More exercises on stochastic interest rates

1. Exercise: The random variable $(1+i)$ follows a log-normal distribution with parameters μ and σ^2. The mean of the rate of interest is 0.08 and the variance is 0.0049.

(a) Find μ and σ^2.

(b) Find the mean and variance of the accumulation of a unit sum of money for 12 years if the rate of interest, in each year, is log-normally distributed as above and is independent of the rate of interest in any other year.

(c) Find the probability that a unit sum of money will accumulate to less than 2.40.

(d) Find the probability that an investor receives a rate of return between 5% and 7% in all the years.

Answer: (a) 0.074865, 0.0041921, (b) 2.51818, 0.32719, (c) 0.46, (d) $(0.112)^{12}$.

2. Find the lower and upper quartiles for the accumulated value at the end of 5 years of an initial investment os 1000 Euros, assuming that the annual growth rates are i.i.d. and have a log-normal distribution with parameters $\mu = 0.075$ and $\sigma = 0.025$.

Answer: $1401, 1511$.

3. Assume now that your goal is to obtain an accumulated value of an investment equal to av_n. The amount of the investment is not known. Further assume that the random varibal i_t will be the return on the investment in year t, with $t = 1, 2, ..., n$, and that $(1 + i_t)$ is lognormally distributed with parameters μ_t and σ_t. The n random variables i_t are mutually independent. In these circumstances, the present value of the investment may be considered a random variable, denote it by X. Show that the present value of av_n (the investment X) is such that

$$X \sim \log\text{-normal}\left(\log\left(av_n\right) - \sum_{t=1}^{n}\mu_t, \sum_{t=1}^{n}\sigma_t^2\right).$$