Advanced Macroeconomics

PhD in Economics

Lisbon School of Economics and Management (ISEG)

The decentralized economy II: Outline

1. Labour Supply
2. Firms
   - Labour Demand without Adjustment Costs
   - Labour Demand with Adjustment Costs
3. General Equilibrium
   - Household and Firm Budget Constraints
   - Labour Market
   - Goods Market
   - Complete Solution

Household labor supply

Model set-up:

1. Work and leisure: $n_t + l_t = 1$
2. Utility: $U(c_t, l_t) = U(c_t, 1 - n_t)$ is increasing and concave, with $U_c > 0, U_l > 0, U_{cc} \leq 0, U_{ll} \leq 0, U_{n,t} = -U_{l,t}$
3. Wage rate $= w_t$, other income $= x_t$, interest rate $= r_t$
4. Budget constraint:

$$a_{t+1} + c_t = w_t n_t + x_t + (1 + r_t)a_t \tag{4.26}$$

Lagrangian:

$$\mathcal{L}_t = \sum_{s=0}^{\infty} \Big\{ \beta^s U(c_{t+s}, 1 - n_{t+s}) + \lambda_{t+s}[w_{t+s}n_{t+s} + x_{t+s} + (1 + r_{t+s})a_{t+s} - a_{t+s+1} - c_{t+s}] \Big\}.$$

First order conditions

$$
\begin{aligned}
\frac{\partial \mathcal{L}_t}{\partial c_{t+s}} &= \beta^s U_{c,t+s} - \lambda_{t+s} = 0, \quad s \geq 0, \\
\frac{\partial \mathcal{L}_t}{\partial n_{t+s}} &= -\beta^s U_{l,t+s} + \lambda_{t+s} w_{t+s} = 0, \quad s \geq 0, \\
\frac{\partial \mathcal{L}_t}{\partial a_{t+s}} &= \lambda_{t+s}(1 + r_{t+s}) - \lambda_{t+s-1} = 0, \quad s \geq 1.
\end{aligned}
$$

and the budget constraint (4.26). The first two conditions gives

$$\frac{U_l(c_t, 1 - n_t)}{U_c(c_t, 1 - n_t)} = w_t. \tag{4.27}$$

The Euler equation is the same as before:

$$\frac{\beta U_c(c_{t+1}, 1 - n_{t+1})}{U_c(c_t, 1 - n_t)}(1 + r_{t+1}) = 1. \tag{4.28}$$

Consumption function

The consumption function, derive from the budget constraint (4.26) with constant interest rate, is similar to (4.17):

$$c_t = \frac{r}{1+r} W_t = r \sum_{s=0}^{\infty} \left[\frac{w_{t+s} n_{t+s}}{(1+r)^{s+1}} + \frac{x_{t+s}}{(1+r)^{s+1}} \right] + r a_t. \qquad (4.29)$$

where wealth or permanent income is

$$W_t = \sum_{s=0}^{\infty} \left[\frac{w_{t+s} n_{t+s}}{(1+r)^{s}} + \frac{x_{t+s}}{(1+r)^{s}} \right] + (1+r) a_t.$$

Equations (4.27), (4.28), and (4.29) are used to solve for the optimal path of c_t, n_t, and a_t.

Example

Let the utility function be

$$U(c_t, l_t) = \frac{c_t^{1-\sigma}}{1-\sigma} + \log l_t, \quad \sigma > 0.$$

Then $U_{c,t} = c_t^{-\sigma}$ and $U_{l,t} = 1/l_t$. Equation (4.27) becomes

$$\frac{c_t^{\sigma}}{1-n_t} = w_t.$$

Labour supply function is then

$$n_t = 1 - \frac{c_t^{\sigma}}{w_t}. \tag{4.31}$$

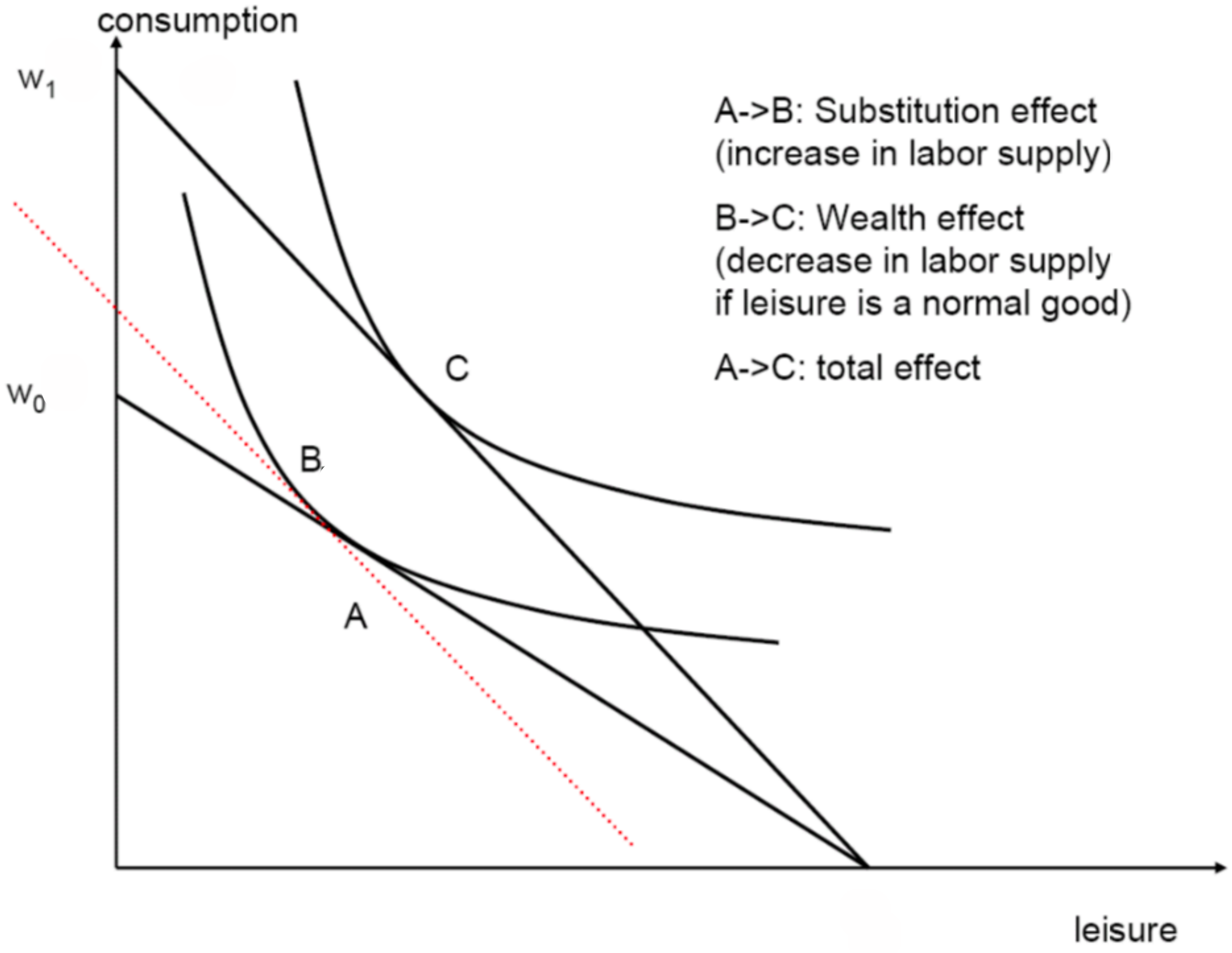
For a *temporary* wage increase in period t, consumption c_t increases slightly since MPC is only $r/(1+r)$ from (4.29). Therefore the labour supply function is upward sloping. For a *permanent* wage increase, however, the net effect on n_t is ambiguous.

Effect of a permanent wage increase on labor supply

How will an increase in the real wage affect labor supply?

- **Substitution effect**: An increase in wage makes leisure more expensive to the agent will work harder
- **Wealth effect**: Higher wage means - for unchanged labor supply - higher income. If consumption and leisure are both normal goods, labor supply must fall
- Thus, the overall impact depends on the relative strength of substitution and wealth effects

Effect of a permanent wage increase on labor supply



A->B: Substitution effect
(increase in labor supply)

B->C: Wealth effect
(decrease in labor supply
if leisure is a normal good)

A->C: total effect

The Frisch elasticity: An important determinant for the behavior of labor supply is the Frisch labor supply elasticity which is defined as the elasticity of labor supply for a constant level marginal utility of wealth. This is the labor supply elasticity that enters the first-order condition for labor supply:

$$
\begin{aligned}
-u_h\left(c_t, h_t\right) &= \lambda_{c,t} w_t \\
\zeta^h &= \frac{dh_t / h_t}{dw_t / w_t}\Big|_{\lambda_{c,t}} = \frac{u_h\left(c_t, h_t\right)}{h_t u_{hh}\left(c_t, h_t\right)}
\end{aligned}
$$

- This parameter determines, for given wealth, the elasticity of the labor supply response to changes in wages and is a key parameter in many macroeconomic theories
- Unfortunately, macroeconomists and microeconomists disagree fundamentally on the appropriate value of this parameter
    - macroeconomists: This elasticity is high (perhaps even infinite)
    - microeconomists: This elasticity is low

Why this disagreement?

- **macroeconomists** find that to account for size of fluctuations in aggregate per capita hours worked, the elasticity must be large. They therefore think about the combined impact of:
  - *the intensive margin*: Hours per worker changes
  - *the extensive margin*: Changes in number of households that work
- **microeconomists** when estimating individual labor supply responses find small elasticities. The extensive margin (mainly for females) also appears inelastic.

Firms: assumptions

1. Objective: maximize the present value of current and future profit.
2. Production decisions: output level y_t and factor inputs k_t and n_t.
3. Financial decisions: level debt financing, b_t (no equity financing)
4. Production function: $y_t = F(k_t, n_t)$
5. Capital accumulation: $k_{t+1} = i_t + (1-\delta)k_t$

Therefore the firm chooses n_{t+s}, k_{t+s+1}, and b_{t+s+1} to maximize

$$\mathcal{P}_t = \sum_{s=0}^{\infty}(1+r)^{-s}\Big\{F(k_{t+s}, n_{t+s}) - w_{t+s}n_{t+s} - k_{t+s+1}$$

$$+ \ (1-\delta)k_{t+s} + b_{t+s+1} - (1+r)b_{t+s}\Big\}.$$

First Order Conditions

$$\begin{aligned}
\frac{\partial \mathcal{P}_t}{\partial n_{t+s}} &= (1+r)^{-s}[F_{n,t+s} - w_{t+s}] = 0, \quad s \geq 0, \\
\frac{\partial \mathcal{P}_t}{\partial k_{t+s}} &= (1+r)^{-s}[F_{k,t+s} + 1 - \delta] - (1+r)^{-(s-1)} = 0, \quad s \geq 1, \\
\frac{\partial \mathcal{P}_t}{\partial b_{t+s}} &= -(1+r)^{-s}(1+r) + (1+r)^{-(s-1)} = 0, \quad s \geq 1.
\end{aligned}$$

From the first equation, we obtain the usual result that given capital, marginal product of labour equals the real wage,

$$F_n(k_t, n_t) = w_t.$$

The third equation is independent of b_t. It means that any debt level is consistent with profit maximization, the firm can choose debt financing or retained earning to finance new investment.

Capital and investment

The second equation gives implicitly the demand for capital:

$$F_k(k_{t+1}, n_{t+1}) = r + \delta.$$

For a given level of labour input, $k_{t+1} = F_k^{-1}(r + \delta)$. The gross investment is

$$i_t = F_k^{-1}(r + \delta) - (1 - \delta)k_t. \tag{4.32a}$$

1. Since $F_{kk} \leq 0$, we get the Keynesian result that investment is decreasing in interest rate.
2. A permanent technology shock raises the optimal capital stock and investment.
3. In the short run marginal product of capital depends on the cost of financing (r).
4. In the long run and general equilibrium $F_k(k, n) = \theta + \delta$.

Modelling labor adjustment costs: assumptions

1. Households choose to whether or not to participate in the work force, total *number* of workers $= n_t$
2. Firms decide the number of working hours for each worker, h_t
3. Production is abstract from capital: $y_t = F(n_t, h_t)$
4. Wages (salary) *per worker* is $W(h_t)$, with $W' \geq 0$ and $W'' \geq 0$
5. Changes in number of workers involve adjustment costs for the firms:

$$\frac{1}{2}\lambda(\Delta n_{t+1})^2, \quad \lambda > 0.$$

The typical firm chooses n_t and h_t to maximize

$$\mathcal{P}_t = \sum_{s=0}^{\infty}(1+r)^{-s}\left\{F(n_{t+s}, h_{t+s}) - W(h_{t+s})n_{t+s} - \frac{1}{2}\lambda(\Delta n_{t+s+1})^2\right\}.$$

First order conditions

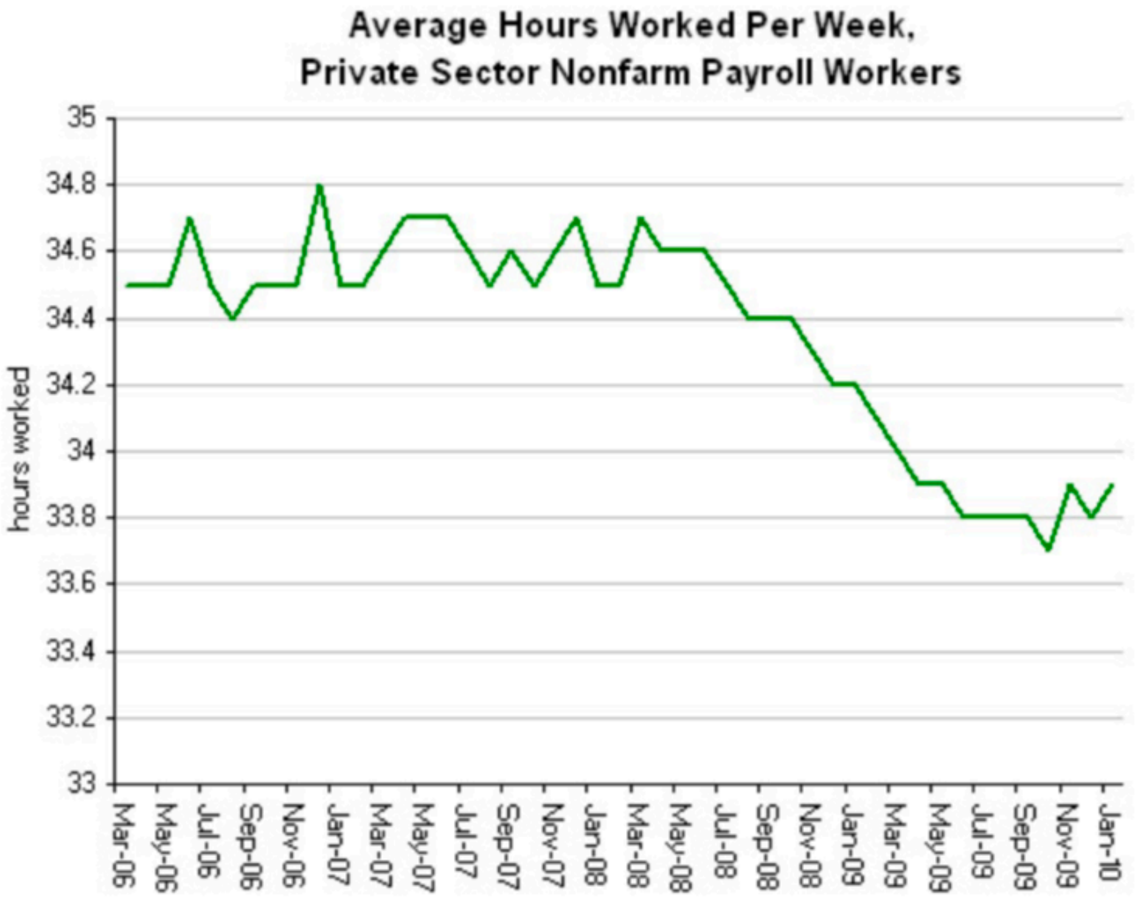
$$\begin{aligned}\frac{\partial \mathcal{P}_t}{\partial n_{t+s}} &= (1+r)^{-s}(F_{n,t+s} - W_{t+s} + \lambda \Delta n_{t+s+1}) \\ &\quad - (1+r)^{-(s-1)}\lambda \Delta n_{t+s} = 0, \quad s \geq 1, \\ \frac{\partial \mathcal{P}_t}{\partial h_{t+s}} &= (1+r)^{-s}(F_{h,t+s} - W'_{t+s} n_{t+s}) = 0, \quad s \geq 0.\end{aligned}$$

The second equation implies that

$$\frac{F_h(n_t, h_t)}{n_t} = W'_t(h), \tag{4.33}$$

which means that the marginal product of working hour per worker is equal to the marginal hourly rate. Given the number of workers n_t, the result gives the number of hours per worker.

Observations (U.S.)



Source: Bureau of Labor Statistics, Current Employment Statistics survey

Level of employment (or unemployment)

From the first FOC,

$$\Delta n_t = \frac{1}{1+r}\Delta n_{t+1} + \frac{1}{\lambda(1+r)}(F_{n,t} - W_t) \tag{4.34}$$

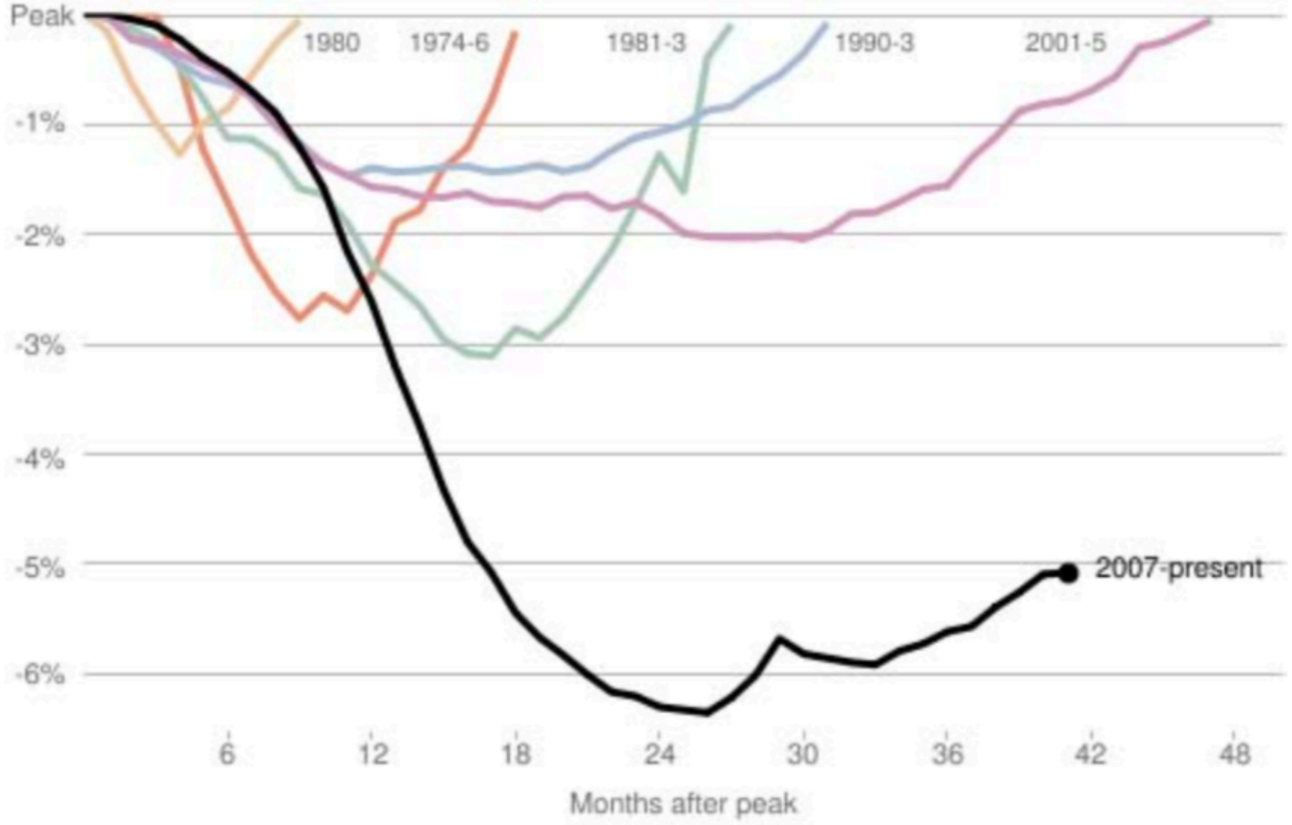
$$= \frac{1}{\lambda(1+r)}\sum_{s=0}^{\infty}(1+r)^{-s}(F_{n,t+s} - W_{t+s}). \tag{4.35}$$

1. Equation (4.35) is forward looking. If marginal product of labour exceeds salary in the current period and/or in the future, the firm will start hiring.
2. Equation (4.34) can be written as (exercise)

$$n_t = \frac{1}{2+r}n_{t+1} + \frac{1+r}{2+r}n_{t-1} + \frac{1}{\lambda(2+r)}(F_{n,t} - W_t). \tag{4.36}$$

This shows that changing employment level takes time. The higher the adjustment cost λ, the slower the process. See the textbook for an example.

Comparing recessions



Vertical axis shows the ratio of that month's nonfarm payrolls to the nonfarm payrolls at the start of recession. Source: The New York Times

General Equilibrium

Supply and demand are in equilibrium in

1. the goods and services market (consumption and firms' output)
2. the labour market
3. financial markets: bonds and equity

Conclusion: the decentralized model gives similar results to the centralized model.

Putting the ingredients together

National income identity:

$$y_t = F(k_t, n_t) = c_t + i_t.$$

Household budget constraint:

$$\Delta a_{t+1} + c_t = w_t n_t + x_t + r_t a_t.$$

Capital accumulation:

$$\Delta k_{t+1} = i_t - \delta k_t.$$

Eliminating c_t and i_t from the three equations gives

$$x_t = F(k_t, n_t) - w_t n_t - \Delta k_{t+1} - \delta k_t + \Delta a_{t+1} - r_t a_t. \tag{4.37}$$

Firms' profit:

$$\Pi_t = F(k_t, n_t) - w_t n_t - \Delta k_{t+1} - \delta k_t + \Delta b_{t+1} - r_t b_t. \tag{4.38}$$

Bond market equilibrium

- Subtracting (4.38) from (4.37) gives

$$x_t - \Pi_t = \Delta(a_{t+1} - b_{t+1}) - r(a_t - b_t).$$

- Bond market equilibrium requires that $a_t = b_t$ in every period.
- Therefore $x_t = \Pi_t$.
- The exogenous incomes of the households are dividends from the firms.

Firms and equity

Recall that for profit maximization, $F_{n,t} = w_t$ and $F_{k,t} = r + \delta$. Equation (4.38) can be written as

$$\begin{aligned}\Pi_t &= F(k_t, n_t) - F_{n,t}n_t - \Delta k_{t+1} - (F_{k,t} - r)k_t + \Delta b_{t+1} - r_t b_t \\ &= F(k_t, n_t) - F_{n,t}n_t - F_{k,t}k_t - \Delta(k_{t+1} - b_{t+1}) + r(k_t - b_t).\end{aligned}$$

Assuming constant returns to scale, Euler theorem implies that

$$F(k_t, n_t) = F_{n,t}n_t + F_{k,t}k_t.$$

Hence

$$\Pi_t = -(k_{t+1} - b_{t+1}) + (1 + r)(k_t - b_t). \qquad (4.39)$$

This is a first-order difference equation in $k_t - b_t$. Since the coefficient $1 + r > 1$, the equation can be solved forward.

Theory of finance

- The solution for equation (4.39) is

$$k_t - b_t = \sum_{s=0}^{\infty} \frac{\Pi_{t+s}}{(1+r)^{s+1}},$$

  which means that the net value of the firm in period t is equal to the present value of its current and future profits.

- As $x_t = \Pi_t$ and $a_t = b_t$, the above equation can be written as

$$r \sum_{s=0}^{\infty} \frac{x_{t+s}}{(1+r)^{s+1}} + ra_t = rk_t.$$

  Asset income in the consumption function (4.29) in equilibrium is in fact rk_t.

Labor supply and labor demand

Take the simple model of labour market without adjustment cost:

- Demand from firms: $F_n(k_t, n_t) = w_t$
- Supply from households (4.27): $-U_n(c_t, 1-n_t)/U_c(c_t, 1-n_t) = w_t$
- In equilibrium,

$$w_t = F_n(k_t, n_t) = -\frac{U_n(c_t, 1-n_t)}{U_c(c_t, 1-n_t)}.$$

This equation of course includes other endogenous variables such as capital and consumption.

Example

1. Production function: $y_t = Ak_t^{\alpha} n_t^{1-\alpha}$. Therefore

$$F_{n,t} = (1-\alpha)A\left(\frac{k_t}{n_t}\right)^{\alpha}.$$

The labour demand function is

$$n_t^d = \left[\frac{w_t}{(1-\alpha)A}\right]^{-1/\alpha} k_t.$$

2. Utility function:

$$U(c_t, l_t) = \frac{c_t^{1-\sigma}}{1-\sigma} + \log(1-n_t).$$

Labour supply, from equation (4.31), is

$$n_t^s = 1 - \frac{c_t^{\sigma}}{w_t}.$$

Example (cont.)

Equilibrium in the labour market is therefore

$$n_t = \left[\frac{w_t}{(1-\alpha)A}\right]^{-1/\alpha} k_t = 1 - \frac{c_t^{\sigma}}{w_t}.$$

The equilibrium wage w_t and labour n_t is therefore functions of c_t and k_t.

Equilibrium in the goods market

Using (4.32a), the aggregate demand is

$$\begin{aligned} y_t^d &= c_t + i_t \\ &= c_t + F_k^{-1}(r+\delta) - (1-\delta)k_t. \end{aligned}$$

For a Cobb-Douglas production function,

$$F_k^{-1}(r+\delta) = \left[\frac{\alpha A}{r+\delta}\right]^{1/(1-\alpha)} n_t.$$

Therefore

$$y_t^d = c_t + \left[\frac{\alpha A}{r+\delta}\right]^{1/(1-\alpha)} n_t - (1-\delta)k_t.$$

Aggregate supply is simply the production function. Therefore equilibrium in the goods market is

$$Ak_t^{\alpha} n_t^{1-\alpha} = c_t + \left[\frac{\alpha A}{r+\delta}\right]^{1/(1-\alpha)} n_t - (1-\delta)k_t.$$

General equilibrium

- In the steady state consumption is

$$c_t = w_t n_t + x_t + ra_t = w_t n_t + rk_t.$$

- Labour market equilibrium is

$$n_t = \left[\frac{w_t}{(1-\alpha)A}\right]^{-1/\alpha} k_t = 1 - \frac{c_t^\sigma}{w_t}.$$

- With the goods market equilibrium equation we can solve for c_t, n_t, w_t, k_t.
- Closed form solutions are often difficult to derive. In practice the model is solved by numerical methods.

Centralized and decentralized models

	Centralized	Decentralized
Capital	$F'(k_{t+1}) = \theta + \delta$	$F_{k,t+1} = r + \delta$
Labour	Fixed	Endogenous
Consumption	Yes	Yes
Interest rate	Implicit	$r = F_{k,t+1} - \delta$
Real wage	Implicit	Endogenous
Debt finance	Not available	Yes
Decision maker	One	Two

Overall, both models provide the key results for a dynamic economy. The decentralized model provides more details and modelling flexibility.

Business Cycles

- Fluctuations in aggregate activity and its components at the business cycle frequencies
- Fluctuations in the economy with a periodicity typically between 2 and 10 years (although there are exceptions)
- To investigate this we will first look at “facts” and then at theory

What are business cycles?

- Here we will use the view that business cycles refer to **fluctuations in main macroeconomic aggregates around their trend**. This is a statistical definition. It does not *necessarily* relate directly to NBER definitions of business cycles or other definitions used more popularly. We will need to make precise what we mean by trend and this will correspond to a view upon the average length of business cycles.
- The NBER definition is "The NBER does not define a recession in terms of two consecutive quarters of decline in real GDP. Rather, a recession is a significant decline in economic activity spread across the economy, lasting more than a few months, normally visible in real GDP, real income, employment, industrial production, and wholesale-retail sales". This is a nice definition but not very applicable for practical purposes. Its main strength is in terms of communicating to the public the state of the economy

Measurement

Detrending: The Hodrick-Prescott filter

Suppose we observe a time-series $\{y_t\}$ for the sample period $t = 1, .., T$. We wish to decompose this into a trend component and a "deviations from trend":

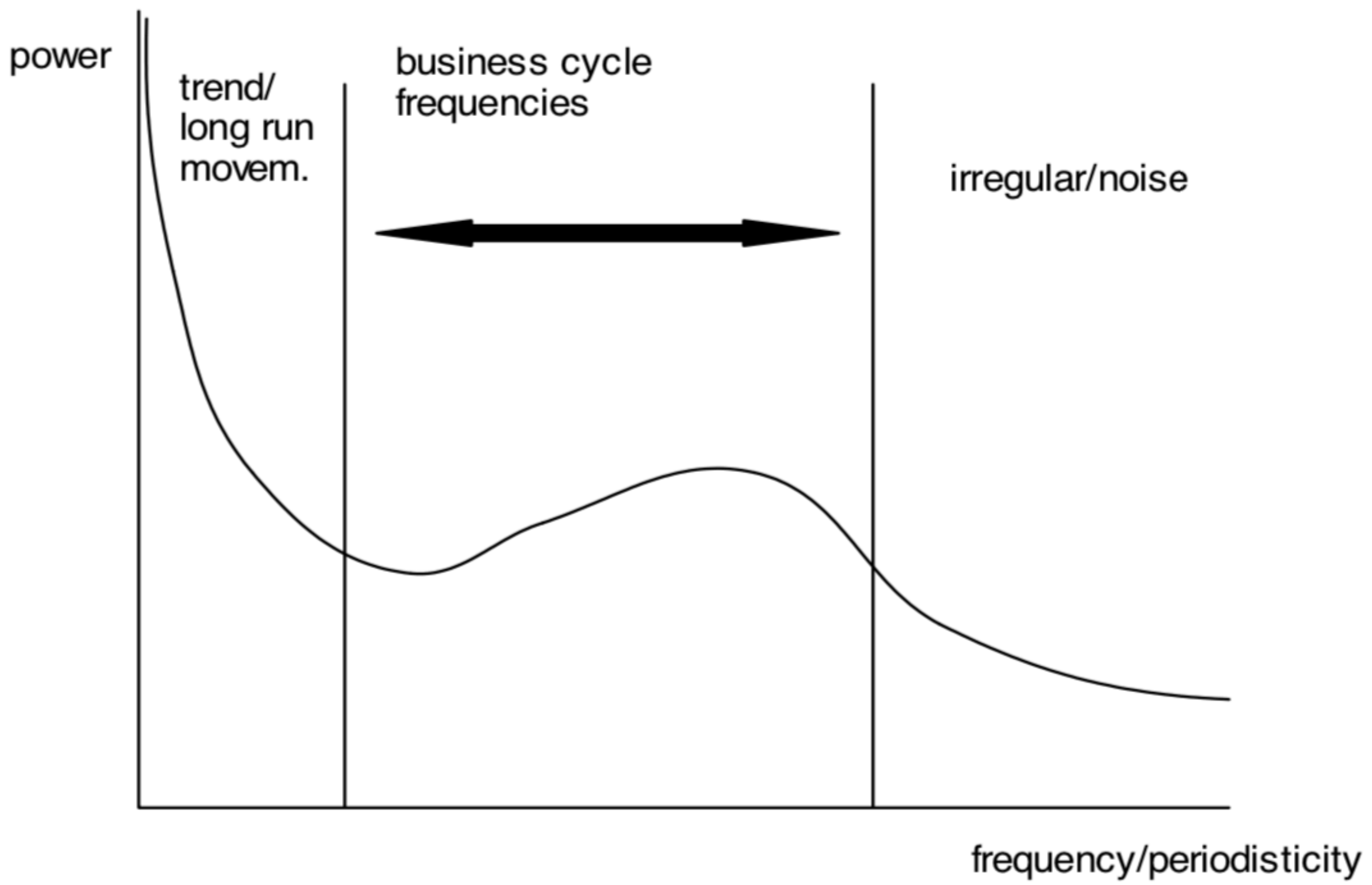
$$y_t = \tau_t + c_t + s_t + e_t$$

τ is the trend component, c is the cyclical component, s is a seasonal component, e is an irregular component

- If we study business cycles, we want to focus attention on the cyclical component - i.e. we want to remove the trend component and possibly seasonals and irregulars
- There are many ways of doing this
- In the business cycle literature it has become common to use the Hodrick-Prescott filter on seasonally adjusted data (i.e. it leaves in irregulars).

Measurement

What we have in mind may be something like the following picture:



The Hedrick-Prescott filter

Specified as:

$$\min_{\{\tau_t\}_{t=0}^{T}} \underbrace{\sum_{t=0}^{T} (y_t - \tau_t)^2}_{\text{“goodness of fit”}} + \lambda \underbrace{\sum_{t=1}^{T-1} [(\tau_{t+1} - \tau_t) - (\tau_t - \tau_{t-1})]^2}_{\text{squared acceleration of trend}} \tag{1}$$

- τ_t is the trend component of y_t
- λ: the smoothing parameter
- The smoothing parameter: Determines the trade-off between fit and variability of the trend
- $\lambda \rightarrow 0$: trend component will become equal to the data series itself
- $\lambda \rightarrow \infty$: trend component will become linear
- Intermediate values of λ : Smooth but non-constant trend

Measurement

How to organize and measure the data

(i) Production inputs: Output - labor input - capital input - inventory stock - plus labor productivity

(ii) Expenditure components: output - consumption - investment - government spending - exports - imports

(iii) Other variables - for example nominal variables such as money supply price level etc.

Which moments to compute:

Volatility as measured by standard deviation

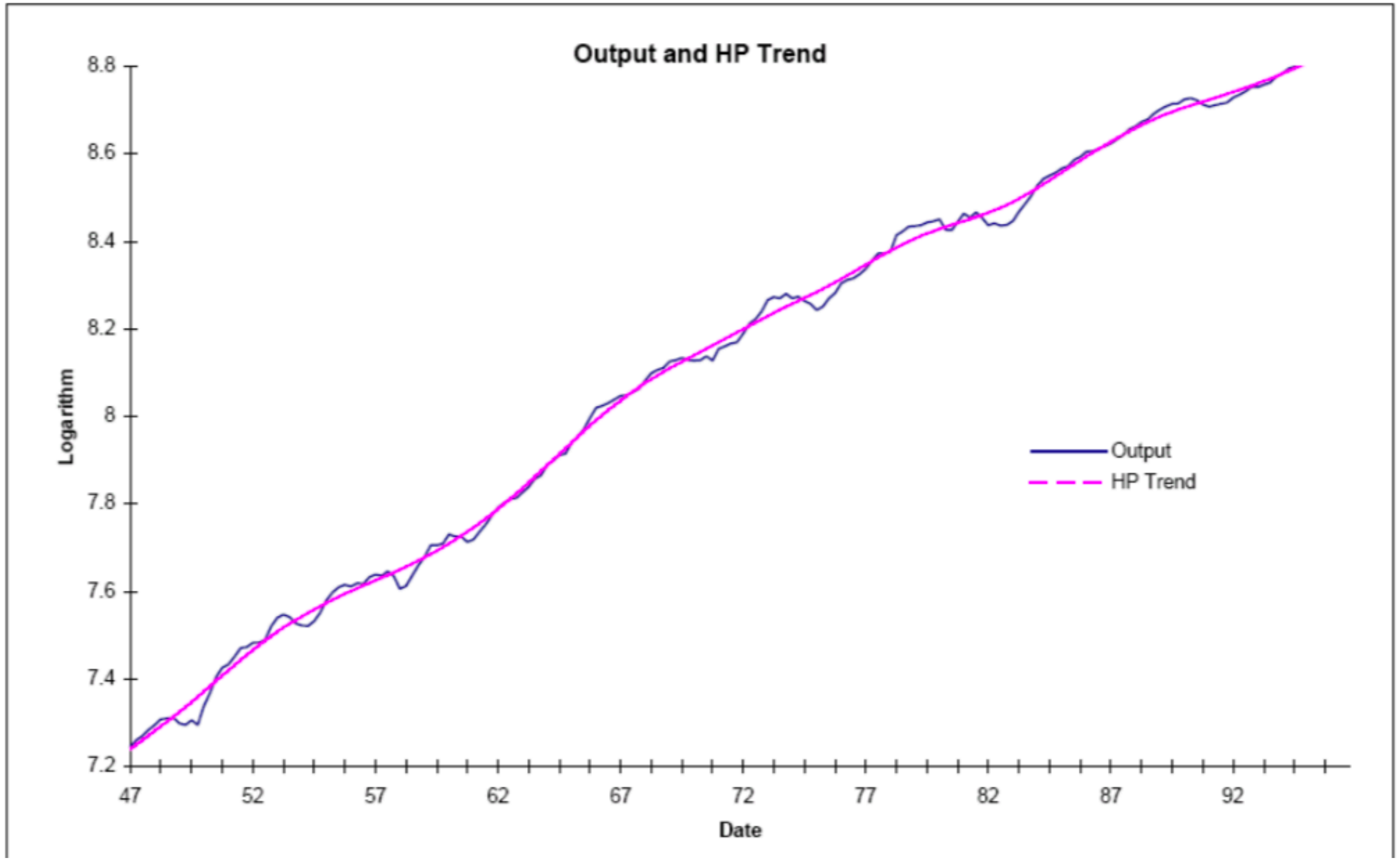
Persistence as measured by autocorrelation

Cyclicality as measured by cross correlation with output

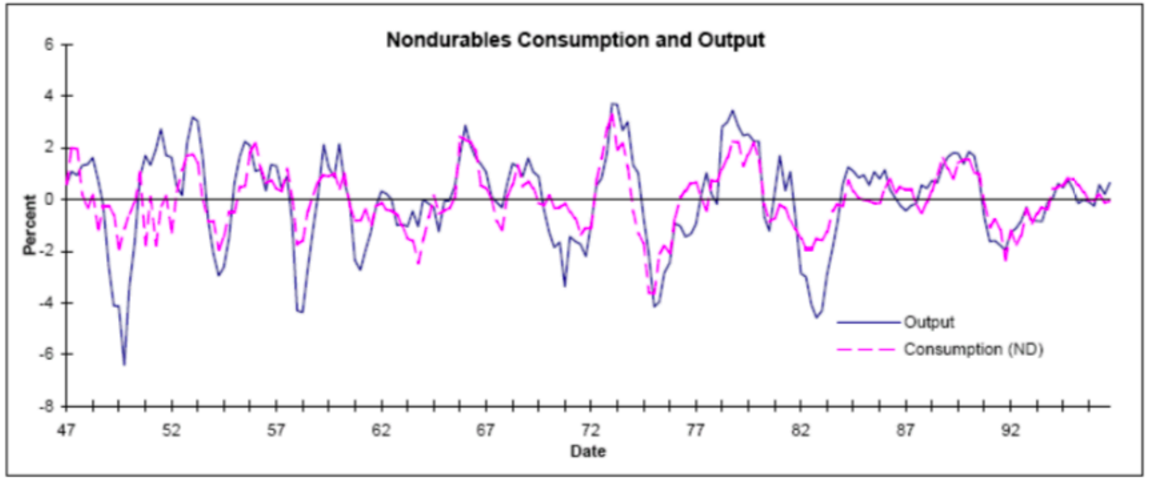
If positive - procyclical variable, if negative - countercyclical, if close to zero - acyclical

US Facts and Figures

- US quarterly data for 1947-1996
- All variables are in constant prices and per capita
- Reported in King and Rebelo (Resuscitating Real Business Cycles)



US Facts and Figures



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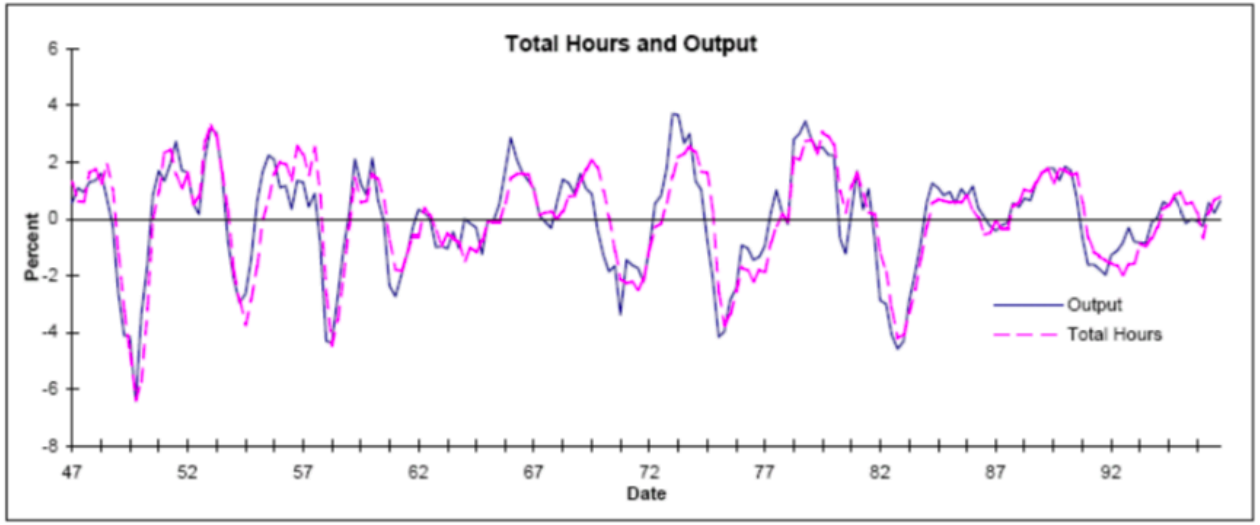

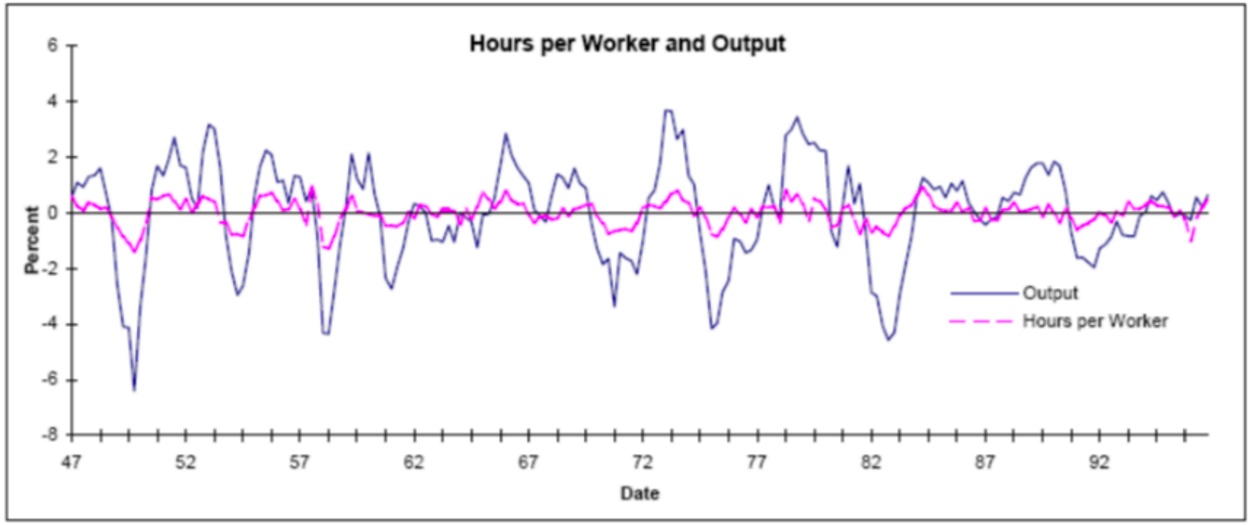

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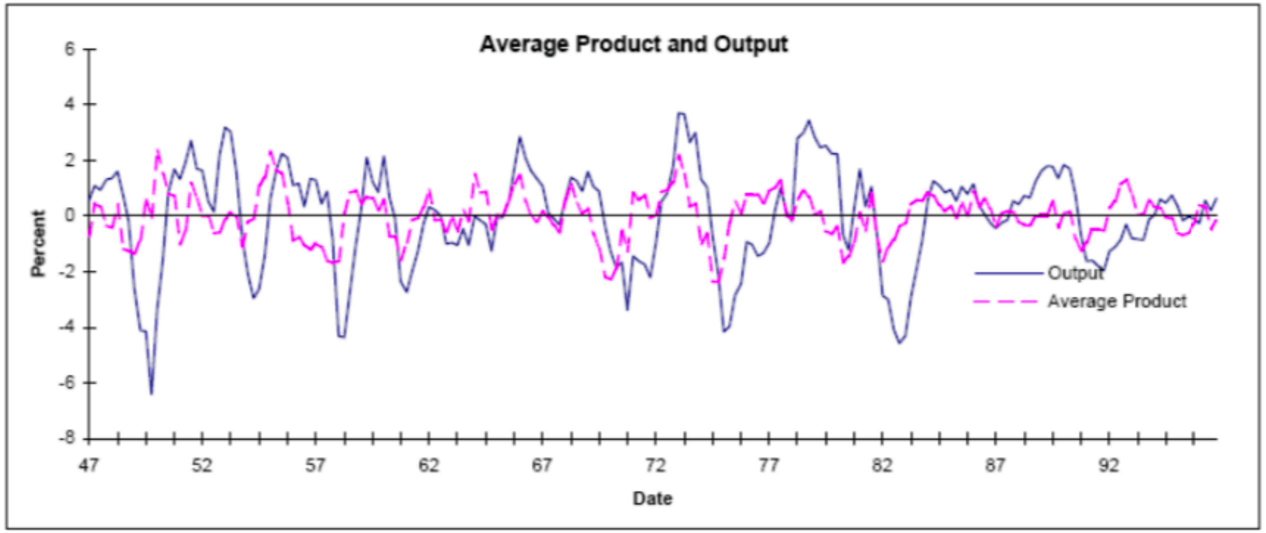

US Facts and Figures

Table 1
Business Cycle Statistics for the U.S. Economy

	Standard Deviation	Relative Standard Deviation	First Order Auto-correlation	Contemporaneous Correlation with Output
Y	1.81	1.00	0.84	1.00
C	1.35	0.74	0.80	0.88
I	5.30	2.93	0.87	0.80
N	1.79	0.99	0.88	0.88
Y/N	1.02	0.56	0.74	0.55
w	0.68	0.38	0.66	0.12
r	0.30	0.16	0.60	-0.35
A	0.98	0.54	0.74	0.78

US Facts and Figures

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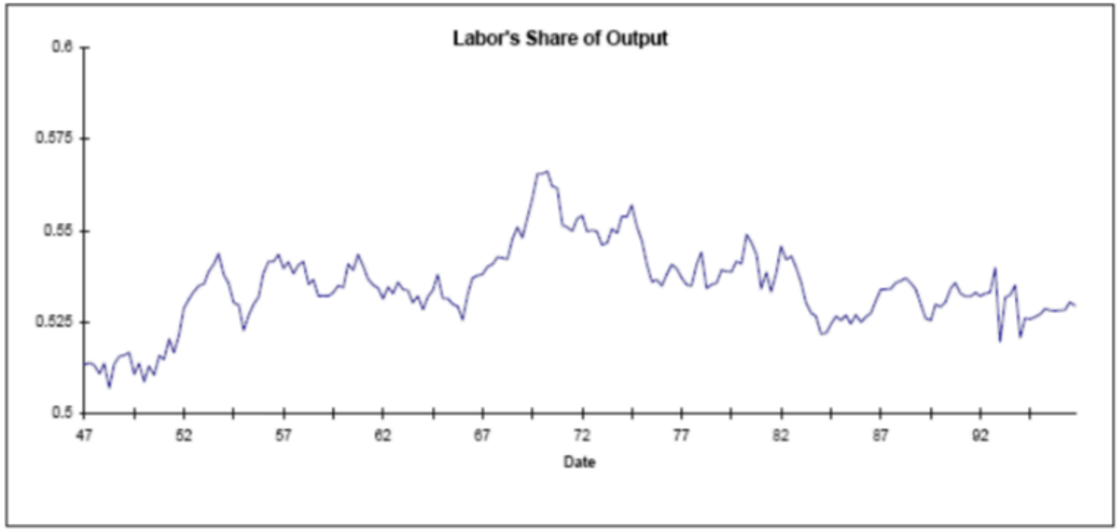
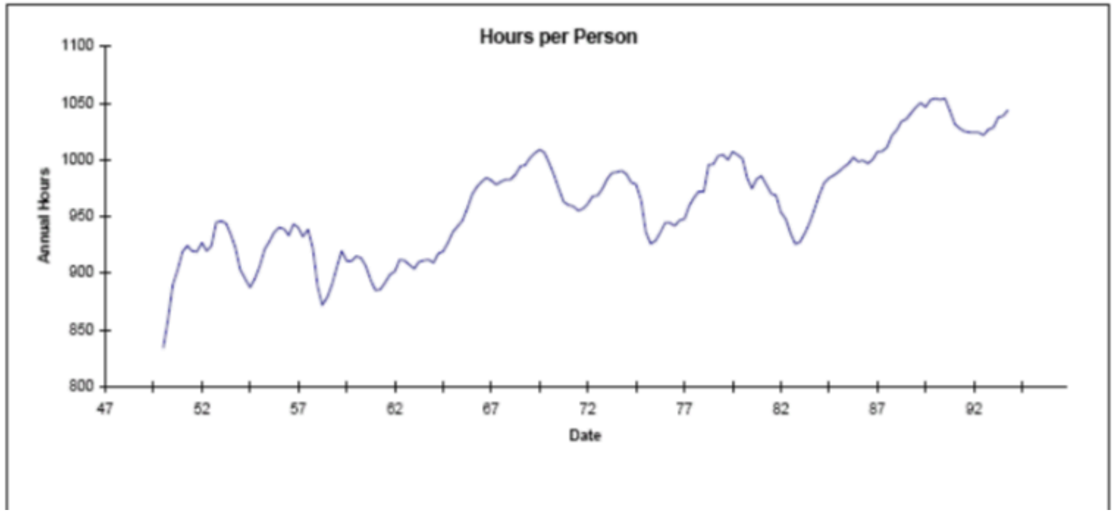
US Facts and Figures

- The “facts” above are useful for evaluating theory
- There’s another set of facts that are useful for building theory - these are facts about the long run
    1. Factor income shares are relatively constant over time and are not trending
    2. The consumption and investment shares of output do not trend
    3. Real wages have grown substantially over time. Aggregate hours worked have not.
    4. Output grows over time

Long run facts





Long run facts