Lecture: Asset Pricing

Advanced Macroeconomics

15/10/2019

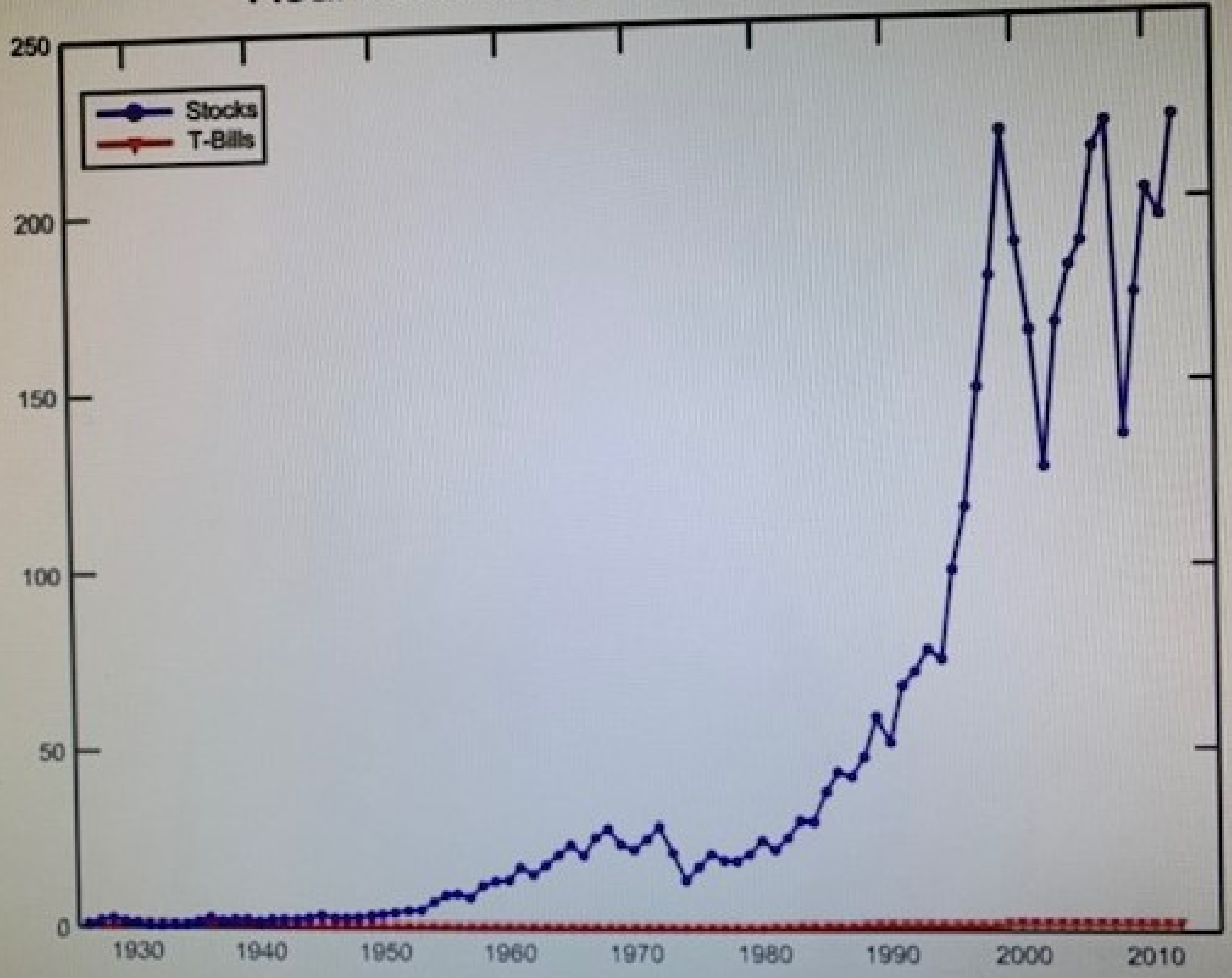
Overview:

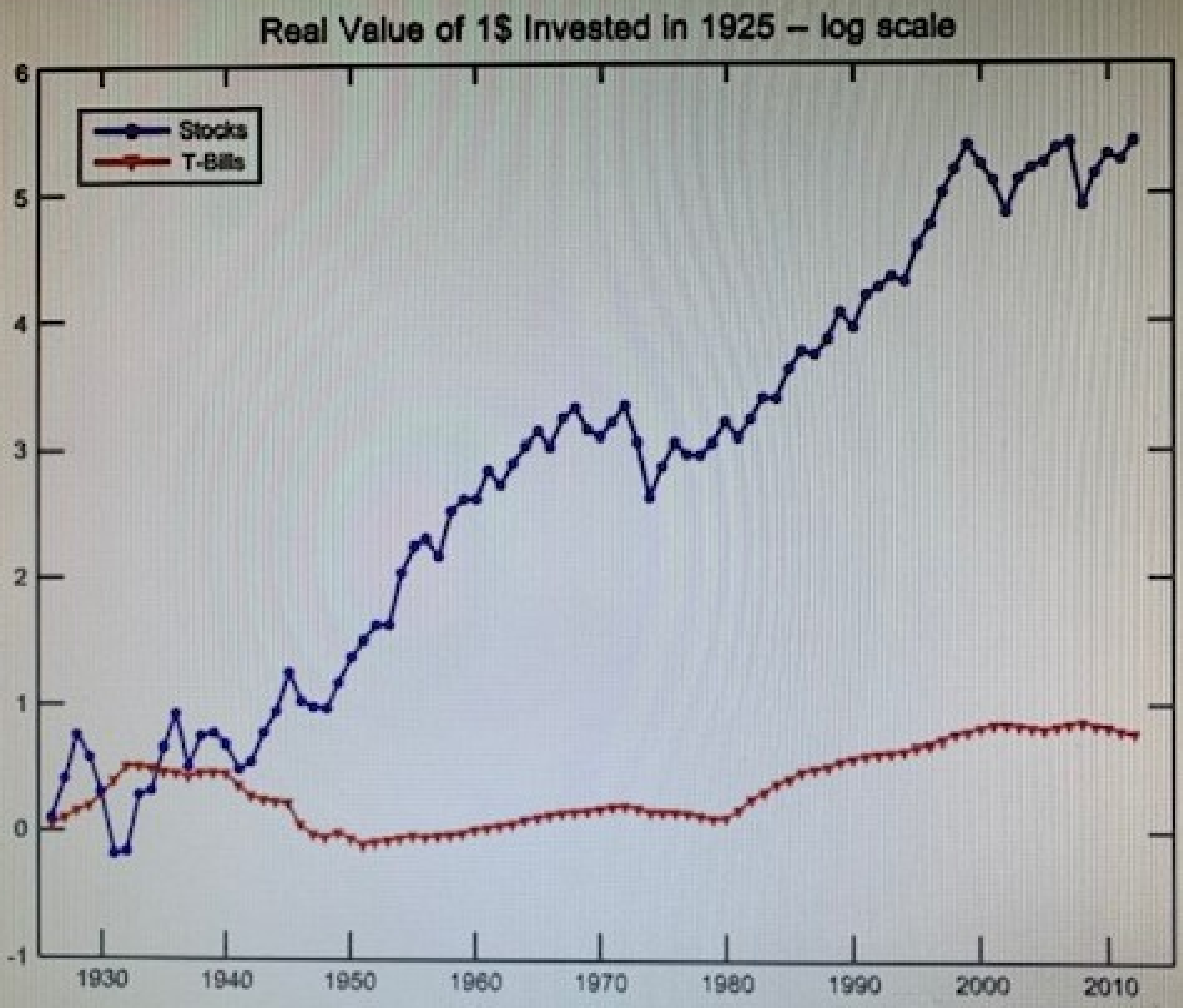
- Some basic facts.
- Study the asset pricing implications of household portfolio choice.
- Consider the quantitative implications of a second-order approximation to asset return equations.
- Reference: Ljungqvist & Sargent, Recursive Macroeconomic Theory, 2nd edition, chapter 13

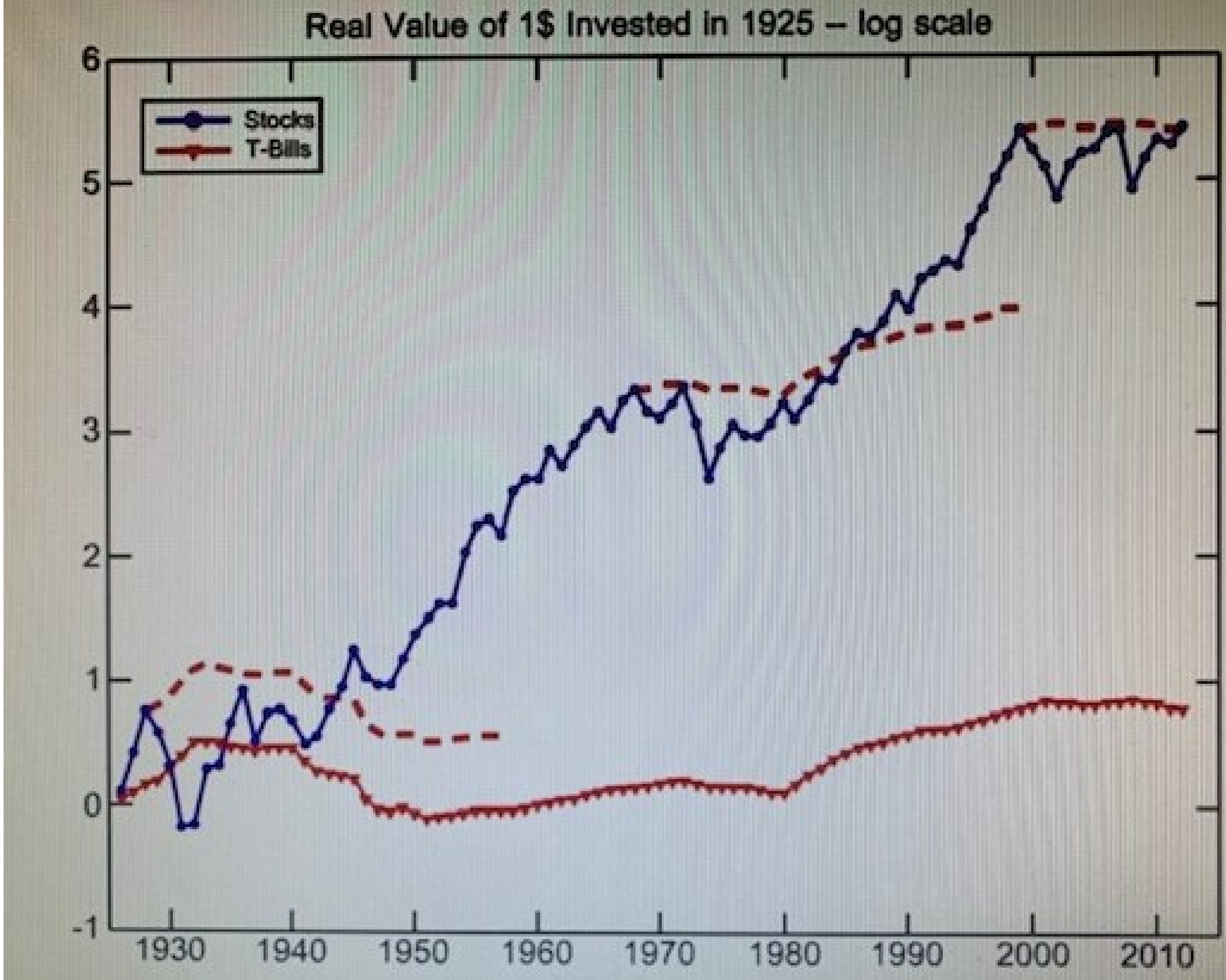
Some facts:

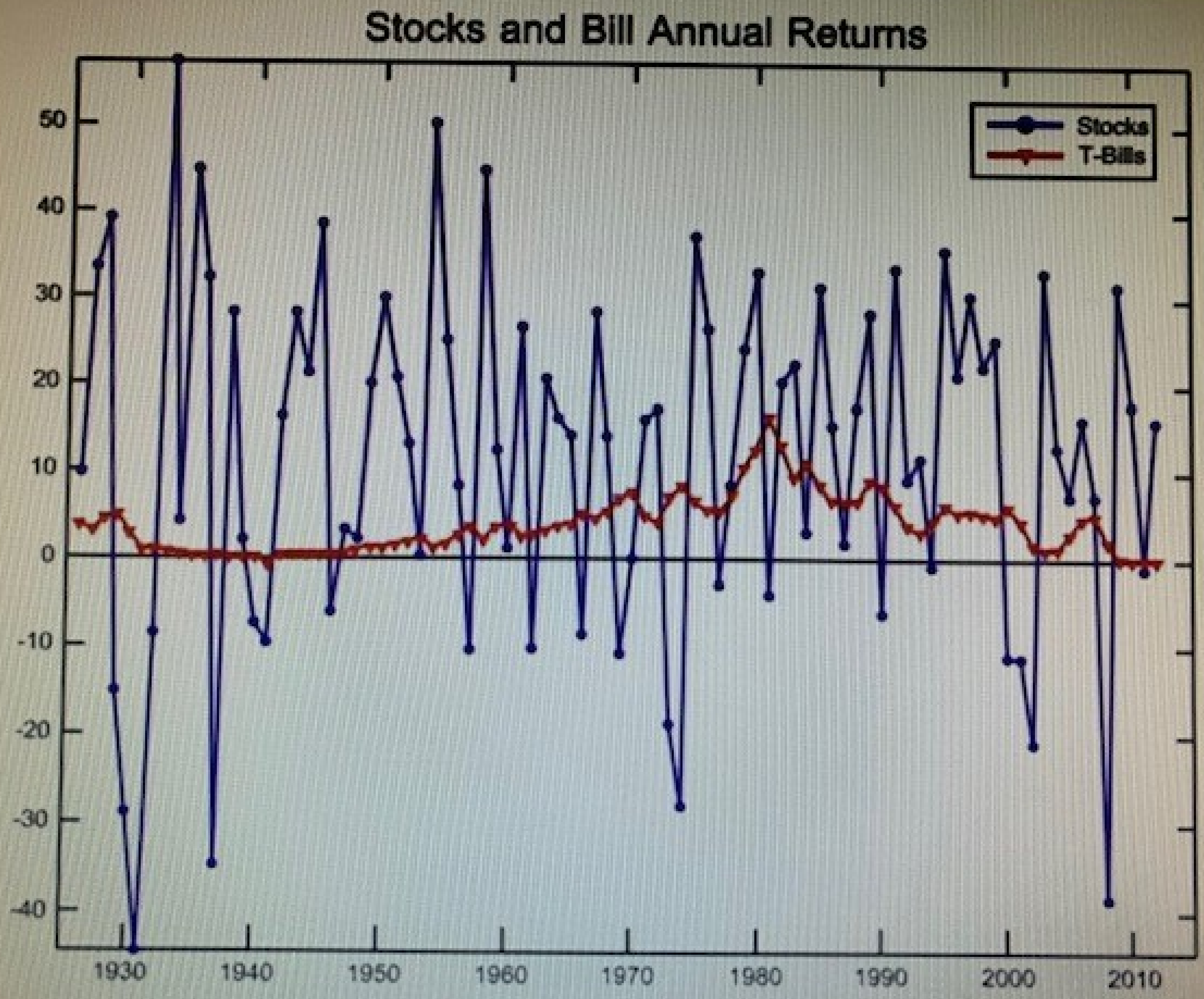
- Stock returns:
- Average real return on SP500 is 8% per year.
- Stock returns are very volatile: σ$(R) = 17\%$ per year.
- Stock returns show very little serial correlation ($= 0.08$ quarterly data, -0.04 annual data).
- Bond returns:
- The average risk free rate is 1% per year (US T-bill - Inflation)
- The risk free rate is not very volatile: σ$(R) = 2\%$ per year but is persistent ($= 0.6$ in annual data).
- These imply that the equity premium is large – 7% per year on an annual basis.

Real Value of $1 Invested in 1925

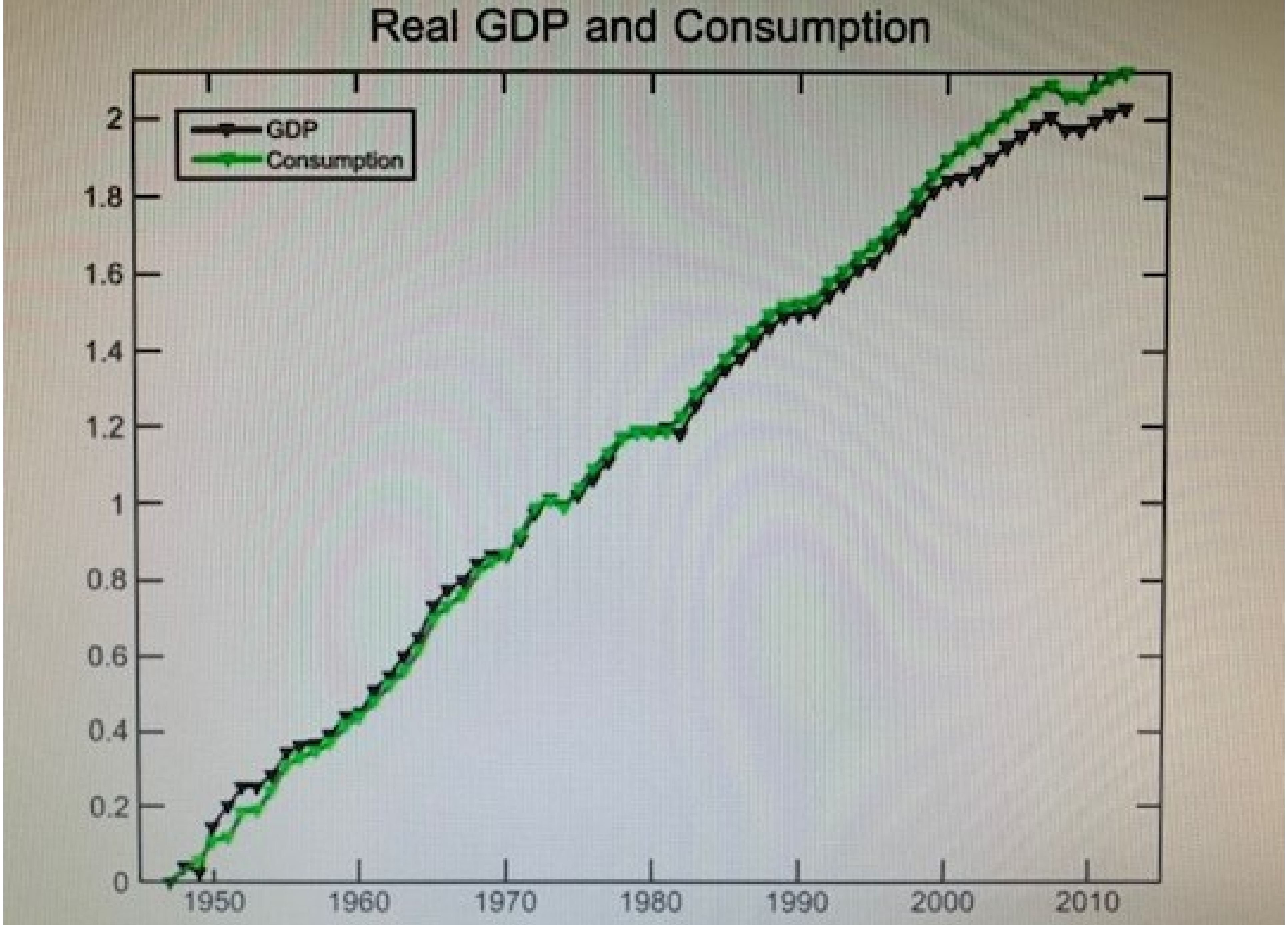


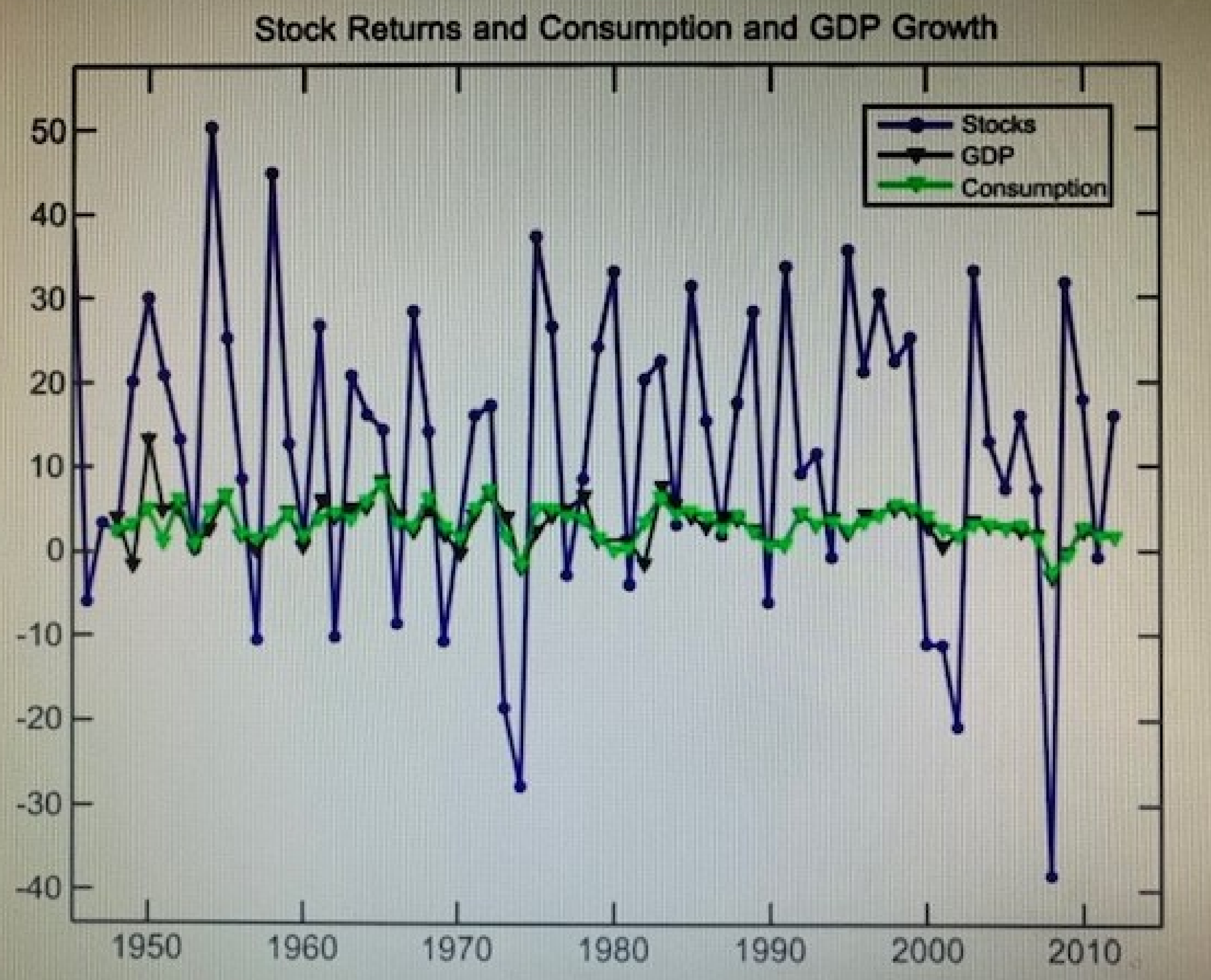

Real Value of 1$ Invested in 1925 – log scale
Stocks
T-Bills
6
5
4
3
2
1
0
-1
1930
1940
1950
1960
1970
1980
1990
2000
2010


Real Value of 1$ Invested in 1925 – log scale
Stocks
T-Bills
6
5
4
3
2
1
0
-1
1930
1940
1950
1960
1970
1980
1990
2000
2010



Stocks and Bill Annual Returns
Stocks
T-Bills
50
40
30
20
10
0
-10
-20
-30
-40
1930
1940
1950
1960
1970
1980
1990
2000
2010


Real GDP and Consumption
GDP
Consumption
2
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
1950
1960
1970
1980
1990
2000
2010



Stock Returns and Consumption and GDP Growth
Stocks
GDP
Consumption
50
40
30
20
10
0
-10
-20
-30
-40
1950
1960
1970
1980
1990
2000
2010

Equity Premium and Risk (all variables measured in real terms)

	Stock Returns	Bond Returns	Stock-Bond	GDP	Consumption
E	8.6	1.3	7.4	3.2	3.3
Stand. Dev	17.6	2.6	18.1	2.6	2.1
Corr	0.99	–0.03	1.00	0.32	0.39

Return Predictability:

Many authors consider the following regression:

$$R_{t+1} = \alpha + \beta X_t + \varepsilon_{t+1}$$

where X_t is a variable that helps in predicting the future return

Regression of returns on lagged returns

Annual data 1927-2008

$$R_{t+1} = a + bR_t + \varepsilon_{t+1}$$

	b	t(b)	R^2	E(R)	$\sigma(E_t(R_{t+1}))$
Stock	0.04	0.33	0.002	11.4	0.77
T bill	0.91	19.5	0.83	4.1	3.12
Excess	0.04	0.39	0.00	7.25	0.91

Return Predictability:

The following regression has been studied:

$$R_{t,t+k} = \alpha + \beta D_t/P_t + \varepsilon_{t+k}$$

where $R_{t,t+k}$ is the realized cumulative return over k periods, and D_t/P_t the dividend price ratio

Table I

Return-Forecasting Regressions

The regression equation is $R^e_{t\to t+k} = a + b \times D_t/P_t + \varepsilon_{t+k}$. The dependent variable $R^e_{t\to t+k}$ is the CRSP value-weighted return less the 3-month Treasury bill return. Data are annual, 1947–2009. The 5-year regression t-statistic uses the Hansen–Hodrick (1980) correction. $\sigma[E_t(R^e)]$ represents the standard deviation of the fitted value, $\sigma(\hat{b} \times D_t/P_t)$.

Horizon k	b	$t(b)$	R^2	$\sigma[E_t(R^e)]$	$\frac{\sigma[E_t(R^e)]}{E(R^e)}$
1 year	3.8	(2.6)	0.09	5.46	0.76
5 years	20.6	(3.4)	0.28	29.3	0.62

OLS Regressions of Excess Returns (value-weighted NYSE—Treasury bill) and Real Dividend Growth on the Value-Weighted NYSE Dividend-Price Ratio

Horizon k (years)	$R^e_{t\to t+k} = a + b\frac{D_t}{P_t} + \varepsilon_{t+k}$			$\frac{D_{t+k}}{D_t} = a + b\frac{D_t}{P_t} + \varepsilon_{t+k}$		
	b	$t(b)$	R^2	b	$t(b)$	R^2
1	4.0	2.7	0.08	0.07	0.06	0.0001
2	7.9	3.0	0.12	−0.42	−0.22	0.0010
3	12.6	3.0	0.20	0.16	0.13	0.0001
5	20.6	2.6	0.22	2.42	1.11	0.0200

Sample 1927–2005, annual data. $R^e_{t\to t+k}$ denotes the total excess return from time t to time $t+k$. Standard errors use GMM (Hansen–Hodrick) to correct for heteroskedasticity and serial correlation.

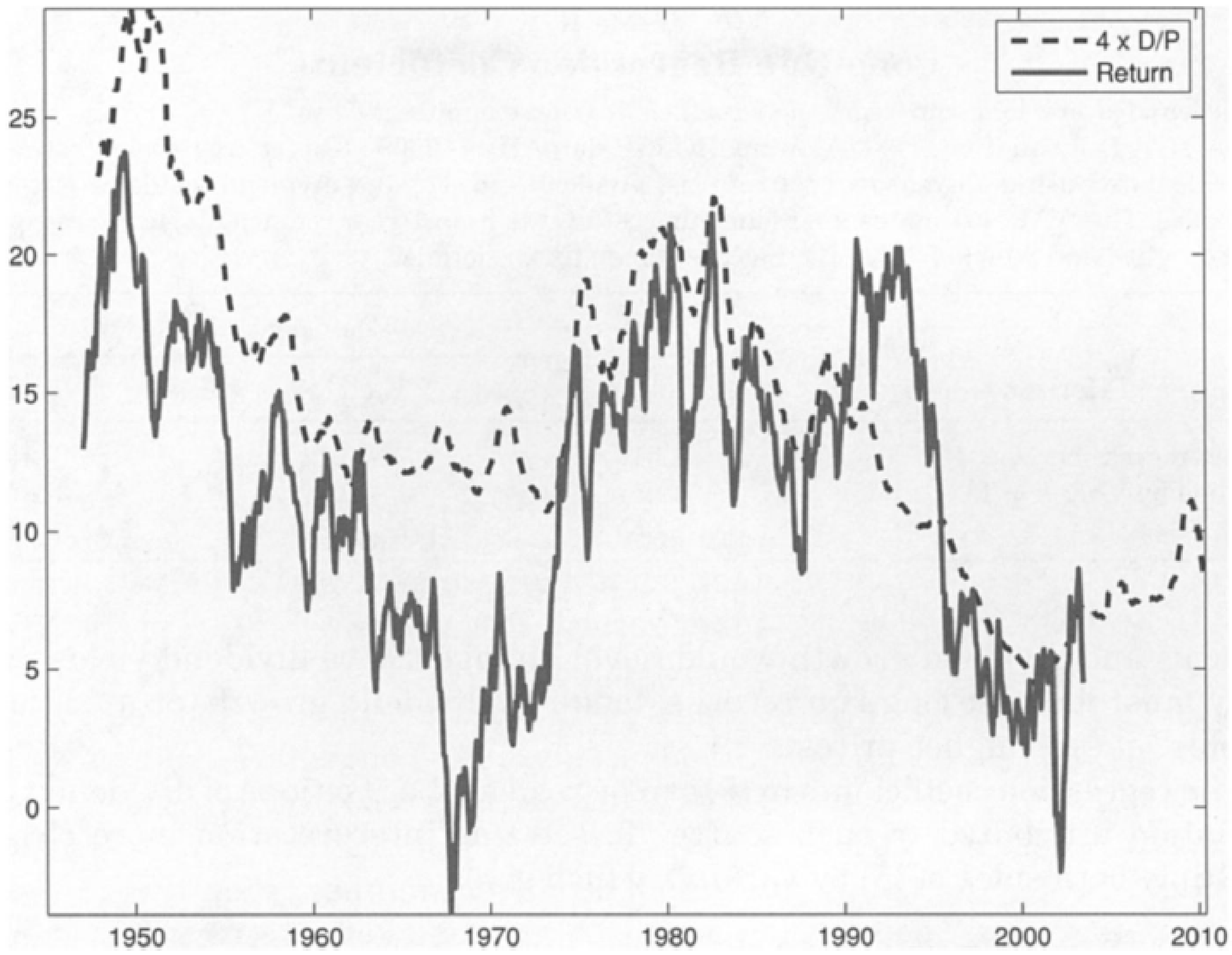


Figure 1. Dividend yield and following 7-year return. The dividend yield is multiplied by four. Both series use the CRSP value-weighted market index.

Comments:

- Past returns do not forecast future returns and the dividend price ratio does not forecast future dividends

- The risk premium is time varying.
  - The conditional expected excess return, $E_t\left(R_{t+1}^e\right)$, varies a lot.
  - The standard deviation of the conditional expected excess return is 5.5%

Comments:

- Returns appear to be predictable: High current price relative to dividends predicts low future returns
  - Prices are high today not because you expect high future dividends, but because you expect low future returns
  - High prices tend to happen in booms when people are willing to take risks, and low prices in recessions when people are not willing to take risks

- Other variables also have predictive power: consumption-to-wealth ratio, term premium, short-term nominal interest rate

Cross sectional evidence:

- Smaller firms have higher returns on average (size premium)
- Firms with low Tobins’ Q (i.e. high book to market value) have higher returns on average (value premium)
  - “Value” stocks have market values that are small relative to the accountant’s book value. Examples: Airlines, Steel Mills or Railroads companies.
  - “Growth” stocks are the opposite of value and have had low average returns. Examples: Google, Apple, Amazon.
- Firms with high recent returns tend to have high returns in near future (momentum anomaly)

Lucas representative agent economy

Preferences:

$$E_0\{\sum_{t=0}^{\infty}\beta^t u(c_t)\},$$

Budget constraints:

$$c_t+\sum_{j=1}^{T} Z_{j,t+1}B_{j,t}+\sum_{i=1}^{N} A_{i,t+1}P_{i,t} = y_t+\sum_{j=0}^{T-1} Z_{j,t}B_{j,t}+\sum_{i=1}^{N} A_{i,t}\left[D_{i,t}+P_{i,t}\right]$$

$P_{i,t}$: price of stock i, y_t : endowment, $B_{j,t}$: price of zero coupon bond that matures at $t+j$, $B_{0,t}=1$, $Z_{j,t}$, and $A_{i,t}$: holdings of j bonds and i shares at the start of t

The $Z_{j,t}$, and $A_{i,t}$ are endogenous choice variables

The $P_{i,t}$, y_t and $B_{j,t}$ are exogenous stochastic processes

The optimal portfolio choice (also known as first order conditions or Euler equations) is:

$$P_{i,t}u'(c_t) = E_t \beta u'(c_{t+1}) \left(D_{i,t+1} + P_{i,t+1}\right), \text{ for } i = 1, ..., N$$

$$B_{j,t}u'(c_t) = E_t \beta u'(c_{t+1}) B_{j-1,t+1}, \text{ for } j = 1, ..., T.$$

These equations can be written as:

$$P_{i,t} = E_t \frac{\beta u'(c_{t+1})}{u'(c_t)} \left(D_{i,t+1} + P_{i,t+1} \right), \text{ for } i = 1, ..., N$$

$$P_{i,t} = E_t \sum_{s=0}^{\infty} \frac{\beta^s u'(c_{t+1+s})}{u'(c_t)} D_{i,t+1+s}, \text{ for } i = 1, ..., N$$

$$B_{j,t} = E_t \frac{\beta u'(c_{t+1})}{u'(c_t)} B_{j-1,t+1}, \text{ for } j = 1, ..., T.$$

Define the rates of return:

$$R_{j,t+1} \equiv \frac{B_{j-1,t+1}}{B_{j,t}}, \text{ for } j = 1, ..., T$$

$$R_{i,t+1} \equiv \frac{D_{i,t+1} + P_{i,t+1}}{P_{i,t}}, \text{ for } i = 1, ..., N$$

In general

$$1 = E_t \frac{\beta u'(c_{t+1})}{u'(c_t)} R_{s,t+1}$$

for the holding returns on all assets $s = i$ or j, $i = 1, ..., N$ or $j = 1, ..., T$.

Assume there is a risk free bond with rate of return R^f_{t+1}

$$1 = R^f_{t+1} E_t \frac{\beta u'(c_{t+1})}{u'(c_t)}$$

Let R_{t+1} be a risky rate of return

$$1 = E_t R_{t+1} \frac{\beta u'(c_{t+1})}{u'(c_t)}$$

Risk neutrality:

Constant $u'(c_t)$

Euler equations imply:

$$E_t R_{t+1} = R^f_{t+1}$$

General framework:

In general the price of any asset is the expected product between the payoff of the asset tomorrow, X_{t+1}, and the pricing kernel (or stochastis discount factor), M_{t+1}.

$$P_t = E_t\left(M_{t+1}X_{t+1}\right)$$

In the case of the Lucas model (or consumption based model)

$$M_{t+1} = \frac{\beta u'(c_{t+1})}{u'(c_t)}$$

and for stocks $X_{t+1} = D_{t+1} + P_{t+1}$.

Implications:

With risk neutrality the price of stock is:

$$P_t = E_t \sum_{s=1}^{\infty} \beta^s D_{t+s}$$

Let $\beta = 1/(1+r)$ and suppose $D_t = (1+g)D_{t-1} + \varepsilon_t$, the expected growth rate of dividends is:

$$E_t D_{t+s} = (1+g)^s D_t$$

then

$$\frac{P_t}{D_t} = \frac{(1+g)}{r-g}$$

For risk-free one-period bond that pays one unit of consumption tomorrow:

$$P_t = E_t M_{t+1}$$

where

$$R_{t+1} = 1/P_t$$

Nominal claims:

$$E_t \left\{ M_{t+1} \frac{X_{t+1}^n}{P_t^n} \frac{1}{1+\pi_{t+1}} \right\} = 1$$

where $R_{t+1}^n = \frac{X_{t+1}^n}{P_t^n}$ is the nominal return, $1+\pi_{t+1} = \frac{P_{t+1}^I}{P_t^I}$, and P_t^I is the price-index (e.g. CPI)

Consumption based asset pricing:

Equating the Euler equations gives:

$$R^f_{t+1} E_t \frac{\beta u'(c_{t+1})}{u'(c_t)} = E_t R_{t+1} \frac{\beta u'(c_{t+1})}{u'(c_t)}$$

Rearranging:

$$-Cov_t \left(R_{t+1}, \frac{\beta u'(c_{t+1})}{u'(c_t)} \right) = \left(E_t R_{t+1} - R^f_{t+1} \right) E_t \frac{\beta u'(c_{t+1})}{u'(c_t)}$$

Risk Premium:

From Euler equation for risk-free asset

$$E_t \frac{\beta u'(c_{t+1})}{u'(c_t)} = \frac{1}{R^f_{t+1}}$$

Therefore:

$$\frac{E_t R_{t+1} - R^f_{t+1}}{R^f_{t+1}} = -Cov_t \left(R_{t+1}, \frac{\beta u'(c_{t+1})}{u'(c_t)} \right)$$

Implications:

- If the risky return covaries positively with tomorrow's consumption, C_{t+1}, then the LHS is positive and the asset return bears a positive premium over the risk free rate.

- If the risky return covaries negatively with tomorrow's consumption then the LHS is negative and the asset return bears a negative premium over the risk free rate.

- Intuition: assets whose returns have a negative covariance with consumption provide a hedge against consumption risk. Households are willing to accept a lower expected return since these assets provide insurance against low future consumption.

Equity premium puzzle:

Assume CRRA:

$$u(c_t) = \frac{c_t^{1-\gamma}}{1-\gamma}$$

The Euler equations are:

$$c_t^{-\gamma} = R_{t+1}^f E_t c_{t+1}^{-\gamma}$$

$$c_t^{-\gamma} = E_t R_{t+1} c_{t+1}^{-\gamma}$$

An approximation to the Euler equations

Let $x_{t+1} = ln(C_{t+1}) - ln(C_t)$; $r_{t+1} = ln(R_{t+1})$, the Euler equation becomes:

$$1 = R_t^f \beta E_t exp(-\gamma x_{t+1})$$

$$1 = \beta E_t exp(-\gamma x_{t+1} + r_{t+1})$$

Assume that consumption growth and asset returns are jointly log-normally distributed

$$\begin{bmatrix} x_{t+1} \\ r_{t+1} \end{bmatrix} \sim N\left(\begin{bmatrix} E\left(x_{t+1}\right) \\ E\left(r_{t+1}\right) \end{bmatrix}, \begin{bmatrix} \sigma^2_{x,t+1} & \sigma^2_{x,r,t+1} \\ \sigma^2_{x,r,t+1} & \sigma^2_{r,t+1} \end{bmatrix} \right)$$

If x is normal distributed then exp(x) is lognormal, and

$$E\,\mathrm{exp}(x) = \mathrm{exp}\left(Ex + \frac{1}{2}Var(x)\right)$$

The Euler equations become

$$1 = \beta exp(-\gamma Ex_{t+1} + r^f_{t+1} + \frac{1}{2}Var(-\gamma x_{t+1}))$$

$$1 = \beta E_t exp(-\gamma Ex_{t+1} + Er_{t+1} + \frac{1}{2}Var(-\gamma x_{t+1} + r_{t+1}))$$

Take logs and equate these equations:

$$\begin{aligned} Er_{t+1} - r^f_{t+1} &= \frac{1}{2}Var(-\gamma x_{t+1}) - \frac{1}{2}Var(-\gamma x_{t+1} + r_{t+1}) \\ &= -\frac{1}{2}Var(r_{t+1}) + \gamma Cov(x_{t+1}, r_{t+1}) \end{aligned}$$

Since $lnE(R_{t+1}) = Er_{t+1} + \frac{1}{2}Var(r_{t+1})$

$$lnE(R_{t+1}) - r^f_{t+1} = \gamma Cov(x_{t+1}, r_{t+1}) = \gamma Corr(x, r)\sigma_x\sigma_r$$

Quantitative implications:

The equity premium is

$$lnE(R_{t+1}) - ln(R^f_{t+1}) = \gamma Corr(x, r)\sigma_x\sigma_r$$

In US data, $\sigma_r = 0.167$; $\sigma_x = 0.036$; $Corr(x, r) = 0.4$ so

If $\gamma = 1$ we have $lnE(R_{t+1}) - ln(R^f_{t+1}) = 0.24\%$

If $\gamma = 10$ we have $lnE(R_{t+1}) - ln(R^f_{t+1}) = 2.4\%$

If $\gamma = 25$ we have $lnE(R_{t+1}) - ln(R^f_{t+1}) = 6\%$

How high is $\gamma = 25$ **?**

Example 1. What would be the interest rate that would make a household that earns $50,000$ euros per year to postpone the annual vacation that costs $3,000$ euros?

$$R^f = \beta E_t \left(\frac{c_{t+1}}{c_t} \right)^{\gamma}$$

taking $\beta = 1, \gamma = 25$ and no uncertainty in the income process get

$$\begin{aligned} R^f_{t+1} &= \left[\frac{\beta u'(c_{t+1})}{u'(c_t)} \right]^{-1} = \left(\frac{53,000}{47,000} \right)^{25} = 20.158 \\ r^f_{t+1} &\approx 1916\% \end{aligned}$$

Example 2. The Consumption Equivalent, CE, of a lottery that gives $50,000$ euros with 50% probability or $100,000$ euros with 50% probability

$$\frac{(CE)^{1-\gamma}}{1-\gamma} = \frac{1}{2}\frac{(50,000)^{1-\gamma}}{1-\gamma} + \frac{1}{2}\frac{(100,000)^{1-\gamma}}{1-\gamma}$$

$$\begin{array}{ll} \gamma = 0 & CE = 75,000 \\ \gamma = 1 & CE = 70,711 \\ \gamma = 2 & CE = 66,246 \\ \gamma = 5 & CE = 58,566 \\ \gamma = 10 & CE = 53,991 \\ \gamma = 20 & CE = 51,858 \\ \gamma = 30 & CE = 51,209 \end{array}$$

The risk free rate:

$$1 = \beta exp(-\gamma E x_{t+1} + r^f_{t+1} + \frac{1}{2} Var(-\gamma x_{t+1}))$$

taking logs

$$0 = \ln \beta - \gamma E x_{t+1} + r^f_{t+1} + \frac{\gamma^2}{2} Var(x_{t+1})$$

rearranging

$$r^f_{t+1} = -\ln \beta + \gamma E x_{t+1} - \frac{\gamma^2}{2} Var(x_{t+1})$$

Quantitative implications:

The risk free rate is:

$$r^f_{t+1} = -\ln\beta + \gamma E x_{t+1} - \frac{\gamma^2}{2} Var(x_{t+1})$$

Suppose $\beta = 0.999$; $E x_{t+1} = 0.015$; $\sigma_x = 0.036$ then we need $\gamma = 0.6$ to obtain $r^f_{t+1} = 1\%$

If $\gamma = 10$ we have $r^f_{t+1} = 22\%$

If $\gamma = 25$ we have $r^f_{t+1} = 78\%$

This is opposite to equity-premium puzzle – we need very low γ to match risk-free rate.

Comments:

1. If consumption growth is iid and homoskedastic, then the risk free rate is constant.

2. Risk free rate is high if the agents are more impatient i.e. have a high relative preference for consumption in the present (low β). In this case agents want to save less, implying a higher interest rate. This is consistent with the equation, since $-\ln\beta$ is decreasing in β.

3. Risk free rate is high when expected consumption growth is high (intertemporal marginal rate of substitution (IMRS)). That is, in order to induce agents to save and consume a lot in the future, the interest rate must be high.

4. Risk free rate is low when conditional consumption volatility is high (precautionary savings or risk aversion). When consumption is more volatile people want to save more driving down the interest rate

5. An higher γ makes r_{t+1}^{f} more sensitive to consumption volatility

To explain these facts, the macro-finance literature explored a wide range of alternative preferences and market structures.

(i) A large differential in the cost of trading between the stock and bond markets; (ii) more general preferences that allow for the separation between risk aversion and IMRS; (iii) incomplete markets, (iv) borrowing constraints, (v) market segmentation (heterogeneity of agents).

(ii) The strategies in the literature boil down to generalize the discount factor to

$$M_{t+1} = \frac{\beta u'(c_{t+1})}{u'(c_t)} Y_{t+1}$$

where the new variable Y_{t+1} does most of the work.