA(12)	-0.887638	0.023475	-37.81162	0.0000

R-squared	0.557799	Mean dependent var	-0.071479
Adjusted R-squared	0.554144	S.D. dependent var	5.378399
S.E. of regression	3.591289	Akaike info criterion	5.411026
Sum squared resid	1560.580	Schwarz criterion	5.456753
Log likelihood	-330.7781	Hannan-Quinn criter.	5.429600
Durbin-Watson stat	2.249783		



Series: Residuals	
Sample 2001M02 2011M04	
Observations 123	
Mean	-0.052075
Median	-0.107363
Maximum	8.061525
Minimum	-10.94672
Std. Dev.	3.576158
Skewness	-0.261054
Kurtosis	3.053888
Jarque-Bera	1.411936
Probability	0.493631

Correlogram of Residuals

Sample: 2001M02 2011M04
Included observations: 123
Q-statistic probabilities adjusted for 2 ARMA term(s)

Lag	AC	PAC	Q-Stat	Prob
1	-0.128	-0.128	2.0665	
2	0.012	-0.005	2.0842	
3	0.232	0.237	8.9770	0.003
4	-0.162	-0.111	12.376	0.002
5	0.013	-0.029	12.399	0.006
6	0.156	0.119	15.601	0.004
7	-0.139	-0.055	18.168	0.003
8	0.032	-0.015	18.308	0.006
9	0.194	0.162	23.366	0.001
10	-0.147	-0.048	26.323	0.001
11	0.084	0.023	27.287	0.001
12	0.221	0.187	34.072	0.000
13	-0.153	-0.030	37.332	0.000
14	0.024	-0.083	37.411	0.000
15	-0.051	-0.152	37.788	0.000
16	-0.073	0.044	38.544	0.000
17	-0.001	-0.051	38.544	0.001
18	-0.033	-0.079	38.706	0.001
19	-0.139	-0.100	41.557	0.001
20	0.103	0.077	43.153	0.001
21	0.033	0.040	43.312	0.001
22	-0.193	-0.160	48.972	0.000
23	0.218	0.158	56.288	0.000
24	-0.204	-0.188	62.736	0.000
25	-0.091	-0.069	64.034	0.000
26	0.087	0.023	65.235	0.000
27	-0.231	-0.078	73.750	0.000
28	0.057	0.049	74.276	0.000
29	0.004	-0.099	74.278	0.000
30	-0.134	0.017	77.241	0.000

20. Consider the following process: $y_t = 2.5 + 0.75y_{t-1} + \varepsilon_t + 0.6\varepsilon_{t-1} - 0.3\varepsilon_{t-2}$ where $\varepsilon_t \sim WN(0, \sigma_\varepsilon^2)$:

a. Given $y_n = 12, \hat{\varepsilon}_n = 1.5$ and $\hat{\varepsilon}_{n-1} = 1$ obtain point forecasts for the next 3 periods.

b. Characterize the forecasting function , $f_{t,n}$ and the prediction error variance, ${\sigma^2}_{n+h|n}$ for the long run (when $h \rightarrow \infty$)

21. Given the following estimation outputs what model you think is best to describe and forecast the AIRPASS time series?

M1

Dependent Variable: D(LOG(AIRPASS),1,12)
Method: Least Squares
Sample (adjusted): 1951M02 1961M06
Included observations: 125 after adjustments
Convergence achieved after 8 iterations
MA Backcast: 1950M01 1951M01

Variable	Coefficient	Std. Error	t-Statistic	Prob.
MA(1)	-0.394813	0.082383	-4.792405	0.0000
SMA(12)	-0.640659	0.072003	-8.897624	0.0000

R-squared	0.369398	Mean dependent var	0.000791
Adjusted R-squared	0.364271	S.D. dependent var	0.046431
S.E. of regression	0.037021	Akaike info criterion	-3.738810
Sum squared resid	0.168575	Schwarz criterion	-3.693557
Log likelihood	235.6756	Hannan-Quinn criter.	-3.720426
Durbin-Watson stat	1.934731		

Sample: 1951M02 1961M06
Included observations: 125

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat
		1	0.030	0.030	0.1118
		2	0.021	0.020	0.1679
		3	-0.144	-0.146	2.8815
		4	-0.124	-0.118	4.8944
		5	0.036	0.050	5.0683
		6	0.038	0.022	5.2572
		7	-0.051	-0.093	5.6111
		8	-0.050	-0.054	5.9560
		9	0.090	0.123	7.0687
		10	-0.081	-0.102	7.9664
		11	0.027	-0.016	8.0647
		12	0.018	0.053	8.1093
		13	0.055	0.069	8.5337
		14	0.034	-0.010	8.6955
		15	0.046	0.049	9.0060

Forecast: AIRPASSF
Actual: AIRPASS
Forecast sample: 1961M07 1961M12
Included observations: 6

Root Mean Squared Error	10.43481
Mean Absolute Error	10.19416
Mean Abs. Percent Error	2.071642

M2

Dependent Variable: D(LOG(AIRPASS),1,12)
Method: Least Squares
Sample (adjusted): 1951M02 1961M06
Included observations: 125 after adjustments
Convergence achieved after 10 iterations
MA Backcast: 1949M11 1951M01

Variable	Coefficient	Std. Error	t-Statistic	Prob.
MA(1)	-0.388933	0.082464	-4.716423	0.0000
MA(3)	-0.190940	0.083017	-2.300027	0.0231
SMA(12)	-0.675069	0.069870	-9.661783	0.0000

R-squared	0.392082	Mean dependent var	0.000791
Adjusted R-squared	0.382116	S.D. dependent var	0.046431
S.E. of regression	0.036497	Akaike info criterion	-3.759444
Sum squared resid	0.162511	Schwarz criterion	-3.691564
Log likelihood	237.9652	Hannan-Quinn criter.	-3.731868
Durbin-Watson stat	1.959840		

Sample: 1951M02 1961M06
Included observations: 125

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat
		1	0.015	0.015	0.0287
		2	0.025	0.025	0.1121
		3	0.033	0.032	0.2537
		4	-0.083	-0.084	1.1514
		5	0.064	0.065	1.6894
		6	0.067	0.069	2.2913
		7	-0.081	-0.083	3.1658
		8	-0.050	-0.063	3.4986
		9	0.103	0.120	4.9397
		10	-0.087	-0.079	5.9877
		11	0.024	-0.002	6.0648
		12	0.049	0.050	6.4049
		13	0.017	0.058	6.4473
		14	0.037	-0.001	6.6433
		15	0.039	0.025	6.8612

Forecast: AIRPASSF
Actual: AIRPASS
Forecast sample: 1961M07 1961M12
Included observations: 6

Root Mean Squared Error	10.57776
Mean Absolute Error	8.880508
Mean Abs. Percent Error	1.783386

22. Suppose that the last five observations of a given time series are:

$x_{96} = 60.4$, $x_{97} = 58.9$, $x_{98} = 64.7$ $x_{99} = 70.4$ and $x_{100} = 62.6$.

Obtain the forecasts of the next four observations for the following estimated models where $\varepsilon_t \sim WN(0, \sigma_\varepsilon^2)$:

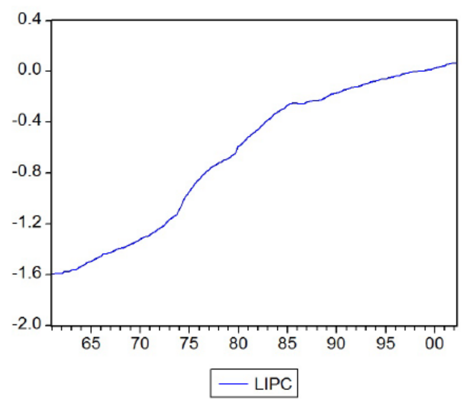
i. $(1 - 0.43L)(1 - L)x_t = \varepsilon_t$

ii. $x_t = 0.2 + 1.8x_{t-1} - 0.81x_{t-2} + \varepsilon_t$

iii. $(1 - 1.4L + 0.8L^2)(1 - L)x_t = \varepsilon_t$

Exercise 4 (chapter 10 Textbook)

23. This exercise makes use of quarterly data from the Belgium Consumer Price Index. The data has been seasonally adjusted and covers the period 1961Q01-2002Q02. In the next figure you will find the time series plot, in logs. The time series will be denoted mathematically as $log\ (IPC_t)$ and in EViews output by LIPC.



a. Before proposing a model, the practitioner needs to study the stationary properties of the data. A possibility is to apply the Augmented Dickey- Fuller (ADF) test to the log of the time series of interest. What made the practitioner apply the log transformation to the time series? And what are the issues that the practitioner needs to be worry to apply the ADF correctly? Justify your answers

b. According to all the figures given below apply ADF tests to the time series $log\ (IPC_t)$. In particular, indicate the null and alternative hypotheses, estimated equations, test statistics, critical regions.

Exogenous: Constant, Linear Trend
Lag Length: 2 (Automatic based on SIC, MAXLAG=13)

		t-Statistic
Augmented Dickey-Fuller test statistic		-0.171180
Test critical values:	1% level	-4.015341
	5% level	-3.437629
	10% level	-3.143037

Augmented Dickey-Fuller Test Equation
Dependent Variable: D(LIPC)
Method: Least Squares
Date: 03/26/13 Time: 10:12
Sample(adjusted): 1961:4 2002:2
Included observations: 163 after adjusting endpoints

Variable	Coefficient	Std. Error	t-Statistic	Prob.
LIPC(-1)	-0.000698	0.004079	-0.171180	0.8643
D(LIPC(-1))	0.389586	0.074688	5.216175	0.0000
D(LIPC(-2))	0.340018	0.075088	4.528247	0.0000
C	0.003237	0.007059	0.458514	0.6472
@TREND(1961:1)	-1.11E-05	5.02E-05	-0.221045	0.8253

R-squared	0.486964	Mean dependent var	0.010180
Adjusted R-squared	0.473976	S.D. dependent var	0.008596
S.E. of regression	0.006235	Akaike info criterion	-7.287211
Sum squared resid	0.006141	Schwarz criterion	-7.192311
Log likelihood	598.9077	F-statistic	37.49267
Durbin-Watson stat	2.148955	Prob(F-statistic)	0.000000

(a) ADF test with constant and trend

Exogenous: Constant
Lag Length: 2 (Automatic based on SIC, MAXLAG=13)

		t-Statistic
Augmented Dickey-Fuller test statistic		-1.783204
Test critical values:	1% level	-3.470679
	5% level	-2.879155
	10% level	-2.576241

Augmented Dickey-Fuller Test Equation
Dependent Variable: D(LIPC)
Method: Least Squares
Date: 03/26/13 Time: 10:09
Sample(adjusted): 1961:4 2002:2
Included observations: 163 after adjusting endpoints

Variable	Coefficient	Std. Error	t-Statistic	Prob.
LIPC(-1)	-0.001578	0.000885	-1.783204	0.0765
D(LIPC(-1))	0.391527	0.073948	5.294619	0.0000
D(LIPC(-2))	0.343006	0.073640	4.657877	0.0000
C	0.001689	0.000900	1.877070	0.0623

R-squared	0.486805	Mean dependent var	0.010180
Adjusted R-squared	0.477123	S.D. dependent var	0.008596
S.E. of regression	0.006216	Akaike info criterion	-7.299172
Sum squared resid	0.006143	Schwarz criterion	-7.223251
Log likelihood	598.8825	F-statistic	50.27468
Durbin-Watson stat	2.151675	Prob(F-statistic)	0.000000

(b) ADF test with constant

Exercises 8 and 9 (chapter 10 Textbook)

24. Consider the process ARIMA(1,1,0) with $\phi = 0.9$. Given $y_n = 100$ and $y_{n-1} = 120$ obtain point forecasts for the next two periods. Comment on the behavior of the forecasting function $f_{t,n}$ and the prediction error variance, $\sigma^2{}_{n+h|n}$ for the long run (when $h \to \infty$) .

25. Given that X_t follows a process ARMA(0,2,1) with $\theta = 0.9$ and that the last observed data is $X_t = 500, X_{t-1} = 490,\ \hat{\varepsilon}_t = -10$ and $\hat{\varepsilon}_{t-1} = 2$ obtain point forecasts for the next 4 periods.

26. According to the simple Keynesian model, the relation between consumption, C_t, and disposable income, Y_t , can be represented by a linear function:

$$\log(C_t) = \beta_0 + \beta_1 \log(Y_t) + \varepsilon_t \quad (1)$$

Where β_1 is the marginal propensity to consume, a quantity of substantial interest and $\varepsilon_t \sim \mathrm{WN}(0, \sigma_\varepsilon^2)$:

In the next figures you will find the graphical representation of the series of Consumption Expenditure (left) and Disposable Income (right) in the US, at constant prices and in logs and the estimation EViews output of equation (1).

Dependent Variable: LRCE
Method: Least Squares
Date: 11/07/12 Time: 14:50
Sample: 1995M01 2012M09
Included observations: 213

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-0.341302	0.056125	-6.081114	0.0000
LRDI	1.029216	0.006187	166.3432	0.0000

R-squared	0.992432	Mean dependent var	8.993582
Adjusted R-squared	0.992396	S.D. dependent var	0.144663
S.E. of regression	0.012615	Akaike info criterion	-5.898585
Sum squared resid	0.033576	Schwarz criterion	-5.867024
Log likelihood	630.1994	Hannan-Quinn criter.	-5.885830
F-statistic	27670.05	Durbin-Watson stat	0.353176
Prob(F-statistic)	0.000000		

a. Using the available information, what can you conclude from the marginal propensity to consume? Motivate your answer.
b. The following figure depicts the result of the ADF test applied to the time series $\log(C_t)$. For this time series should you apply the ADF test with a constant and a trend or only the constant term? Justify your answer.
c. According to the EViews output, is it possible to conclude that $\log(C_t)$ has a unit root? Indicate the null and alternative hypotheses, test statistic, significance level, critical value and the test conclusion.

Null Hypothesis: LRCE has a unit root
Exogenous: Constant, Linear Trend
Lag Length: 1 (Automatic - based on SIC, maxlag=14)

		t-Statistic
Augmented Dickey-Fuller test statistic		-0.616883
Test critical values:	1% level	-4.002142
	5% level	-3.431265
	10% level	-3.139292

*MacKinnon (1996) one-sided p-values.

Augmented Dickey-Fuller Test Equation
Dependent Variable: D(LRCE)
Method: Least Squares
Date: 11/07/12 Time: 16:05
Sample (adjusted): 1995M03 2012M09
Included observations: 211 after adjustments

Variable	Coefficient	Std. Error	t-Statistic
LRCE(-1)	-0.003970	0.006436	-0.616883
D(LRCE(-1))	-0.228968	0.067366	-3.398880
C	0.039599	0.056317	0.703143
@TREND(1995M01)	-1.04E-05	1.52E-05	-0.683498

R-squared	0.113574	Mean dependent var
Adjusted R-squared	0.100727	S.D. dependent var
S.E. of regression	0.003699	Akaike info criterion
Sum squared resid	0.002832	Schwarz criterion
Log likelihood	884.1650	Hannan-Quinn criter.
F-statistic	8.840660	Durbin-Watson stat
Prob(F-statistic)	0.000015	

Exercises 5 and 6 (chapter 13 Textbook)

Exercises 5 and 6 (chapter 14 Textbook)

27. For each of the following ARCH/GARCH models decide if they are stationary , compute the unconditional variance and obtain the prediction of the conditional variance for the next three periods:

a. $\sigma_t^2 = \alpha_0 + 0.65e_{t-1}^2 + 0.25e_{t-2}^2 + 0.10e_{t-3}^2$

b. $\sigma_t^2 = \alpha_0 + 0.20e_{t-1}^2 + 0.20e_{t-2}^2 + 0.50\sigma_{t-1}^2$

c. $\sigma_t^2 = \alpha_0 + 0.10e_{t-1}^2 + 0.20e_{t-2}^2 + 0.60e_{t-3}^2$

d. $\sigma_t^2 = \alpha_0 + 0.10e_{t-1}^2 + 0.90\sigma_{t-1}^2$

28. Given the correlograms, presented above, of the log returns of a financial series what stylized characteristics can be observed? Define the order of an ARCH model to fit the conditional variance of the series of returns.

Correlogram of $(1-B)\ln(y_t)$

Included observations: 952

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat
		1	-0.001	-0.001	0.0015
		2	0.007	0.007	0.0436
		3	0.037	0.037	1.3636
		4	0.042	0.042	3.0663
		5	0.059	0.059	6.4413
		6	-0.041	-0.042	8.0183
		7	0.027	0.023	8.7098
		8	-0.018	-0.024	9.0378
		9	-0.022	-0.025	9.5155
		10	-0.018	-0.020	9.8171
		11	0.041	0.046	11.401
		12	-0.084	-0.085	18.152
		13	-0.002	0.005	18.157
		14	0.027	0.027	18.843
		15	0.052	0.057	21.416
		16	-0.030	-0.030	22.262
		17	-0.010	0.001	22.369
		18	-0.008	-0.025	22.424
		19	-0.058	-0.059	25.674
		20	0.042	0.040	27.426

Correlogram of squared $(1-B)\ln(y_t)$

Included observations: 952

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat
		1	0.011	0.011	0.1093
		2	0.126	0.126	15.376
		3	0.124	0.124	30.202
		4	0.097	0.083	39.244
		5	0.162	0.138	64.519
		6	0.057	0.028	67.642
		7	0.060	0.008	71.051
		8	0.081	0.033	77.313
		9	0.029	-0.011	78.122
		10	0.062	0.016	81.842
		11	0.048	0.021	84.098
		12	0.067	0.042	88.450
		13	0.076	0.049	94.102
		14	0.046	0.022	96.122
		15	0.091	0.056	104.08
		16	0.002	-0.036	104.08
		17	0.073	0.026	109.27
		18	0.003	-0.037	109.28
		19	0.087	0.055	116.63
		20	0.077	0.053	122.48

29. True or False? Correct the sentence and justify when appropriate.
    a. Volatility clustering is one of the most prominent features of financial returns. Time series analysis reproduces this stylized fact using the ARMA model with white noise errors.
    b. An ARCH(2) model is equivalent to an AR(2) model for the squared returns.
    c. A GARCH(1,1) model is equivalent to a MA(2) model for the squared returns.
    d. The ARMA-GARCH model only generates forecasts for the variance.
    e. Usually the final ARMA-GARCH model uses the same ARMA model that was fitted before modelling the volatility.
    f. The GARCH model is able to describe adequately the dynamic properties of volatility of standard financial time series with less parameters than the ARCH model.

30. Consider the estimation output presented below:

.

Dependent Variable: D(LOG(PB))
Method: ML - ARCH (Marquardt) - Normal distribution

Sample (adjusted): 1/04/2000 4/04/2011
Included observations: 2935 after adjustments
Convergence achieved after 10 iterations
Presample variance: backcast (parameter = 0.7)
GARCH = C(2) + C(3)*RESID(-1)^2 + C(4)*RESID(-2)^2 + C(5)*RESID(-3)^2
+ C(6)*RESID(-4)^2 + C(7)*RESID(-5)^2 + C(8)*RESID(-6)^2

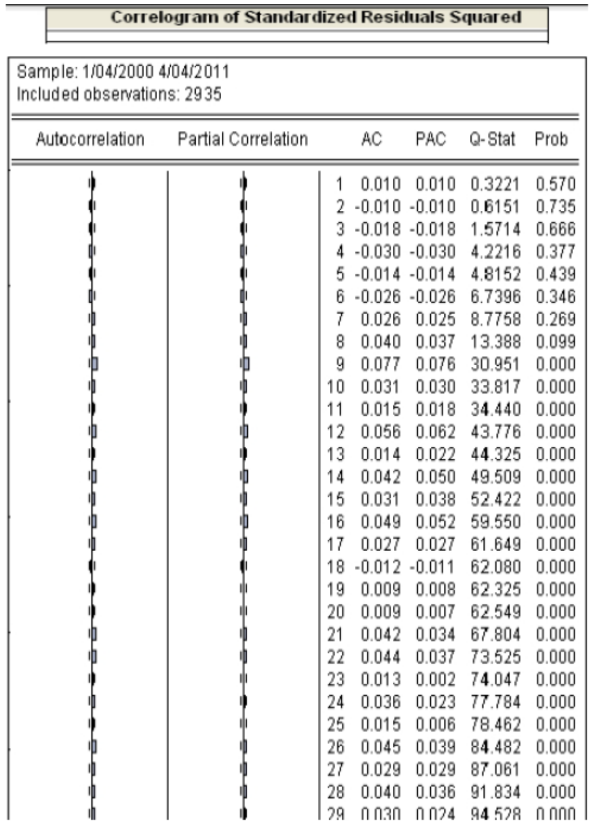
Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	0.001321	0.000396	3.337769	0.0008
		Variance Equation		
C	0.000280	1.41E-05	19.87422	0.0000
RESID(-1)^2	0.089043	0.014307	6.223588	0.0000
RESID(-2)^2	0.068808	0.017353	3.965242	0.0001
RESID(-3)^2	0.082096	0.017453	4.703768	0.0000
RESID(-4)^2	0.100291	0.016906	5.932440	0.0000
RESID(-5)^2	0.092199	0.019404	4.751628	0.0000
RESID(-6)^2	0.054665	0.016496	3.313866	0.0009

R-squared	-0.001094	Mean dependent var	0.000543
Adjusted R-squared	-0.003488	S.D. dependent var	0.023535
S.E. of regression	0.023576	Akaike info criterion	-4.754119
Sum squared resid	1.626930	Schwarz criterion	-4.737807
Log likelihood	6984.670	Hannan-Quinn criter.	-4.748245
Durbin-Watson stat	1.952501		

a. Write explicitly the estimated equation.

b. Obtain the estimate for the unconditional variance of the error of the series.

c. Comment on the correlogram of the standardized squared residuals presented below.



Correlogram of Standardized Residuals Squared

Sample: 1/04/2000 4/04/2011
Included observations: 2935

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.010	0.010	0.3221	0.570
		2	-0.010	-0.010	0.6151	0.735
		3	-0.018	-0.018	1.5714	0.666
		4	-0.030	-0.030	4.2216	0.377
		5	-0.014	-0.014	4.8152	0.439
		6	-0.026	-0.026	6.7396	0.346
		7	0.026	0.025	8.7758	0.269
		8	0.040	0.037	13.388	0.099
		9	0.077	0.076	30.951	0.000
		10	0.031	0.030	33.817	0.000
		11	0.015	0.018	34.440	0.000
		12	0.056	0.062	43.776	0.000
		13	0.014	0.022	44.325	0.000
		14	0.042	0.050	49.509	0.000
		15	0.031	0.038	52.422	0.000
		16	0.049	0.052	59.550	0.000
		17	0.027	0.027	61.649	0.000
		18	-0.012	-0.011	62.080	0.000
		19	0.009	0.008	62.325	0.000
		20	0.009	0.007	62.549	0.000
		21	0.042	0.034	67.804	0.000
		22	0.044	0.037	73.525	0.000
		23	0.013	0.002	74.047	0.000
		24	0.036	0.023	77.784	0.000
		25	0.015	0.006	78.462	0.000
		26	0.045	0.039	84.482	0.000
		27	0.029	0.029	87.061	0.000
		28	0.040	0.036	91.834	0.000
		29	0.030	0.024	94.528	0.000

32. Suppose that the return series of a given stock, r_t, is well described by the following model:

$$r_t = \varepsilon_t = \sigma_t z_t\,, \qquad z_t \overset{iid}{\rightarrow} D(0,1)$$

$$\sigma_t^2 = 1 + 0.4\varepsilon_t^2 + 0.2\varepsilon_{t-1}^2 + 0.3\sigma_{t-1}^2$$

a. Derive the forecast equations that are used to obtain the forecasts, $\sigma_{T+s|T}^2$ with origin at time T.

b. Suppose that the last two observations for the returns are $r_{T-1} = 0.03$ and $r_T = 0.06$. Using $\sigma_T^2 = 1$ obtain the forecasts for $\sigma_{T+1}^2, \sigma_{T+2}^2$ and σ_{T+3}^2 with origin at time T.