# Lecture: Liquidity Provision by Intermediaries and Bank Runs

Advanced Macroeconomics 5/11/2019

## **Some Characteristics of Financial Intermediaries**

- 1. Borrow from one group of economic agents and lend to another.
- 2. Well-diversified with respect to both assets and liabilities.
- 3. Transform assets.
- 4. Diamond-Dybvig (1983) "Bank Runs, Deposit Insurance, and Liquidity", JPE

# The Diamond-Dybvig banking model

- Three periods, 0, 1, and 2.
- Two types of consumers: *impatient* (consume in period 1) and *patient* (consume in period 2)
- In period 0 they do not know their type. They learn their type in period 1
- Asymmetric information: Types are consumers private information
- Efficient economic arrangement is for consumers to set up a bank in order to share risk.
- Given the bank's deposit contract, the bank is open to a *run*, which is a bad equilibrium.

# Assumptions (I)

Each consumer has 1 unit of the good in period **O**.

Production technology takes 1 unit of good in period 0 and converts into (1 + r) units of the consumption good in period 2.

However, this production technology can also be interrupted in period **1**.

If interruption occurs in period 1, then 1 unit of consumption goods can be obtained for each unit of the good invested in period 0.

## **Assumptions (II)**

In period 0, each consumer knows that he or she has a probability *t* of being an early consumer and probability **1** - *t* of being a late consumer

In period 1, tN consumers learn that they are early consumers and (1 - t)N consumers learn that they are late consumers. We have 0 < t < 1.

In period 0 consumers maximize expected utility:

 $u(c_1, c_2) = u(c_1)$  with prob. *t* if impatient or  $u(c_1, c_2) = u(c_1 + c_2)$  with prob. 1-*t* if patient

# Marginal rate of substitution

• The marginal rate of substitution of early consumption for late consumption is

$$MRS_{c_1,c_2} = \frac{tMU_{c_1}}{(1-t)MU_{c_2}},$$

#### The preferences of a Diamond-Dybvig consumer



c<sub>1</sub> = Early Consumption

#### Autarcy

Invests all of his or her unit of endowment in the technology in period **0**.

Then, in period 1, if he or she is an early consumer, then he or she interrupts the technology and is able to consume  $c_1 = 1$ 

If he or she is a late consumer, then the technology is not interrupted and the consumer gets  $c_2 = 1 + r$  in period 2 when the investment matures.

Want to show that a bank allows all consumers to do better than this.

#### **Constraints on the Deposit Contract**

Let x be the fraction of the investment to interrupt

 $Ntc_1 = xN$  $N(1-t)c_2 = (1-x)N(1+r).$ 

Combine the two constraints to get one:

$$tc_1 + \frac{(1-t)c_2}{1+r} = 1,$$

#### **Rewrite the constraint**

$$c_2 = -\frac{t(1+r)}{1-t}c_1 + \frac{1+r}{1-t},$$

$$c_1 = 0 \text{ implies } c_2 = \frac{1+r}{1-t}$$
$$c_2 = 0 \text{ implies } c_1 = \frac{1}{t}$$

#### **Incentive compatibility constraint**

The patient consumer should not want to withdraw earlier

$$c_1 < c_2$$

## **Characterizing the optimal alocation**

Optimal allocation: First ignore the (IC) constraint (will verify that it will hold)

The problem is to maximize the expected utility s.t. the resource constraint. The FOC is:

$$u'\!\left(\!c_1^*\right)\!=\!(1\!+\!r)u'\!\left(\!c_2^*\right)$$
 This implies:  $c_1^* < c_2^*$ 

It follows the (IC) constraint is automatically satisfied The optimal allocation equates the MRS to the technological price

#### The equilibrium contract offered by the Diamond Dybvig bank



### **Optimality of banks**

Let the utility function be : 
$$\log(c) = \frac{c^{1-\sigma}}{1-\sigma}$$
, with  $\sigma > 1$ 

$$MU_c = c^{-\sigma}$$

$$\frac{MU_{\mathcal{C}_{1}}}{MU_{\mathcal{C}_{2}}} = \left(\frac{c_{2}}{c_{1}}\right)^{\sigma}$$

in autarcy the expression above is  $(1+r)^{\sigma}$ 

### **Autarcy vs Optimality**

The slope of the indifference curve at B is: 
$$-\frac{t}{1-t}$$

The slope of the ind. curve at D is: 
$$-\frac{t}{1-t}(1+r)^{\sigma}$$

The slope of the curve at A is: 
$$-\frac{t}{1-t}\left(\frac{c_2}{c_1}\right)^{\sigma}$$
  
At the optimum:  $\left(\frac{c_2}{c_1}\right)^{\sigma} = (1+r)$ 

For  $\sigma = 1$ , log utility the optimum is the autarcy. If  $\sigma > 1$  then the bank provides **liquidity insurance.** Why?

## **Optimal contract**

- Point *D* is autarcy (1, 1 + r)
- Point A is the optimal point which lies to the southeast of point D
- By accepting the banking contract, the consumer is able to consume more in period 1 at the expense of lower consumption in period 2
- The Diamond-Dybvig bank has some of the properties of financial intermediaries
- The bank holds illiquid assets and is able to convert these assets into liquid deposits

## Bank Runs in the Diamond-Dybvig Model

- Suppose that a late consumer believes that all other late consumers will go to the bank to withdraw in period 1.
- If all late consumers think that then there will be a **bank run**.
- Proof: Since  $c_1 > 1$  at point A, the amount left for period 2 would be

 $\max\{N - (N - 1)c_1, 0\} < 1$ 

• Thus, the individual late consumer prefers to go to the line hoping that he can get paid, this is a **bank-run** 

#### How to avoid a bank run?

- Suspension of Convertibility: At date 1 only serve the first *tN* depositors
- Deposit insurance: The government guarantees that the promised return will be paid to all depositors