Microeconomics

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Chapter 4:
Partial equilibrium

Market demand

We let $I \equiv 1, \ldots, I$ index the set of individual buyers and $q^i(p, \mathbf{p}, y^i)$ be i's non-negative demand for good q as a function of its own price p, income y^i, and prices $\mathbf{p}$ for all other goods. **Market demand** for q is simply the sum of all buyers' individual demands

$$q^d(p) \equiv \sum_{i \in I} q^i(p, \mathbf{p}, y^i).$$

Market supply

We let $J \equiv 1, \ldots, J$ index the firms in the market and are able to be up and running by acquiring the necessary variable inputs. The **short-run market supply** function is the sum of individual short-run supply functions $q^j(p, w)$:

$$q^s(p) \equiv \sum_{j \in J} q^j(p, w).$$

Short-run competitive equilibrium

Market demand and market supply together determine the price and total quantity traded. We say that a competitive market is in **short-run equilibrium** at price p^* when $q^d(p^*) = q^s(p^*)$.

Long-run competitive equilibrium

In a **long-run equilibrium**, we require not only that the market clears but also that no firm has an incentive to enter or exit the industry. Two conditions characterise long-run equilibrium in a competitive market:

$$q^d(\hat{p}) = \sum_{j=1}^{\hat{J}} q^j(\hat{p}),$$
$$\pi^j(\hat{p}) = 0,\ j = 1, \ldots, \hat{J}.$$

Monopoly

The monopolist's problem is:

$$Max_q \pi(q) \equiv p(q)q - c(q) \text{ s.t. } q \geq 0.$$

If the solution is interior,

$$mr(q^*) = mc(q^*).$$

Equilibrium price will be $p^* = p(q^*)$, where $p(q)$ is the inverse market demand function.

Monopoly

Alternatively, equilibrium satisfies:

$$p(q^*)\left[1 + \frac{1}{\epsilon(q^*)}\right] = mc(q^*) \geq 0,$$

or:

$$\frac{p(q^*) - mc(q^*)}{p(q^*)} = \frac{1}{|\epsilon(q^*)|}.$$

Cournot oligopoly

Suppose there are J identical firms, that entry by additional firms is effectively blocked, and that each firm has identical cost, $C(q^j) = cq^j$, $c \geq 0$ and $j = 1, \ldots, J$.
Firms sell output on a common market price that depends on the total output sold by all firms in the market. Let inverse market demand be the of linear form,

$$p = a - b \sum_{j=1}^{J} q^j,$$

where $a > 0$, $b > 0$, and we require $a > c$.
Firm j's problem is:

$$Max_{q^j} \pi^j(q^1, \ldots, q^J) = \left(a - b \sum_{k=1}^{J} q^k\right) q^j - cq^j \text{ s.t. } q^j \geq 0.$$

Bertrand oligopoly

In a simple Bertrand duopoly, two firms produce a homogeneous good, each has identical marginal costs $c > 0$ and no fixed cost. For easy comparison with the Cournot case, we can suppose that market demand is linear in total output Q and write:

$$Q = \alpha - \beta p,$$

where p is the market price.
Firm 1's problem is:

$$Max_{p^1} \pi^1(p^1, p^2) = \begin{cases} (p^1 - c)(\alpha - \beta p^1), & c < p^1 < p^2, \\ \frac{1}{2}(p^1 - c)(\alpha - \beta p^1), & c < p^1 = p^2, \\ 0, & \text{otherwise.} \end{cases}$$

Monopolistic competition

Assume a potentially infinite number of possible product variants $j = 1, 2, \dots$. The demand for product j depends on its own price and the prices of all other variants. We write demand for j as $q^j = q^j(p)$, where $\partial q^j/\partial p^j < 0$ and $\partial q^j/\partial p^k > 0$ for $k \neq j$, and $p = (p^1, \dots, p^j, \dots)$. In addition, we assume there is always some price $\tilde{p}^j > 0$ at which demand for j is zero, regardless of the prices of the other products.
Firm j's problem is:

$$Max_{p^j} \pi^j(p) = q^j(p)p^j - c^j(q^j(p)).$$

Two classes of equilibria can be distinguished in monopolistic competition: short-run and long-run.

Short-run equilibrium

Let $j = 1, \ldots, \bar{J}$ be the active firms in the short run.
Suppose $\bar{p} = (\bar{p}^1, \ldots, \bar{p}^j)$ is a Nash equilibrium in the short run. If $\bar{p}^j = \tilde{p}^j$, then $q^j(\bar{p}) = 0$ and firm j suffers losses equal to short-run fixed cost, $\pi^j = -c^j(0)$. However, if $0 < \bar{p}^j < \tilde{p}^j$, then firm j produces a positive output and $\bar{p}$ must satisfy the first-order conditions for an interior maximum:

$$\frac{\partial q^j(\bar{p})}{\partial p^j}[mr^j(q^j(\bar{p})) - mc^j(q^j(\bar{p}))] = 0.$$

Long-run equilibrium

Let that p^* be a Nash equilibrium vector of long-run prices. Then the following two conditions must hold for all active firms j:

$$\frac{\partial q^j(p^*)}{\partial p^j}[mr^j(q^j(p^*) - mc^j(q^j(p^*)] = 0.$$
$$\pi_j(q_j(p^*)) = 0.$$