Advanced Macroeconomics

PhD in Economics

Lisbon School of Economics and Management (ISEG)

The decentralized economy

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2. Consumption
   - The Consumption Decision
   - The Intertemporal Budget Constraint
   - Interpreting the Euler Equation
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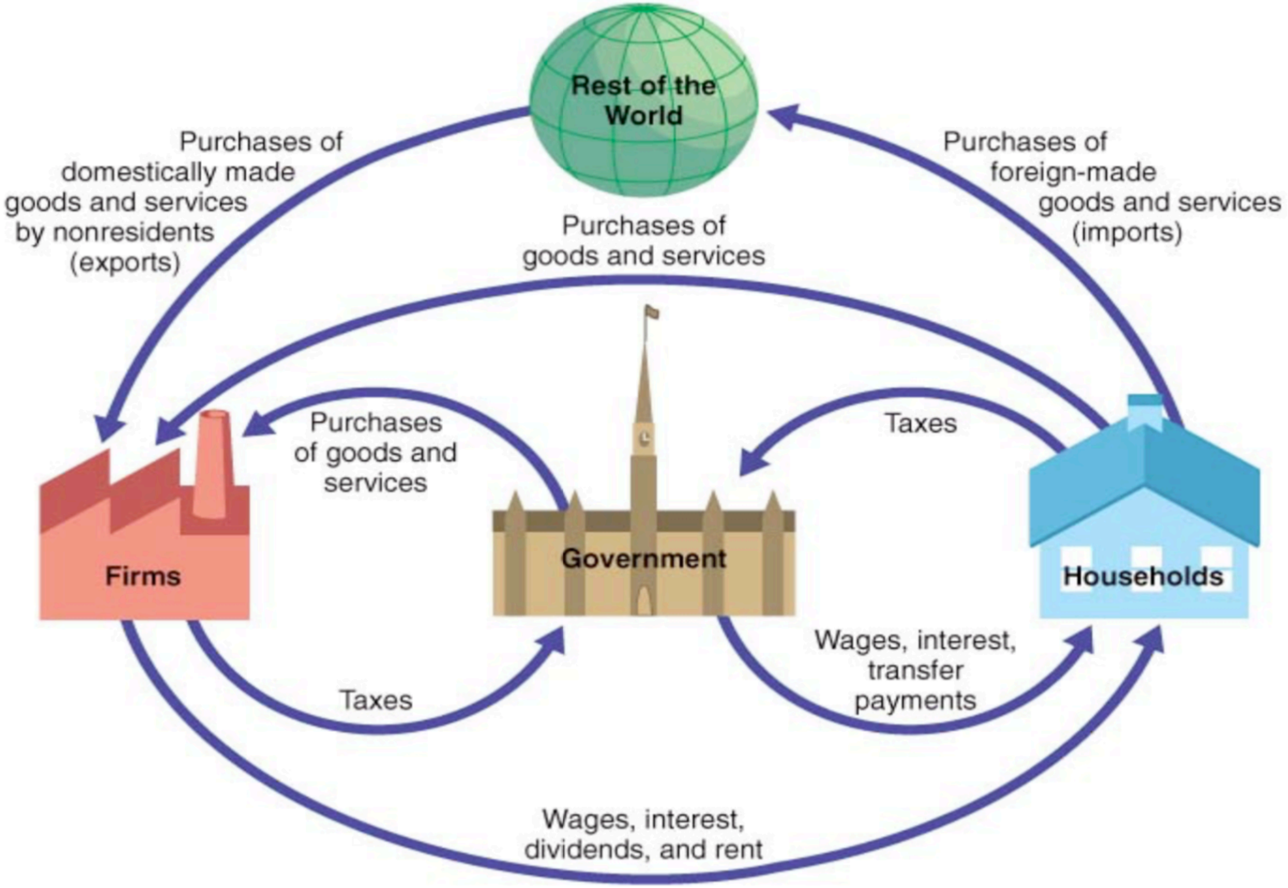
Households and firms

Instead of a social planner making all decisions, a more realistic model involves households and firms making their own decisions with different objectives.



Consumption

- Consumption makes up for around 60 percent of total aggregate spending
- Important to understand its determinants
- Fortunately, there is lots of data on consumption
    - Aggregate data
    - Household level data
- Tests based on Euler equations and tests based on consumption functions

Intertemporal utility maximization problem

- The representative household's problem is

$$\max_{c_{t+s}, a_{t+s+1}} \sum_{s=0}^{\infty} \beta^s U(c_{t+s}), \qquad (4.1)$$

subject to the budget constraints

$$\Delta a_{t+s+1} + c_{t+s} = x_{t+s} + r_{t+s} a_{t+s}, \quad s = 0, 1, \ldots, \qquad (4.2)$$

where a_t, x_t, and r_t are exogenous asset, income, and interest rate respectively in period t. The choice variables are c_t and a_{t+1}.

- Once c_t is chosen, a_{t+1} is determined by the budget constraint. Therefore the household is effectively choosing the consumption path $\{c_t, c_{t+1}, c_{t+2}, \ldots\}$.

Optimization

The Lagrangian is

$$\mathcal{L}_t = \sum_{s=0}^{\infty} \left\{ \beta^s U(c_{t+s}) + \lambda_{t+s}[x_{t+s} + (1 + r_{t+s})a_{t+s} - c_{t+s} - a_{t+s+1}] \right\} \tag{4.3}$$

The first-order conditions are

$$\begin{aligned} \frac{\partial \mathcal{L}_t}{\partial c_{t+s}} &= \beta^s U'(c_{t+s}) - \lambda_{t+s} = 0, \quad s \geq 0, \\ \frac{\partial \mathcal{L}_t}{\partial a_{t+s}} &= \lambda_{t+s}(1 + r_{t+s}) - \lambda_{t+s-1} = 0, \quad s \geq 1, \end{aligned}$$

and the budget constraint (4.2). The Euler equation is

$$\frac{\beta U'(c_{t+1})}{U'(c_t)}(1 + r_{t+1}) = 1. \tag{4.4}$$

Two-period budget (another way of expressing the households problem)

- Consider the budget constraints in periods t and $t+1$:

$$\begin{aligned} a_{t+1} + c_t &= x_t + (1+r_t)a_t, \\ a_{t+2} + c_{t+1} &= x_{t+1} + (1+r_{t+1})a_{t+1}. \end{aligned}$$

- The two equations can be combined to eliminate a_{t+1} to give

$$\frac{a_{t+2}}{1+r_{t+1}} + \frac{c_{t+1}}{1+r_{t+1}} + c_t = \frac{x_{t+1}}{1+r_{t+1}} + x_t + (1+r_t)a_t. \qquad (4.5)$$

- The left-hand side is the present value of total expenditure in consumption and investment.
- The right-hand side is the present value of total incomes.

Extension to n-1 periods

- By successive substitution, the present value of total expenditure is

$$W_t = \frac{a_{t+n}}{\prod_{s=1}^{n-1}(1 + r_{t+s})} + \sum_{s=1}^{n-1} \frac{c_{t+s}}{\prod_{u=1}^{s}(1 + r_{t+u})} + c_t. \tag{4.7}$$

- Similarly, the present value of total income is

$$W_t = \sum_{s=1}^{n-1} \frac{x_{t+s}}{\prod_{u=1}^{s}(1 + r_{t+u})} + x_t + (1 + r_t)a_t. \tag{4.8}$$

- The infinite horizon budget constraint, assuming that interest rate is constant at r, is obtained as $n \to \infty$:

$$W_t = \sum_{s=0}^{\infty} \frac{c_{t+s}}{(1 + r)^s} = \sum_{s=0}^{\infty} \frac{x_{t+s}}{(1 + r)^s} + (1 + r)a_t. \tag{4.9}$$

Permanent income

- W_t is the present value of life-time income of the household and is often called permanent income.
- Equation (4.9) can be used a the alternative single budget constraint in the optimization problem.
- One more necessary condition called the transversality condition:

$$\lim_{s\to\infty} \beta^s a_{t+s} U'(c_{t+s}) = 0. \tag{4.10}$$

This ensures that the terminal value of the asset has no utility. In particular, the household cannot finance extra consumption indefinitely by borrowing ($a_t < 0$). Therefore it is sometimes called "no-Ponzi-scheme". See Kamihigashi (2006) for details.

Intertemporal substitution

The Euler equation has the usual meaning that it prescribes the intertemporal substitution of consumption between to consecutive periods.

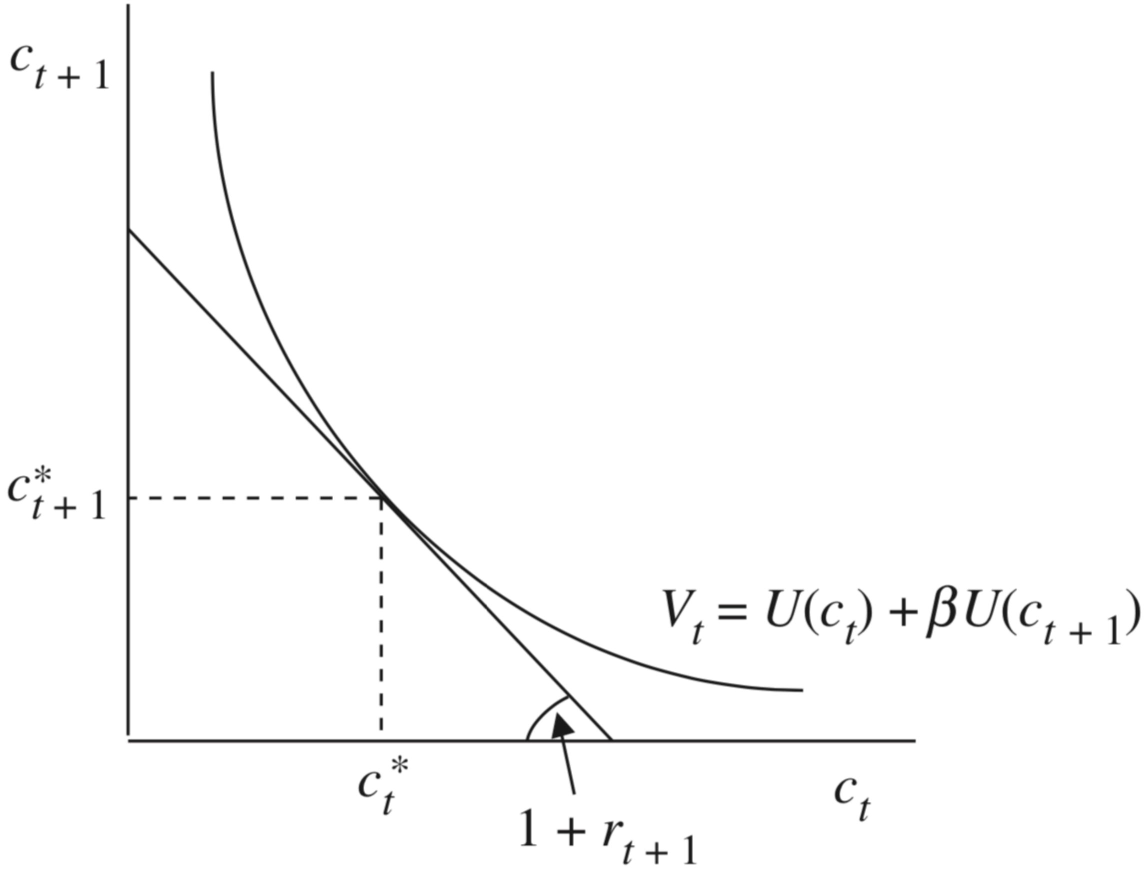


Figure 4.1. Two-period solution.

An increase in the interest rate (assuming $a_t = 0$)

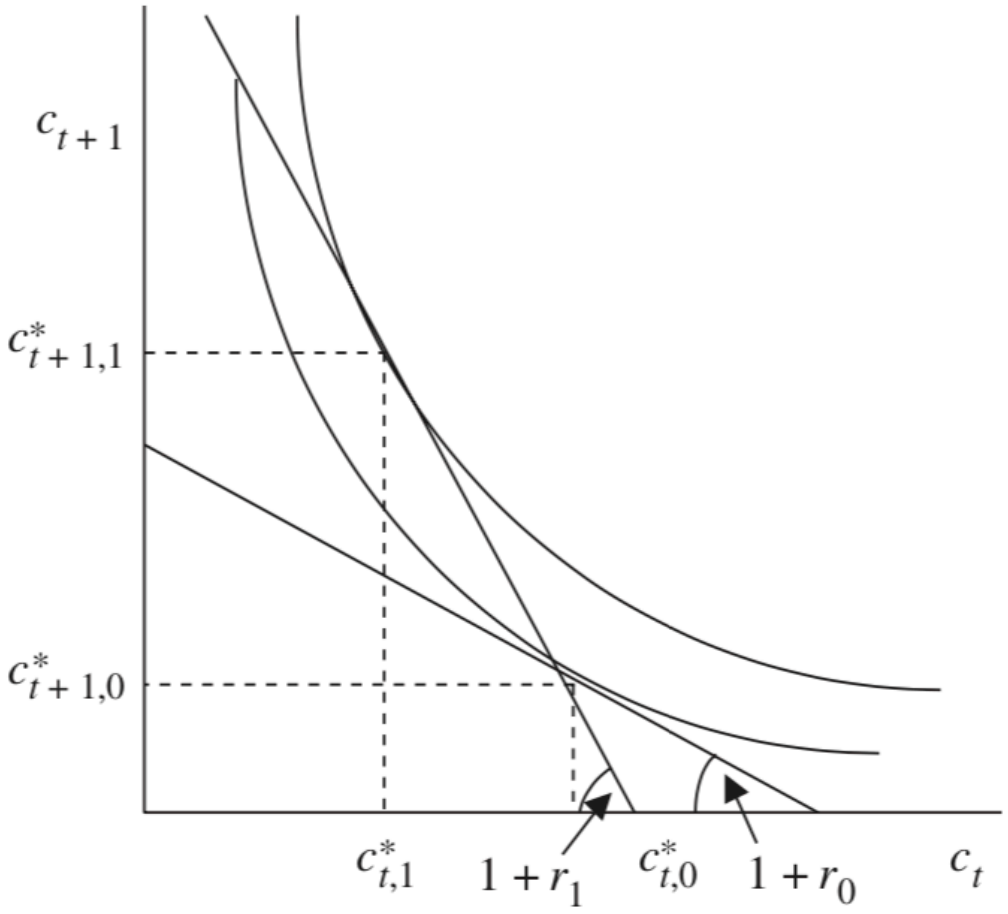


Figure 4.2. The effect of an increase in the interest rate.

Optimal consumption path

- Using a first-order Taylor approximation of $U'(c_{t+1})$ about c_t, we have

$$\frac{U'(c_{t+1})}{U'(c_t)} \simeq \frac{U'(c_t) + U''(c_t)(c_{t+1} - c_t)}{U'(c_t)}$$
$$= 1 + \frac{U''(c_t)}{U'(c_t)}\Delta c_{t+1} = 1 - \sigma \frac{\Delta c_{t+1}}{c_t}, \tag{4.15}$$

  where $\sigma = -cU''/U'$ is the coefficient of relative risk aversion (see Jehle and Reny, 2011, p. 123).
- Putting (4.15) into the Euler equation (4.4) gives

$$\frac{\Delta c_{t+1}}{c_t} = \frac{1}{\sigma}\left[1 - \frac{1}{\beta(1 + r_{t+1})}\right] = \frac{r_{t+1} - \theta}{\sigma(1 + r_{t+1})} \tag{4.16}$$

- If $r = \theta$, consumption growth is zero.

Consumption function

From (4.9) with constant interest rate and the steady-state consumption c_t, we have

$$\begin{aligned} W_t &= \sum_{s=0}^{\infty} \frac{c_{t+s}}{(1+r)^s} = \frac{1+r}{r} c_t \\ &= \sum_{s=0}^{\infty} \frac{x_{t+s}}{(1+r)^s} + (1+r) a_t \end{aligned}$$

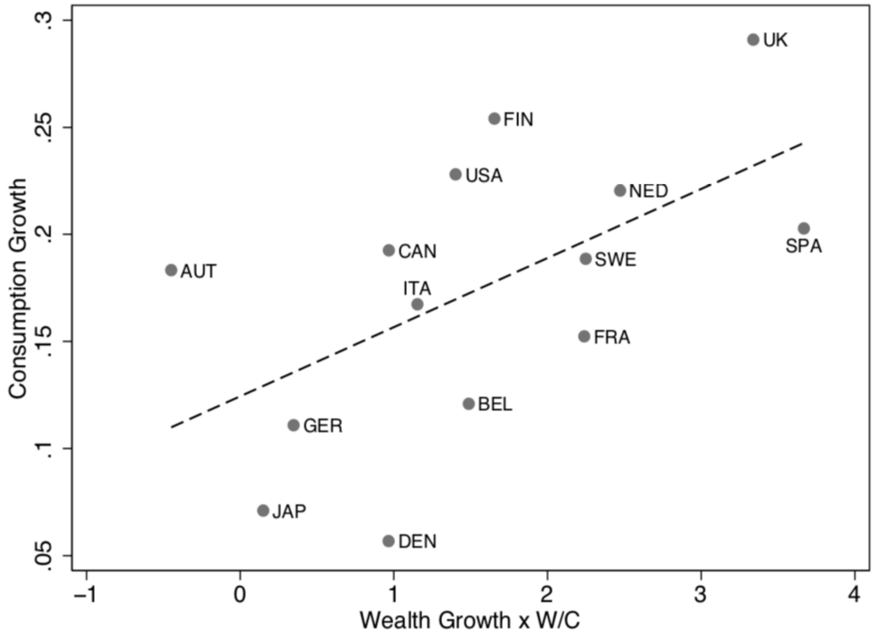
Therefore the consumption function is

$$c_t = \frac{r}{1+r} W_t = r \sum_{s=0}^{\infty} \frac{x_{t+s}}{(1+r)^{s+1}} + r a_t. \qquad (4.17)$$

This means that in steady state consumption in each period is a fixed portion of total wealth.

Empirical evidence

Figure 1: Consumption Growth and Wealth Growth 1994–2002



Note: Consumption growth and rescaled wealth growth between 1994Q4 and 2002Q4; wealth growth is rescaled by multiplying with the wealth–consumption ratio of 1994Q4. Slope of the regression line, $\text{MPC}_w^{\text{LR}} = 0.032$, t-statistic: 2.36, p-value: 0.018.

Source: Jiri Slacalek (2009) "What Drives Personal Consumption? The Role of Housing and Financial Wealth," *The B.E. Journal of Macroeconomics*, 9(1), Article 37.

Permanent and temporary income shocks

- Suppose income x_t is constant in every period. Then (4.17) becomes

$$c_t = x_t + ra_t. \tag{4.18}$$

  A permanent shock in income will therefore have the same effect on consumption, which is the Keynesian idea of marginal propensity to consume (MPC) equals to 1.

- To see the effect of a temporary shock, rewrite (4.17) as

$$c_t = \frac{r}{1+r}x_t + \frac{r}{(1+r)^2}x_{t+1} + \cdots + ra_t.$$

  A temporary income shock in period t, therefore only have a MPC equals to $r/(1+r)$.

- Therefore the effects of a permanent tax cut and a temporary tax cut are very different.

Permanent and temporary interest rate shocks

1. Permanent shocks — Recall equation (4.18):

$$c_t = x_t + ra_t.$$

Consumption therefore changes depending on the household has asset ($a_t > 0$) or debt ($a_t < 0$). MPC $= 1$.

2. Temporary shocks — Recall equation (4.8):

$$W_t = \sum_{s=1}^{n-1} \frac{x_{t+s}}{\prod_{u=1}^{s}(1 + r_{t+u})} + x_t + (1 + r_t)a_t.$$

The effect of a temporary change in period t again depends on whether a_t is positive or negative. The effect on consumption, however, is small due to intertemporary substitution. MPC $= r/(1 + r)$.

3. In practice, consumption depends of expectations of future asset returns. See Shiller (2009).

Anticipated and unanticipated income shocks

- Confusion about (4.16) and (4.17):

$$\frac{\Delta c_{t+1}}{c_t} = \frac{r_{t+1} - \theta}{\sigma(1 + r_{t+1})},$$

$$c_t = r \sum_{s=0}^{\infty} \frac{x_{t+s}}{(1+r)^{s+1}} + r a_t.$$

- The first equation does not contain information on income. If $r_{t+1} = \theta$, then $c_{t+1} = c_t$.
- But if income change is anticipated, the information is embedded in c_t. That is ,

$$c_{t+1} = c_t + e_{t+1},$$

where e_{t+1} is a random shock ($E_t[e_{t+1}] = 0$).

- In this case consumption is called a martingale process ($E_t[c_{t+1}] = c_t$).

Saving

For constant interest rate, saving is

$$s_t = x_t + ra_t - c_t.$$

Substitute $ra_t - c_t$ from (4.17),

$$\begin{aligned} s_t &= x_t - r\sum_{s=0}^{\infty}\frac{x_{t+s}}{(1+r)^{s+1}} \\ &= -\frac{r}{1+r}\sum_{s=1}^{\infty}\frac{x_{t+s}-x_t}{(1+r)^s} \\ &= -\sum_{s=1}^{\infty}\frac{\Delta x_{t+s}}{(1+r)^{s+1}} \end{aligned}$$

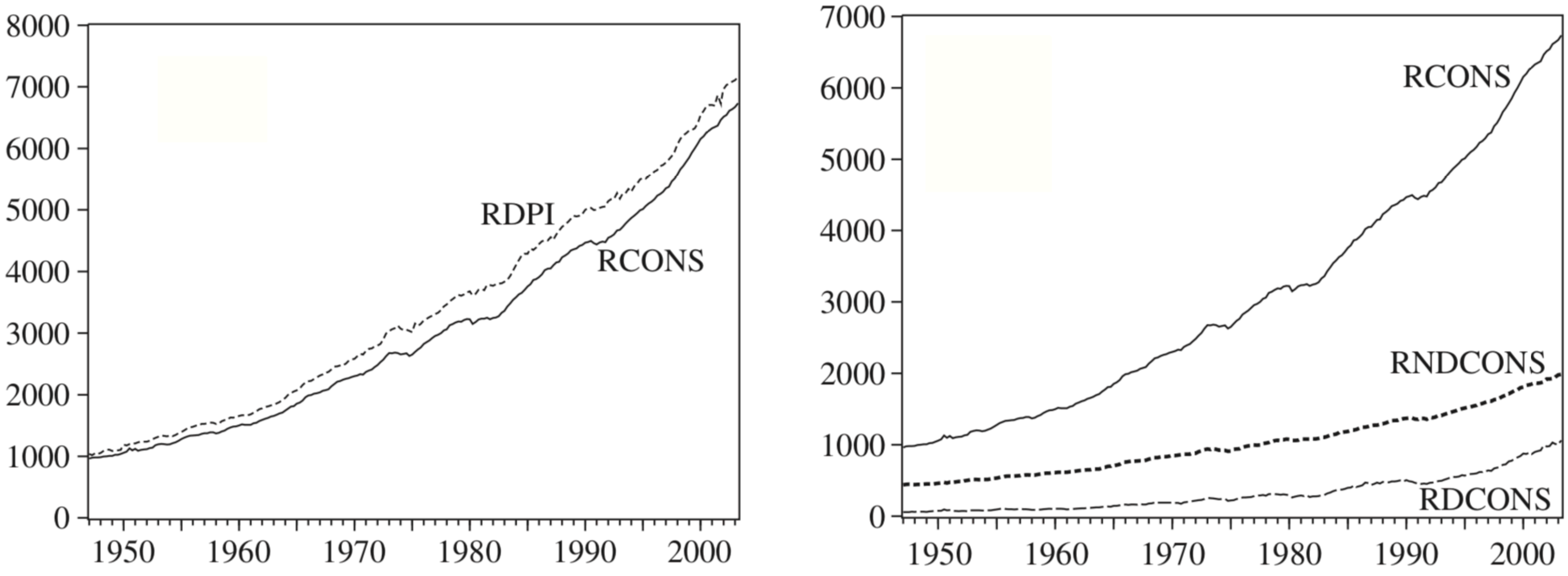
This means that saving is used to offset anticipated future change in income.

Life-Cycle Theory

Up to now we assumed that households are identical and live forever.

- Our models so far do not address the problem of consumer heterogeneity.
- Young households need to borrow, middle-age households want to save, and mature households just spend.
- Our consumption function smooth expenditure throughout the life cycle.
- Assumptions:
  1. Life is infinite and predictable
  2. No borrowing constraint
- What are the empirical evidences?

Income and consumption in the US



RDPI = real disposable income, RCONS = real consumption,
RNDCONS = nondurables, RDCONS = durables

Durables and non-durables

Table 4.1. Standard deviations in growth rates.

	%
Disposable income	1.12
Total consumption	0.74
Nondurables and services	1.22
Nondurables	0.60
Durables	17.13

Source: Census Bureau via Haver Analytics

DURABLE GOODS SHIPMENTS AND ORDERS

	PERCENTAGE CHANGE FIRST HALF '08 to FIRST HALF '09	
MILITARY	SHIPMENTS	ORDERS
Aircraft	+23%	+18%
Other military	+10	– 6
TOTAL MILITARY	**+15**	**+ 3**
NONMILITARY		
Primary metals	–40	–44
Fabricated metals	–12	–18
Machinery	–18	–29
Computers	–18	–20
Communication equipment	–33	–18
Semiconductors	–33	na
Electrical equip. & appliances	–20	–26
Motor vehicles & parts	–30	–31
Aircraft	– 7%	–65%
TOTAL NONMILITARY	**–20**	**–27**

Model of Perpetual Youth

- The probability of death in each period is ρ (independent of age).
- The probability of staying alive is $1 - \rho$.
- The probability of having a life of s periods long is

$$f(s) = \rho(1-\rho)^{s-1}, \quad s = 1, 2, \ldots$$

- Life expectancy is

$$E[s] = \sum_{s=1}^{\infty} s\rho(1-\rho)^{s-1} = \rho^{-1}.$$

- $E[s] \to \infty$ if $\rho \to 0$.

The power of posterity

- Under uncertainty, a household in period t plans ahead with the probability of being alive in period $t+s$ equals to $(1-\rho)^s$.
- Life time expected utility is therefore

$$
\begin{aligned}
E\left[\sum_{s=0}^{\infty} \beta^s U(c_{t+s})\right] &= \sum_{s=0}^{\infty} \beta^s (1-\rho)^s U(c_{t+s}) \\
&= \sum_{s=0}^{\infty} \tilde{\beta}^s U(c_{t+s}),
\end{aligned}
$$

where $\tilde{\beta} = \beta(1-\rho)$.

- Our models still work, with a different interpretation of β.
- See Brooks (2009) for an alternative explanation.

Durable consumption

- Durable goods are similar to capital, providing consumption services over time with depreciation (δ).
- Let D_t be stock of durables, d_t new investment in durables, and c_t consumption of nondurables.
- Durable accumulation equation:

$$D_{t+1} = d_t + (1-\delta)D_t. \tag{4.21}$$

- The household budget constraint is

$$a_{t+1} + c_t + p_t^D d_t = x_t + (1+r_t)a_t,$$

where p_t^D is the price of durables relative to nondurables.

Utility maximization problem

The household's problem is

$$\max \sum_{s=0}^{\infty} \beta^s U(c_{t+s}, D_{t+s})$$

$$\text{subject to} \quad a_{t+s+1} + c_{t+s} + p^D_{t+s}[D_{t+s+1} - (1-\delta)D_{t+s}] = x_{t+s} + (1 + r_{t+s})a_{t+s}. \qquad (4.22)$$

Notes:

- $U(c, D)$ is assumed to be differentiable, increasing, and concave.
- The control variables are c_{t+s}, D_{t+s+1}, and a_{t+s+1}.

First order conditions

The Lagrangian is

$$\mathcal{L}_t = \sum_{s=0}^{\infty} \Big\{ \beta^s U(c_{t+s}, D_{t+s}) + \lambda_{t+s} \big[x_{t+s} + (1 + r_{t+s}) a_{t+s} - c_{t+s} - p^D_{t+s}[D_{t+s+1} - (1-\delta) D_{t+s}] - a_{t+s+1} \big] \Big\}.$$

First-Order Conditions:

$$\frac{\partial \mathcal{L}_t}{\partial c_{t+s}} = \beta^s U_{c,t+s} - \lambda_{t+s} = 0, \quad s \geq 0,$$

$$\frac{\partial \mathcal{L}_t}{\partial D_{t+s}} = \beta^s U_{D,t+s} + \lambda_{t+s} p^D_{t+s} (1-\delta) - \lambda_{t+s-1} p^D_{t+s-1} = 0, \quad s \geq 1,$$

$$\frac{\partial \mathcal{L}_t}{\partial a_{t+s}} = \lambda_{t+s}(1 + r_{t+s}) - \lambda_{t+s-1} = 0, \quad s \geq 1.$$

Euler equation

- The first and the third FOCs give the Euler equation

$$\frac{\beta U_c(c_{t+1}, D_{t+1})}{U_c(c_t, D_t)}(1 + r_{t+1}) = 1,$$

  which relates the intertemporal marginal rate of time preference in nondurable consumption to β and r.
- Notice that it depends on the levels of the durable stock.
- Examples are ice cream and refrigerators, gasoline and cars, Internet services and computers, data plans and smart phones, cable services and HDTVs, fire insurance and houses, gym memberships and degrees etc. (These are all complements, can you think of some substitutes?)

Substitution between durables and non-durables

- Combining all three FOCs gives

$$U_D(c_{t+1}, D_{t+1}) = \frac{U_c(c_t, D_t)}{\beta} \left[p_t^D - \frac{1-\delta}{1+r_{t+1}} p_{t+1}^D \right]. \qquad (4.23)$$

- Equation (4.23) can be written as

$$\frac{\beta U_D(c_{t+1}, D_{t+1})}{U_c(c_t, D_t)} = p_t^D - \frac{1-\delta}{1+r_{t+1}} p_{t+1}^D.$$

- The left hand side defines the marginal rate of substitution between durable in period $t+1$ and nondurable in period t.
- The right hand side is the relative price. The first term is the purchasing price of durable. The second term is resale value in the next period net of depreciation and interest cost.

Parametric analysis

- Let $U(c_t, D_t) = c_t^{\alpha} D_t^{1-\alpha}$.
- The Euler equation is

$$\beta \left(\frac{c_{t+1}/D_{t+1}}{c_t/D_t} \right)^{-(1-\alpha)} (1 + r_{t+1}) = 1. \tag{4.24}$$

- Equation (4.23) becomes

$$\left(\frac{c_{t+1}}{D_{t+1}} \right)^{\alpha} = \frac{\alpha}{\beta(1-\alpha)} \left(\frac{c_t}{D_t} \right)^{\alpha-1} \left[p_t^D - \frac{1-\delta}{1+r_{t+1}} p_{t+1}^D \right]. \tag{4.25}$$

Therefore an increase in interest rate r_{t+1} reduces the purchases of durables D_{t+1} relative to nondurables c_{t+1}.

Steady-state solution

- In the steady state,

$$\begin{aligned} c_{t+1} &= c, \\ D_{t+1} &= D, \\ p^D_{t+1} = p^D_t &= p^D, \\ r_{t+1} &= \theta. \end{aligned}$$

  Hence (4.25) becomes

$$\frac{c}{p^D D} = \frac{\alpha}{1-\alpha}(\theta + \delta).$$

- In terms of investment of durables,

$$\frac{c}{p^D d} = \frac{\alpha}{1-\alpha}\frac{\theta + \delta}{\delta}.$$

Short-run dynamics

In the short-run the stock of durables is fixed. Multiply equation (4.21) by p_t^D / c_t, we get

$$\begin{aligned}\frac{p_t^D d_t}{c_t} &= \frac{p_t^D D_{t+1}}{c_t} - (1-\delta)\frac{p_t^D D_t}{c_t} \\ &= \frac{c_{t+1}}{c_t}\left(\frac{c_t/D_t}{c_{t+1}/D_{t+1}}\right)\left(\frac{p_t^D D_t}{c_t}\right) - (1-\delta)\frac{p_t^D D_t}{c_t} \\ &= \left[\frac{c_{t+1}}{c_t}\left(\frac{c_t/D_t}{c_{t+1}/D_{t+1}}\right) - 1 + \delta\right]\frac{p_t^D D_t}{c_t}\end{aligned}$$

From the Euler equation (4.24) we have

$$\frac{c_t/D_t}{c_{t+1}/D_{t+1}} = [\beta(1+r_{t+1})]^{-1/(1-\alpha)} = \left(\frac{1+r_{t+1}}{1+\theta}\right)^{-1/(1-\alpha)}.$$

Short-run dynamics

In the short-run the stock of durables is fixed. Multiply equation (4.21) by p_t^D/c_t, we get

$$
\begin{aligned}
\frac{p_t^D d_t}{c_t} &= \frac{p_t^D D_{t+1}}{c_t} - (1-\delta)\frac{p_t^D D_t}{c_t} \\
&= \frac{c_{t+1}}{c_t}\left(\frac{c_t/D_t}{c_{t+1}/D_{t+1}}\right)\left(\frac{p_t^D D_t}{c_t}\right) - (1-\delta)\frac{p_t^D D_t}{c_t} \\
&= \left[\frac{c_{t+1}}{c_t}\left(\frac{c_t/D_t}{c_{t+1}/D_{t+1}}\right) - 1 + \delta\right]\frac{p_t^D D_t}{c_t}
\end{aligned}
$$

From the Euler equation (4.24) we have

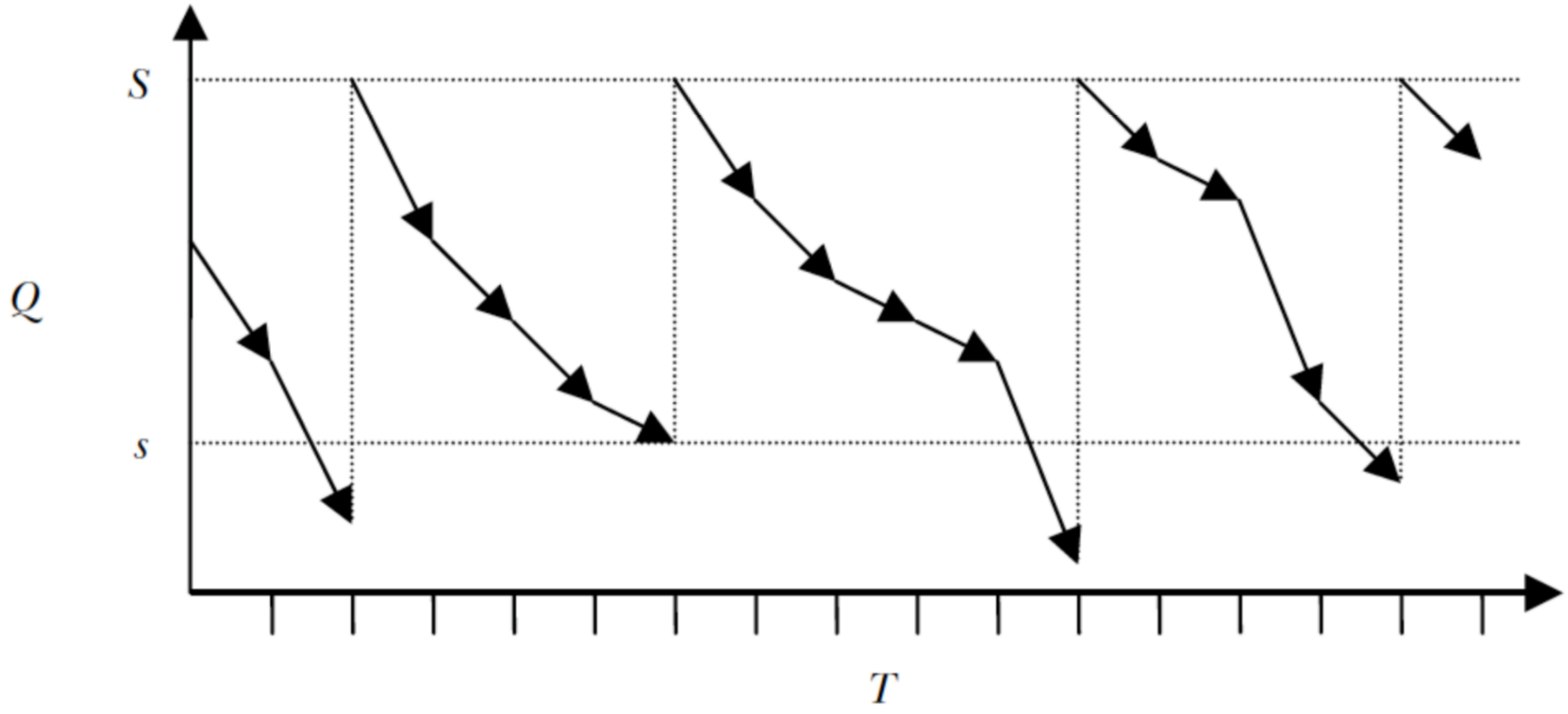
$$
\frac{c_t/D_t}{c_{t+1}/D_{t+1}} = [\beta(1+r_{t+1})]^{-1/(1-\alpha)} = \left(\frac{1+r_{t+1}}{1+\theta}\right)^{-1/(1-\alpha)}.
$$

Empirical evidence

- Under the permanent income hypothesis consumption is a martingale.
- Using the above framework, Mankiw (1982) shows that durable consumption follows an ARMA(1, 1) process.
- He tests the model with U.S. post-war data using an additively separably utility function in c and D, the model was rejected. In particular, the estimated depreciation rate δ is 1.038.
- Other economists suggest that consumers do not constantly adjust their stock of durable goods. For example, we buy houses, cars, computers, etc. only occasionally.
- Purchases of these items follow what is called an (S, s) rule.

(S,s) Policy

Operation of an (*S*, *s*) Policy with Upper and Lower Barriers

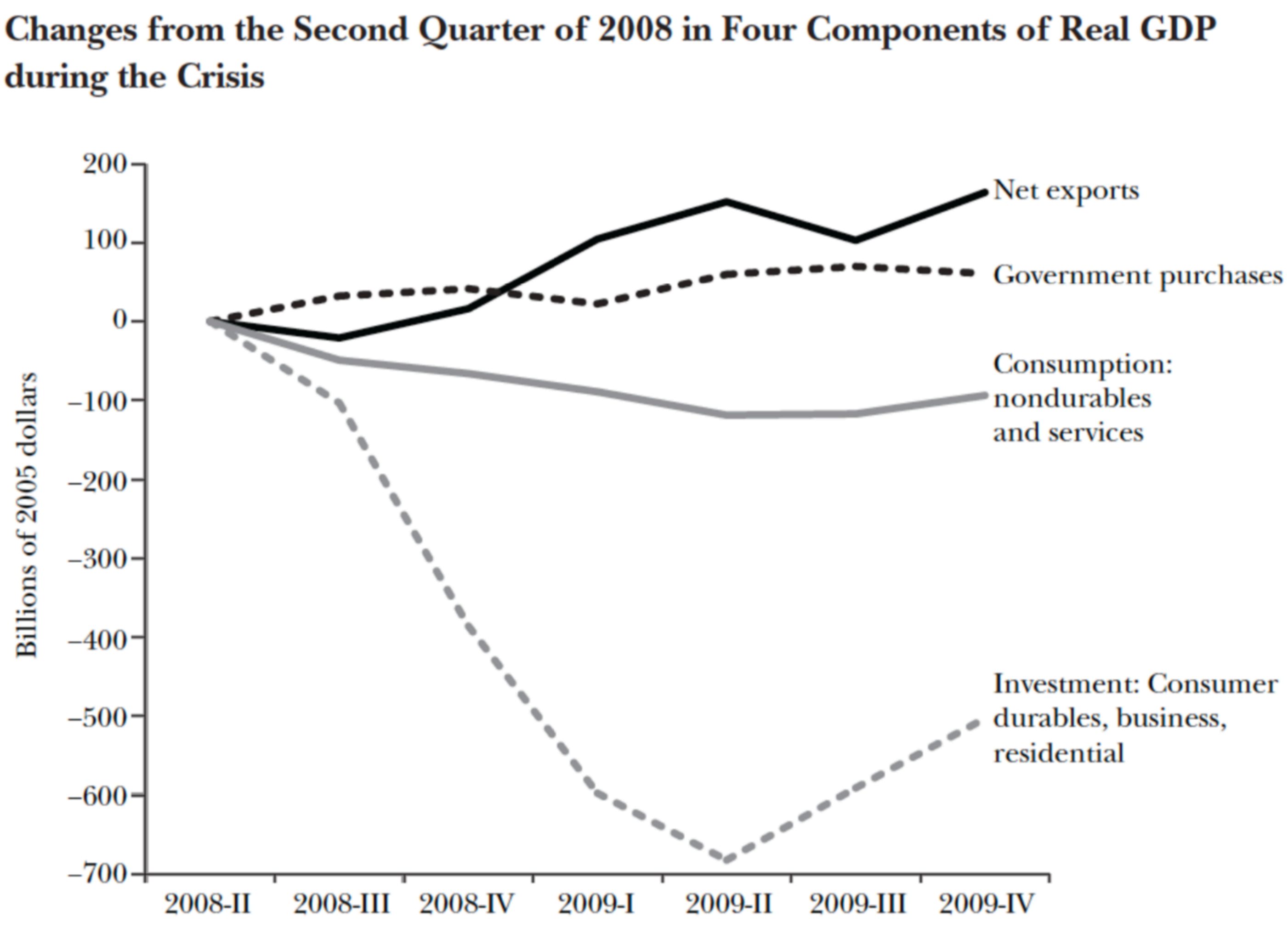


Source: Caplin and Leahy (2010)

Mechanics of the (S,s) model

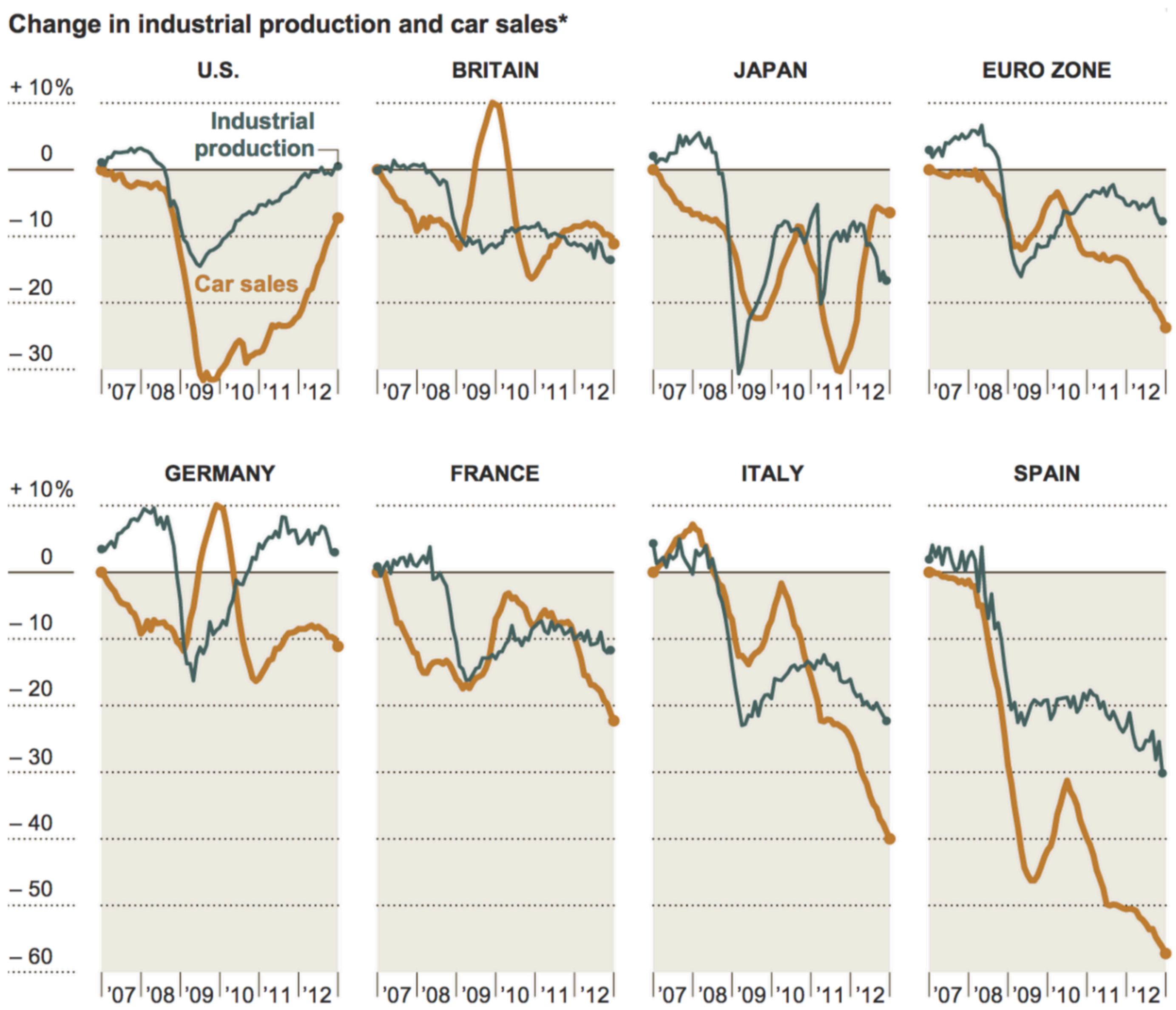
- A household has an optimal level of durable goods within an interval (s, S). But it does not buy new goods until the stock falls below the lower bound s.
- This is due to the presence of a fixed cost when buying new durable goods.
- Some writers argue that the aggregate demand may smooth out the individual household cycles. But the effect of a large aggregate income shock depends on the distribution of existing stock and may create complicated dynamic pattern such as coupled oscillations.
- The (S, s) model has been applied to other problems such as inventory control, money demand, capital investment, marketing, household finance, and monetary policy analysis.
- See Caplin and Leahy (2010) for a survey.

Evidence from the Great Recession



Source: Hall (2010)

Car sales



Change in industrial production and car sales*
U.S.
BRITAIN
JAPAN
EURO ZONE
+ 10%
0
– 10
– 20
– 30
Industrial production
Car sales
'07 '08 '09 '10 '11 '12
GERMANY
FRANCE
ITALY
SPAIN
+ 10%
0
– 10
– 20
– 30
– 40
– 50
– 60
'07 '08 '09 '10 '11 '12

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