

# Advanced Macro

ISEG

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## Lecture I:

# Foundations of Fiscal Policy Analysis

# Bibliography

- ❑ Ljungqvist & Sargent, *Recursive Macroeconomic Theory*, 2ª edição, capítulo 10\*
- ❑ Woodford, M., “Simple Analytics of the Government Expenditure Multiplier”, *American Economic Journal*, 3, pp.1-35, 2011\*
- ❑ Ilzetzki, E., Mendoza, E, & Vegh, C. “How Big (Small?) are Fiscal Multipliers?”, NBER working paper 16479, 2010.\*
- ❑ Ramey, Valerie, “Can Government Purchases Stimulate the Economy”? JEL, 2011
- ❑ Bohn, H. “The Behavior of US Public Debt and Deficits”, *Quarterly Journal of Economics*, 113, pp.949-964, 1997.

# Ricardian Equivalence

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## A Basic Set-up (Ljungqvist & Sargent, ch. 10)

Household preferences (over a single consumption good):

$$\sum_{t=0}^{\infty} \beta^t u(c_t) \quad (1.1)$$

where  $\lim_{c \rightarrow 0} u'(c) = \infty$  (Inada's condition).  $c \geq 0$  throughout.

Budget constraint: 
$$c_t + q_t b_{t+1} \leq y_t + b_t \quad (1.2)$$

## Ricardian Equivalence (cont.)

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where:  $b$  = risk-free government (or foreign) bond

$q_t = 1/R =$  (time-invariant) bond price, with  $R > 1$ .

Further assume: A1)  $\beta R = 1$  (to eliminate trended consumption)

A2)  $y_t$  is deterministic and  $\sum_{t=0}^{\infty} \beta^t y_t < \infty$

A3)  $b_0$  is given.

This is our basic set-up on the household side.

# Ricardian Equivalence (cont.)

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- The ball game at this point is to impose restrictions on and see what happens to household consumption,  $c_t$ , when government enters the picture.

$$\{b_t\}_0^\infty$$

Key: the government will not face the same restrictions on borrowing as the household, so its intervention (e.g. through changes in taxation path) can change  $c_t$ .

## Ricardian Equivalence (cont.)

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But before introducing government, let's develop some intuition as to what restrictions on the sequence of asset (bond) holdings  $\{b_t\}_0^\infty$  do to the path of consumption under various scenarios for endowment income ( $y_t$ ).

As in L-S, consider two forms of borrowing constraints:

- i) agents can never borrow, i.e.,  $b_t \geq 0, \quad \forall t.$
- ii) agents can borrow up to a “natural borrowing limit”,  $\bar{b}_t$

## Ricardian Equivalence (cont.)

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where  $\bar{b}_t = -\sum_{j=0}^{\infty} R^{-j} y_{t+j}$ , with Ponzi schemes ruled out.

Hence, under the natural borrowing limit, households can actually borrow in net terms, this implies a less stringent borrowing constraint than (i).

To see the implications, consider the FOC using (1.1) & (1.2):

$$u'(c_t) \geq \beta R u'(c_{t+1}) \quad (1.3)$$

$\beta R=1$  and (1.2) imply that  $c_t=c_{t+1}$  when  $b_{t+1}>0 \rightarrow c_t$  is smoothed!



## Ricardian Equivalence (cont.)

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But when  $b_{t+1}=0$ , the borrowing constraint will bind, so

$$u'(c_t) > \beta R u'(c_{t+1}) \because c_t < c_{t+1}$$

Since then  $c_t = y_t + b_t$ , household's consumption is constrained. In particular, if  $b_0=0$ ,  $c_0 = y_0$ , so consumption smoothing is not warranted.

Proposition 1.1: Under strict no-borrowing constraint  $b_t \geq 0$ ,  $\forall t$

the household will **not** be able to stabilize consumption under all possible endowment paths,  $\{y_t\}_{t=0}^{\infty}$ .

## Ricardian Equivalence (cont.)

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Illustration of Proposition 1.1 (L-S ch. 10, ex. 2):

Let  $b_0=0$  and  $\{y_t\}_{t=0}^{\infty} = \{y_l, y_h, y_l, \dots\}$ . Recall that if the household faces a non-borrowing constraint,  $b_1=0$ . From (1.2)

$c_0 = y_l < PV \{y_t\}_0^{\infty}$ . So, consumption in  $t=0$  will be smaller than life-time income, and the household will not be able to smooth consumption for all  $t$ . ▣

But full consumption smoothing is achieved if the sequence

$\{y_t\}_{t=0}^{\infty} = \{y_h, y_l, y_h, \dots\}$ . **Homework: go through the derivation in L-S!**

# Ricardian Equivalence (cont.)

## Introducing Government

Let fiscal policy be one in which the path of government spending (per household),  $\{g_t\}_{t=0}^{\infty}$ , is fixed and that of lump-sum taxation,  $\{\tau_t\}_{t=0}^{\infty}$  can vary.

The government's budget constraint is:

$$B_t + g_t = \tau_t + R^{-1}B_{t+1} \quad (1.4)$$

Solving forward & ruling out Ponzi schemes thus yields:

$$B_t = \sum_{j=0}^{\infty} R^{-j} (\tau_{t+j} - g_{t+j}) \quad (1.5)$$

## Ricardian Equivalence (cont.)

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The household budget constraint is now:

$$c_t + R^{-1}b_{t+1} \leq y_t - \tau_t + b_t$$

Solving forward thus yields: 
$$b_t = -\sum_{j=0}^{\infty} R^{-j} (y_{t+j} - c_{t+j} - \tau_{t+j}) \quad (1.6)$$

Consider now again the natural borrowing limit with government. Set  $c_t=0$  for all  $t$ , and the debt limit will be:

$$b_t \geq -\sum_{j=0}^{\infty} R^{-j} (y_{t+j} - \tau_{t+j})$$

Which is clearly absolutely lower than the one without taxes. **So, households will typically more constrained in dis-saving!**

## Ricardian Equivalence (cont.)

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This sets us ready for a key Ricardian proposition:

**Under the natural debt limit**, given  $(b_0$  and  $B_0)$ , if  $\{\bar{c}_t, \bar{b}_{t+1}, \bar{g}_t, \bar{\tau}_t, \bar{B}_t\}$  is an equilibrium, there is also an equilibrium where  $\{\hat{c}_t, \hat{b}_{t+1}, \bar{g}_t, \hat{\tau}_t, \hat{B}_t\}$  provided that 
$$\sum_{j=0}^{\infty} R^{-j} \hat{\tau}_{t+j} = \sum_{j=0}^{\infty} R^{-j} \bar{\tau}_{t+j}.$$

Intuition of the proof: Under the natural debt limit the household budget set depends only on the present value of taxes, rather than on the current tax rate (cf. l.4). Since the present value of taxes is unchanged, so will be consumption for a given path of income.  $b$  and  $B$  will adjust minus one to one with  $\tau$ , so  $c$  stays put, i.e.,  $\tau_1 > \tau_0$ ,  $b_{t+1} < b_{t+0}$  .

## Ricardian Equivalence (cont.)

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But things change under the stricter no-borrowing constraint, i.e.,  $b_t \geq 0$  for all  $t$ . Now the household budget varies period by period, i.e., with  $b_{t+1}=0$ , we have:

$$c_t = y_t - \tau_t + b_t$$

For  $c$  to remain unaltered given  $y$ , then changes in  $b$  will have to offset changes in  $\tau$ . But  $b_t \geq 0$  requirement means that there is a limit to this offset: some values of  $\tau$  may require  $c$  to change!

In general: if borrowing constraints are tougher than the natural one, Ricardian eq. is less likely to hold.

## Ricardian Equivalence (cont.)

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But note that when  $b_{t+1} > 0$ , the RE results can be recovered. In particular, if the agent starts with positive assets, RE will hold for tax changes that do not lead to the corner of  $b_{t+1} = 0$

### Homework:

- 1) Show proposition 2 of ch. 10 of L-S
- 2) Show why RE does not hold with finite horizon but is recovered with a bequest motive that is stringent enough.

# Ricardian Eq. & Fiscal Multipliers

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- The recent global recession has re-kindled the debate on the neutrality of fiscal policy.
- Under RE, fiscal policy is neutral. E.g. Lowering T today means higher T in the future so that the present value of tax revenues does not change (i.e.,  
$$\sum_{j=0}^{\infty} R^{-j} \hat{\tau}_{t+j} = \sum_{j=0}^{\infty} R^{-j} \bar{\tau}_{t+j}$$
- But this means that households will save more by:

$$\bar{c}_t + \frac{b_{t+1}^{\uparrow}}{R} = y_t - \tau_t^{\downarrow} + b_t$$



# Ricardian Eq. & Fiscal Multipliers

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- But in the aggregate we have  $c_t + g_t = Y_t$ , if neither  $c$  nor  $g$  move, then output remains the same.
- Hence the economy cannot be jump-started by a deficit resulting of lowering taxes  $\rightarrow$  the fiscal multiplier is zero!
- But how about changes in  $G$ ? And how about if  $R$  is no longer constant as previously assumed?
- Clearly, one needs to look at this from a general equilibrium (GE) perspective.

# The Government Spending Multiplier

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## A simple G.E. framework for gauging the spending multiplier

(based on Woodford, 2011)

Preferences: 
$$\sum_{t=0}^{\infty} \beta^t [u(C_t) - v(N_t)] \quad (1.7)$$

where  $u' > 0, u'' < 0, v' > 0, v'' > 0$

Let's put some standard functional forms into (1.7):

$$u(C) = \frac{C^{1+\sigma}}{1+\sigma}, \quad v(N) = \frac{N^{1+\rho}}{1+\rho}$$

# The Government Spending Multiplier

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Production: 
$$Y_t = f(N_t) = A_t N_t \quad (1.8)$$

To simplify, normalize  $A_t=1$ , so  $Y_t=N_t$ .

MRS: 
$$\frac{v'}{u'} = \frac{W_t}{P_t} \quad (1.9)$$

Perfect Competition in factor markets: 
$$f'(N_t) = \frac{W_t}{P_t} = A_t = 1 \quad (1.10)$$

# The Government Spending Multiplier

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Combine (1.9) and (1.10) to obtain:

$$u'(C_t) = v'(Y_t) \quad (1.11)$$

But in the closed economy, recall that:

$$Y_t = C_t + G_t \quad (1.12)$$

(1.12) into (1.11):

$$u'(Y_t - G_t) = v'(Y_t)$$

We are almost there.. Now differentiate:

# The Government Spending Multiplier

$$u'' dY_t - u'' dG_t = v'' dY_t$$

Dividing through by  $u'$  and recalling that  $u' = v'$ :

$$\frac{u''}{u'} dY_t - \frac{u''}{u'} dG_t = \frac{v''}{v'} dY_t$$

$$\therefore dY_t \left[ \frac{v''}{v'} - \frac{u''}{u'} \right] = -\frac{u''}{u'} dG_t \quad (1.13)$$

# The Government Spending Multiplier

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From the chosen functional forms and  $Y=N$ , we have:

$$\frac{u''}{u'} = \frac{-\sigma}{C}, \quad \frac{v''}{v'} = \frac{\rho}{N} = \frac{\rho}{Y}$$

Substituting into (I.12):

$$dY_t \left[ \frac{\rho}{Y_t} + \frac{\sigma}{C_t} \right] = \frac{\sigma}{C_t} dG_t$$

Dividing it through by C and arranging yields:

$$\frac{dY}{dG} = \frac{\sigma}{\sigma + \rho(\bar{C} / \bar{Y})} < 1 \quad (\text{I.14})$$

# The Government Spending Multiplier

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The multiplier is thus lower the lower the  $\sigma$  and the higher  $\rho$ .

- Role of  $\sigma$ : the less risk averse the representative household, the lower the multiplier. Since with CARA utility, the degree of inter-temporal substitution in consumption is  $1/\sigma$ , this is equivalent to saying that **the higher the degree of inter-temporal substitution, the lower the multiplier.**

This is intuitive: if households don't care much about whether they consume now vs. later, they will cut consumption more when government spending is higher, so there is greater "Ricardian offset". *Lower  $\sigma$  gets us closer to Ricardian equivalence!*

# The Government Spending Multiplier

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- Role of  $\rho$ : it is also intuitive that higher degree of labor disutility,  $\rho$ , gets us closer to Ricardian equivalence.

To see this, recall that  $(1/\rho)$  is the elasticity of labor supply. If labor is less elastic, ie.  $\rho$  is higher, workers will demand higher wages per unit of employment. So, higher  $G$  will raise more the marginal cost of production, crowding out employment. Since  $Y=f(N)$ ,  $Y$  will be lower; given  $A$ , the multiplier will decline on  $\rho$ .

*Hence, Lower labor supply elasticity ( $=1/\rho$ ) also gets us closer to Ricardian equivalence!*



# The Government Spending Multiplier

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- Let's now consider what  $dY/dG$  would roughly be for standard calibrations found in the real business cycle (RBC) literature. E.g.:  $C/Y=0.8$ ,  $\beta=2$ ,  $\alpha=3$ .

$$\frac{dY}{dG} = \frac{2}{2 + 3 * 0.8} \approx 0.45$$

- So, below 1 but not so low!

# The Government Spending Multiplier

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## Extensions & Modifications to the above neo-classical setting

- Introducing monopolistic competition in goods markets: No change  
(but do check the formalization in Woodford, 2011 pp.4-6)

Intuition: monopolistic competition introduces a wedge (mark-up) in the relation between prices and marginal costs; if this wedge is fixed, it will wash out in the differentiation.

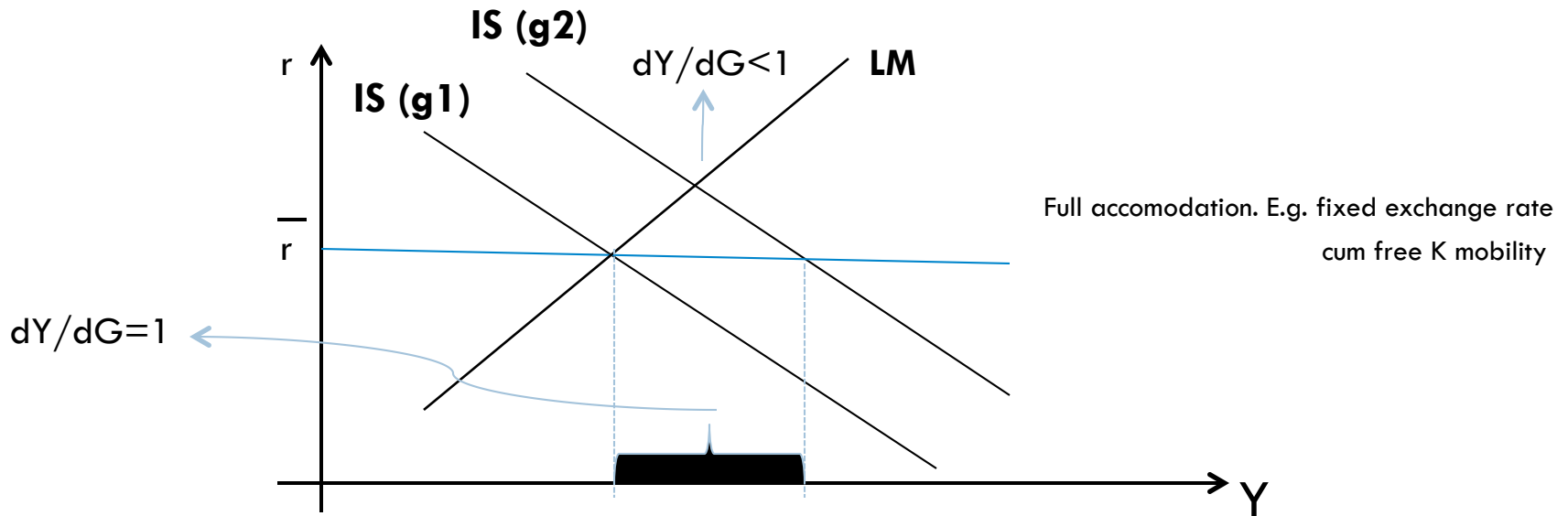
# The Government Spending Multiplier

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## Extensions & Modifications to the neo-classical setting (cont.)

- Allowing for sticky prices and distinct monetary accommodation:

The good old IS-LM



# The Government Spending Multiplier

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- When there is full accommodation:  $r = \bar{r}$
- With  $r$  unchanged and  $\beta R = 1$ ,  $C_t = C_{t+1} = \bar{C}$ . Hence,  $Y_t = \bar{C} + G_t$  and the multiplier is thus  $dY/dG = 1$ .
- This is the standard Keynesian textbook case: there is no crowding out of private expenditure, but there is also no additional stimulus of additional private consumption.
- For private spending to react positively, you need  $dY/dG > 1$ .

# The Government Spending Multiplier

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Some interesting features about this familiar result in an optimizing setting.

- One is that it is independent from the degree of wage and price rigidity. *It only matters that there is some rigidity, so as to enable a central bank to stabilize  $r$  despite rising  $G$ .*
- If prices are fully flexible, then when  $G$  rises, inflation will go up, and to stabilize prices the central bank will have to increase  $i$  by more than  $\pi$  (as per the Taylor rule), raising  $r$ .
- We are then back to the neo-classical setting where  $dY/dG < 1$

# The Government Spending Multiplier

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- Another important point is that the new Keynesian model with price rigidity can also generate  $dY/dG < 1$  and in fact  $dY/dG \ll 1$ !
- *That is, the new Keynesian model can produce multipliers larger as well as smaller than in the neo-classical model!*
- All will depend on the degree of monetary policy accommodation of the fiscal expansion.
- In the zero bound:  $i=0$ , higher  $G$  will raise  $E(\pi)$ . Hence  $r=i-E(\pi)$ .  
So, now  $C_t > C_{t-1}$ , i.e.,  $dY/dG > 1$ !



# The Fiscal Multiplier: Empirical Evidence

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- Highly topical, hotly debated issue.
- Very complex too, so one can get easily confused with too many analytical layers.
- So, a good illustration for the kind of analytical and practical problems faced by the economic analyst in using theory to make sense of data...
- and for the policy maker trying to distill implications for policy design.

# The Fiscal Multiplier: Empirical Evidence

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- First analytical cut: Spending vs. the Tax multiplier
- Second analytical cut: Short vs. Long-Run Multiplier
- Third analytical cut: Average vs. Peak Multiplier
- Fourth analytical cut: Length of the fiscal stimulus and implications for the sustainability of fiscal policy. If unsustainable,  $r$  higher and the multiplier smaller.
- Fifth analytical cut: Closed vs. Open Economy (2<sup>nd</sup> half)



# The Fiscal Multiplier: Empirical Evidence

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## Summary of Findings of Existing Studies

- Estimates for tax multipliers (over both short and long run) have large variance: -0.5 to -5!
- Estimates for spending multipliers are also disparate (again over both short and long run) but usually within a narrower range: 0.5 to 2.
- Length of the fiscal stimulus matters: “Long run” (cumulative multipliers) are often larger than short-run ones
- A higher long-run multiplier is consistent with textbook Keynesian model  $dY/dG = 1 / (1 - mgpc)$ ,  $mgpc$  higher in long-run.

# The Fiscal Multiplier: Empirical Evidence

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- Interestingly (as noted in Ramey, 2011, p.679), the range of estimates *within* studies is almost as wide as *across* studies.
- Hence studies concur that estimation is imprecise but spending multipliers are not trivially low, nor crazily high.
- Also consistent with theory, spending multipliers tend to be lower when financed by distortionary taxation.
- Because of the complex effects of distortionary taxes on the multiplier (e.g. effects on labor supply decisions), some studies control for taxation changes. Ramey then gets  $dY/dG \sim 1$ .

# The Fiscal Multiplier: Empirical Evidence

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- As often in Economics, a key difficulty in pinning down  $dG \rightarrow dY$  is reverse causality, esp in advanced countries where  $G$  increases as  $Y$  goes down (e.g. unemployment and social benefits) .
- So, a common approach is to set-up a VAR of the form:

$$Y_{n,t} = \sum_{k=1}^K A_k Y_{n,t-k} + Bu_{n,t} \quad (I.15)$$

where

$$Y_{n,t} = (g_{n,t}, y_{n,t}, \text{others})$$

# The Fiscal Multiplier: Empirical Evidence

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□ Short-run (“impact”) multiplier:  $\equiv \frac{\Delta Y_0}{\Delta G_0}$

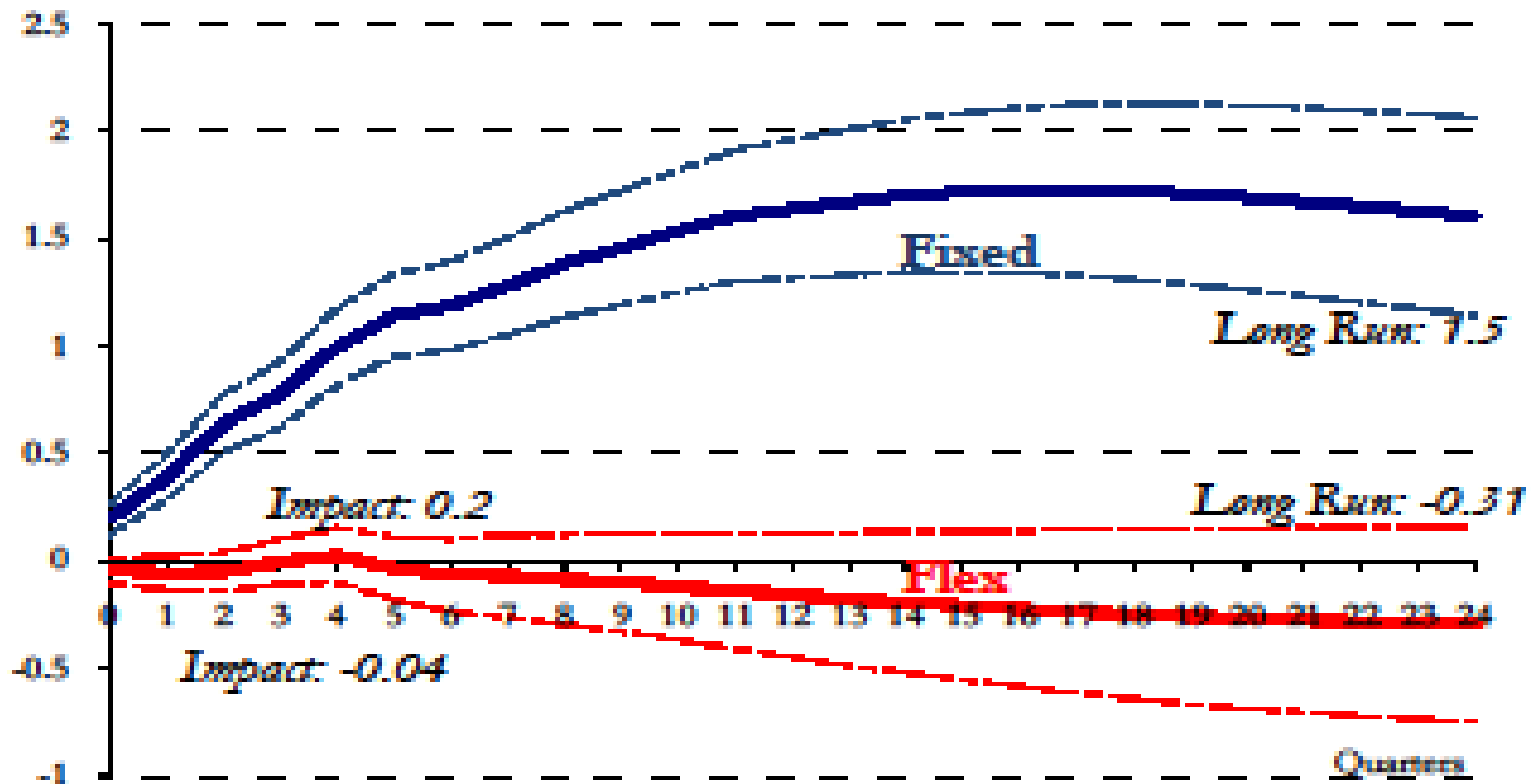
□ Long-run multiplier:  $\equiv \frac{\sum_{j=0}^N \Delta Y_{t+j}}{\sum_{j=0}^N \Delta G_{t+j}}$

□ Peak Multiplier:  $\equiv \max_N \frac{\Delta Y_{t+N}}{\Delta G_t}$

# The Fiscal Multiplier: Empirical Evidence

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**Figure 1: Fiscal Expenditure Multipliers Across Monetary Regimes**  
(from Itzezlki, Mendoza and Vegh, NBER WP, 2010)



# The Fiscal Multiplier: Empirical Evidence

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- But two main problems with this VAR approach.
- One is identify the “autonomous”  $g$  shock: Ilzetzki, Mendoza and Vegh (2010) use lags and Cholesky identification schemes, but these are strong assumptions
- Another way is to look for exogenous drivers (“instruments”) of  $G$ . One is military spending (Ramey, 2011 and Barro & Redlick, 2011). Another is the “narrative approach” of Romer and Romer. See the you tube video by Valerie Ramey: <http://www.youtube.com/watch?v=eSQN-mMjJd4>
- Another problem is what to put in “others”, e.g. the kind of monetary policy or regime will influence  $dY/dG$ , as just seen.

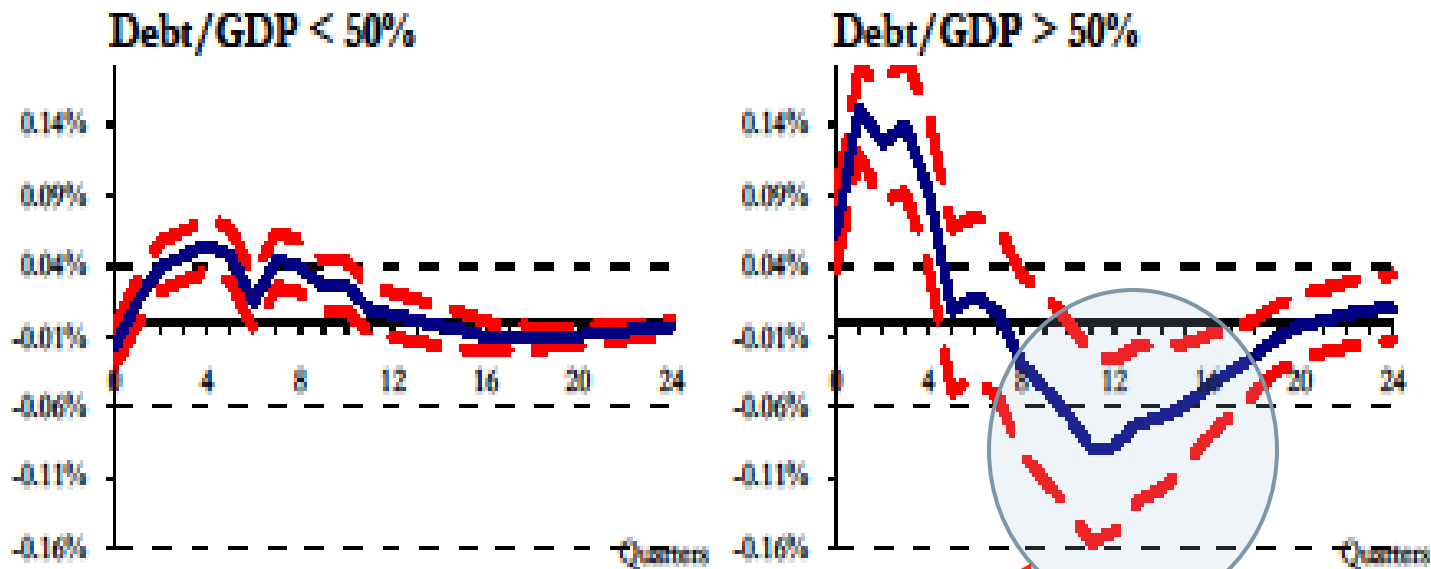
# The Fiscal Multiplier: Empirical Evidence

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- The other is that if the fiscal stimulus is sufficiently recurrent and persistent, debt will built-up.
- This may raise the risk of government insolvency (more on solvency and tests thereof in a few minutes).
- Greater solvency/default risk will raise  $r$ : as we saw this is like having a steeper LM curve, reducing the multiplier.
- In short: one might expect the multiplier to be lower (or even negative) for more indebted countries.

# The Fiscal Multiplier: Empirical Evidence

**Figure 2: Fiscal Expenditure Multipliers under Lower vs. Higher Debt**  
(from Itzezki, Mendoza and Vegh, NBER WP, 2010)



Negative multiplier



## The Fiscal Multiplier: Some Bottom Line

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- Bottom-line: multiplier not zero – so full-fledge Ricardian equivalence fails -- but not  $>1$  to many estimates.
- In many empirical/simulation applications (as we will see in the second half of the course), it is common to assume or impose a “*Ricardian offset*” of around 0.5.
- That is, if government consumption rises by one dollar, private consumption declines only by 50 cents.
- There is also concern that multipliers may be negative (as seen in Figure 2) if fiscal sustainability is jeopardized by prolonged fiscal stimuli. We turn to this next.

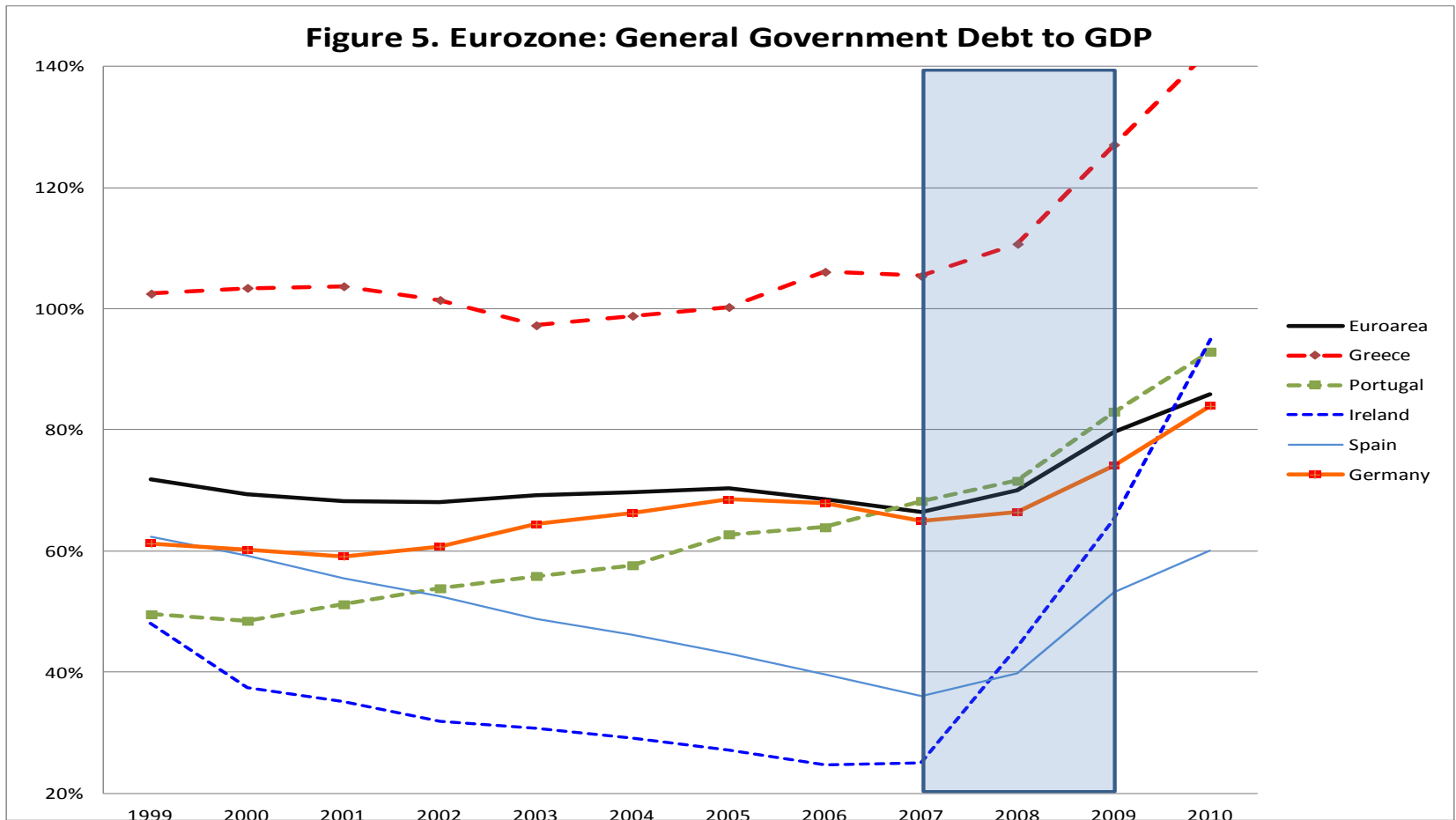
# Public Debt and Fiscal Sustainability

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- A main problem with persistent fiscal stimulus is the build-up of public debt.
- If debt/GDP ratio is too high, markets start doubting government solvency.
- If the risk of a default on public bonds rises, then markets will demand higher interest rates, i.e., a higher *spread* over the “risk-free” interest rate (the so-called “default” or “risk” premium).

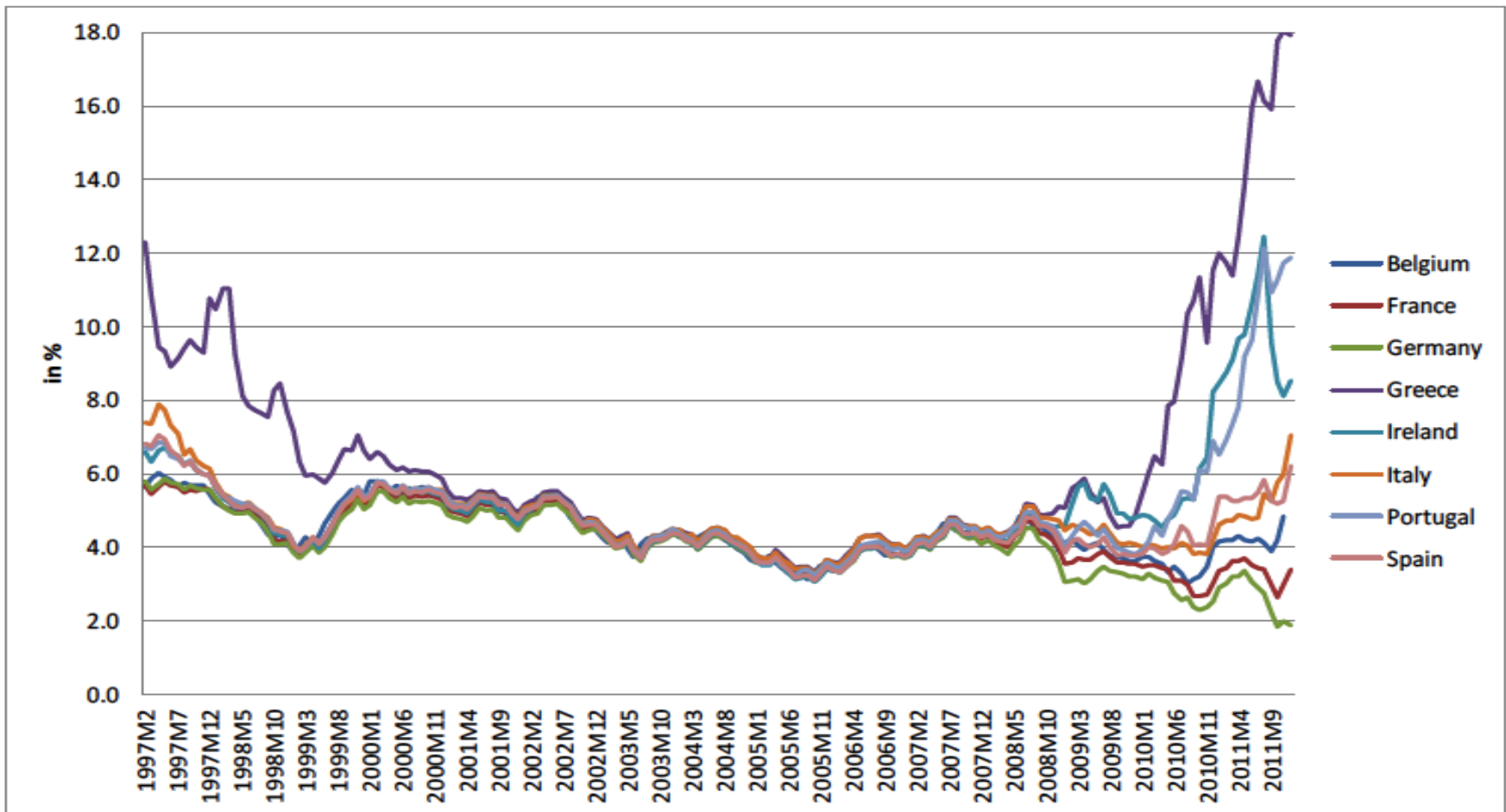
# Public Debt and Fiscal Sustainability

**Figure 3. Public Debt in the Eurozone**  
(from Catão, Fostel, Ranciere, 2012)



# Public Debt and Fiscal Sustainability

**Figure 4. Interest Rates on Public Bonds in Selected Eurozone Countries**  
(from Catão, Fostel, Ranciere, 2012)



# Public Debt and Fiscal Sustainability

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- To examine government solvency, a first step is to start with the government budget constraint.
- To simplify, assume away money (“seignorage”) financing. (We will discuss that later), so as in (1.4):

$$B_t + G_t^P - T_t = R_{t+1}^{-1} B_{t+1} \quad (1.16)$$

where  $G^P$  is government primary expenditure (total  $G$  - interest payments on public debt,  $B$ ) in nominal euros or dollars,  $T$  stands for general tax revenues and  $R=(1+r)$ .

# Public Debt and Fiscal Sustainability

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## Beware of Notation and Measurement Units!

- If all variables in (1.16) are expressed in nominal terms, then  $r$  is the **nominal** interest rate.
- If all variables in (1.16) are expressed in terms of units of a good, i.e., inflation free, then  $r$  is the **real** interest rate. This is the notation in Ljungqvist and Sargent!
- Often, people denote the nominal interest rate as  $\tilde{z}_t$ . This is the notation in Walsh's textbook.

# Public Debt and Fiscal Sustainability

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Also careful how you denote “t” for stock variables!

- In Ljungqvist and Sargent, “t” means the stock variable (e.g.  $B$ ) at the beginning of the year and “t+1” at the end of the year.
- In Walsh  $B_t$  is public debt at the **end** of the year and  $B_{t-1}$  at the **beginning** of the year.
- Finally, different authors use the interest rate capitalization differently.

# Public Debt and Fiscal Sustainability

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- For instance, Walsh and many others write the budget constraint as:

$$G_t^P + i_{t-1}B_{t-1} = T_t + (B_{t+1} - B_t)$$

$$\therefore (1 + i_{t-1})B_t + G_t^P - T_t = B_{t+1}$$

- Compare that with (1.16):

$$B_t + G_t^P - T_t = R_{t+1}^{-1} B_{t+1} \therefore$$

$$R_{t+1}(B_t + G_t^P - T_t) = B_{t+1}$$

- The capitalization factor  $R_{t+1}$  is applied on the G-T flow too!



# Public Debt and Fiscal Sustainability

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- Back to (I.16): It is useful to express fiscal variables and the government budget constraint as ratios to GDP ( $Y$ ):


$$\frac{B_t}{Y_t} + \frac{G_t^P}{Y_t} - \frac{T_t}{Y_t} = R_t^{-1} \frac{B_{t+1}}{Y_{t+1}} \frac{Y_{t+1}}{Y_t}$$

- Calling  $B/Y = d$ , the expression above can be re-expressed as:

$$g_t^P - \tau_t + d_t = \frac{(1 + \Delta y_{t+1})}{(1 + r_{t+1})} d_{t+1} \quad (\text{I.16a})$$

Bohn's (1998) eq. 1

Or:

$$d_{t+1} = \frac{(1 + r_{t+1})}{(1 + \Delta y_{t+1})} [g_t^P - \tau_t + d_t] = x_{t+1} [d_t - s_t] \quad (\text{I.16.b})$$


# Public Debt and Fiscal Sustainability

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- where  $s=t-g$  is the government's *primary* surplus as a ratio to GDP.
- Integrating forward and imposing non-Ponzi the inter-temporal budget constraint (**IBC**) is:

$$d_t = \sum_{j=0}^{\infty} \frac{1}{x^j} s^{t+j} = s_t + \sum_{j=1}^{\infty} \frac{1}{x^j} s^{t+j} \quad (\text{I.17})$$

$$\therefore s_t = d_t - \sum_{j=1}^{\infty} \frac{1}{x^j} s^{t+j} \quad (\text{I.18})$$

- Taking expectations at  $t$  yields:

# Public Debt and Fiscal Sustainability

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$$s_t = d_t - E_t \sum_{j=1}^{\infty} \frac{1}{x^j} s^{t+j} \quad (1.19)$$

- This says that the primary surplus this year (say) will respond to the stock of debt at the beginning of the period ( $d_t$ ) and the expected path of the discounted value of primary surpluses.
- Note that this sequence is only bound if  $x > 1$  and so  $r > g$ .
- If so, Bohn (1998, 2007) argues that if a regression of  $s$  on  $d$  yields a positive coefficient on  $d$ , then this is a *sufficient* condition for fiscal solvency.

# Public Debt and Fiscal Sustainability

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- In particular, Bohn (1998) runs the following regression:

$$s_t = \alpha_0 + \rho d_t + \mathbf{\alpha}'_g \mathbf{z}_t + \varepsilon_t$$

where  $\mathbf{z}$  is a vector of additional “controls” that he calls GVAR and YVAR.

- He then finds for historical US data,  $\rho \sim 0.05$ . That is, a rise in the public debt of 20 percentage points of GDP (i.e. from 80% to 100%) requires an increase in the primary surplus of 1% of GDP. [he gives his calculation in dollar terms]

# Public Debt and Fiscal Sustainability

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- Another important application of expression (I.16) is to use it to compute the required primary surplus to stabilize the debt to GDP ratio.
- To compute this, set  $d_{t+1} = d_t$  to obtain:

$$d_{t+1} - d_t = 0 = (x_{t+1} - 1)d_t - x_{t+1}s_t$$

- This implies:  $(1 - 1/x_{t+1})d_t = s_t$   
 $\therefore s_t \approx (r_{t+1} - \Delta y_{t+1})d_t$  (I.20)

# Public Debt and Fiscal Sustainability

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- Two salient implications.
- One is that if  $d$  is high, small increases in  $r$ , especially if combined with reduction in GDP growth rate, can require a large increase in the primary fiscal surplus to prevent  $D/Y$  from soaring.
- Since in many countries  $r$  and  $g$  are negatively correlated, fiscal solvency can be put at risk during periods of low growth.
- For periods in which  $g > r$ , debt stabilization is compatible with a primary deficit.

# Public Debt and Fiscal Sustainability

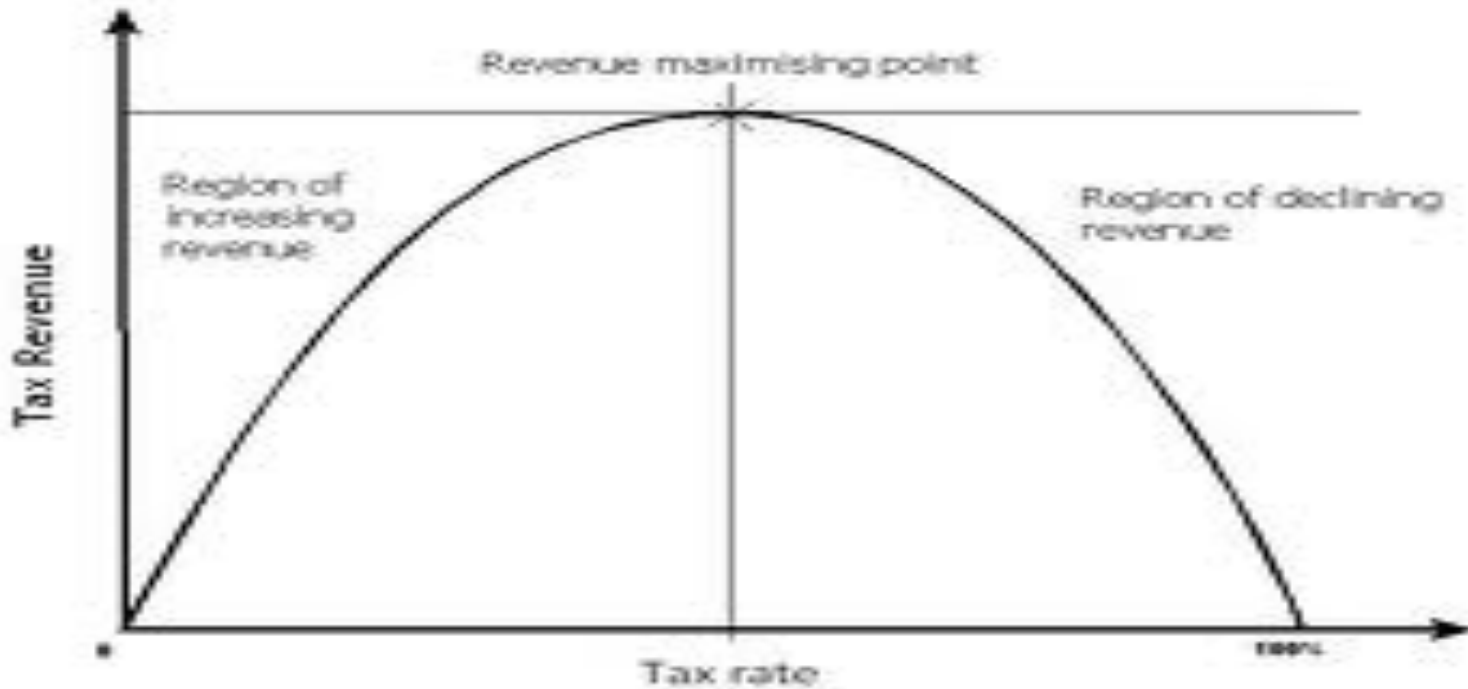
55

- For countries which have high debt and face high interest rate, possibly compromising fiscal sustainability, this discussion has been silent as to whether the required improvement in  $s$  should come from revenue improvement and/or spending cuts.
- There is widespread view that tax increases make it costly collect revenues.
- That is, if the fiscal authority hikes up tax rates, evasion will rise and the government may end up collecting less tax revenues, perversely as it may seem.

# Public Debt and Fiscal Sustainability

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- This idea is embedded in the so-called **Laffer curve**:





# Public Debt and Fiscal Sustainability

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- Barro, 1979: An influential formalization of the idea that, once the top of the Laffer curve is reached, the government should not move too much around with tax rates.
- That is, tax smoothing should be a desirable feature of fiscal policy.
- A formalization is as follows.

# Tax Smoothing and Public Debt

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- Let the cost of tax collection be given by:

$$C(t) = u_1 \tau_t + \frac{u_2}{2} \tau_t^2 \quad (1.21)$$

- The government seeks to minimize the cost of tax collection:

$$E \sum_{t=0}^{\infty} \beta^t C(t)$$

- s.t (1.16), where (to simplify let growth be zero so  $x_t = R_t = R$ ):

$$d_{t+1} = R[g_t - \tau_t + d_t]$$

# Tax Smoothing and Public Debt

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- The solution to this stochastic dynamic programming problem is:

$$C'(\tau_t) = R\beta E_t[C'(\tau_{t+1})]$$

- Since  $C'(\tau) = u_1 + u_2\tau$

- The solution yields:  $u_1 + u_2\tau = R\beta[u_1 + u_2 E_t(\tau_{t+1})]$

- Under the familiar assumption of  $R\beta=1$  :  $E_t(\tau_{t+1}) = \tau_t$  (1.22)

**⇒ So, the tax rate should be optimally constant overtime!**

## Tax Smoothing and Public Debt

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- With government expenditure following an exogenous process, say  $g = \bar{g}$  and  $T = \tau Y$ , this means that as  $Y$  goes down, so will overall tax revenues  $T$  and the fiscal deficit will widen.
- Hence governments should “optimally” build up debt during recessions, and surpluses during “good times”.
- This is sometimes observed, but not always.
- Yet, sometimes the downfall in activity is so sharp, that fiscal solvency requires government spending to be cut too.

## Tax Smoothing and Public Debt

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- Yet, if the fiscal spending multiplier is large, then this may aggravate the drop in  $Y$ , reducing further revenue collection, and thus worsening further the fiscal balance.
- These trade-offs are non-trivial.
- Whether one opts for drastic “fiscal consolidation” or allow public debt to build up rapidly will depend on economy-specific fiscal multiplier parameters.
- Will also depend on the expected severity/length of the recession, as well as other considerations.

## Lecture II:

# Fiscal and Monetary Theories of Inflation

## Bibliography

Ljungqvist & Sargent, *Recursive Macroeconomic Theory*, 2ª edição, capítulo 24\*

- Wash, Carl, *Monetary Theory and Policy*, 2ª edição, capítulo 4\*
- Sargent, T. & Wallace, N, “Some unpleasant Monetarist Arithmetic”, FED Minneapolis *Quarterly Review*, pp. 1-17, 1981.
- Fisher, S., Sahay, R and Vegh, C., “Modern Hyper- and High Inflations”, *Journal of Economic Literature* 40, pp. 837-880, 2002.
- Catão, L.A.V. and Terrones, M. “Fiscal Deficits and Inflation”, *Journal of Monetary Economics* 52, pp. 529-554, 2005.

# Fiscal-Monetary Policy Links

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## Two Polar Regimes

- “Ricardian” Regime: Fiscal policy adjusts to ensure government’s solvency (IBC). Monetary policy sets interest rates and/or money supply consistent with inflation objective.
- Non-Ricardian Regime: Fiscal policy sets  $g$  and  $t$  inconsistently with IBC. The price level adjusts so as to ensure that IBC holds.
  - case of “**fiscal dominance**”: monetary policy typically can only choose between inflation now vs. inflation later.



# Fiscal-Monetary Policy Links

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## Basic government accounting with central bank

Take Eq. (I.16) and add central bank “receipts” (RBC):

$$B_t^T + G_t^P - T_t = R_{t+1}^{-1} B_{t+1}^T + RBC_t \quad (I.23)$$
$$\therefore G_t^P = R_{t+1}^{-1} B_{t+1}^T - B_t^T + T_t + RBC_t$$

Central bank transfer to  
Treasury

Where the superscript “T” accounts for total government bonds.

# Fiscal-Monetary Policy Links

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## Central Bank Accounting:

Typical Central Bank Balance Sheet

Assets	Liabilities
International Reserves ("NFA")	High Powered Money ("H" or "M")
Net Domestic Assets ("NDA")	

Divide by  $p$  only if everything is expressed in real terms

$$\underbrace{R_{t+1}^{-1} B_{t+1}^M - B_t^M}_{\text{Change in Government bond holdings in the hands of the central bank (central bank financing of Treasury)}} + RBC_t = \underbrace{(M_{t+1} - M_t)}_{\text{Central bank finances its spending with issuance of high powered money}} / p_t \quad (1.24)$$

Change in Government bond holdings in the hands of the central bank (central bank financing of Treasury)

Central bank finances its spending with issuance of high powered money

## Fiscal-Monetary Policy Links

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Government bond holdings in the hands of households is of course total government bond issuance less the stock of government bonds sitting in the central bank balance sheet (under the item “NDA”). Hence:

$$B = B^T - B^M$$

Solving (I.24) for RBC, plugging into (I.23) and using the above, we end up with the **consolidated budget for the government** (i.e. Treasury + Central Bank):

$$B_t + G_t^P - T_t = R_{t+1}^{-1} B_{t+1} + (M_{t+1} - M_t) / p_t \quad (\text{I.25})$$

$$\therefore G_t^P - T_t = R_{t+1}^{-1} B_{t+1} - B_t + (M_{t+1} - M_t) / p_t$$

## Fiscal-Monetary Policy Links

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Eq. (1.25) says that the **consolidated** government's primary deficit can now be financed with either net bond issuance (i.e. discounted of interest payments) to the households plus money issuance – the so-called “**seignorage**” financing.

- Clearly, bond financing can be expensive: the government has to pay interest rate  $r$  on its bond issuance.
- And we have seen in Figure 4, that  $r$  can be high!

# Fiscal-Monetary Policy Links

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- But this doesn't mean (as we will see more shortly) that seignorage financing is not costly!
- To start examining this, re-write (l.25) as:

$$G_t^P - T_t = R_{t+1}^{-1} B_{t+1} - B_t + \frac{p_{t+1}}{p_t} \frac{M_{t+1}}{p_{t+1}} - \frac{M_t}{p_t}$$

$$= R_{t+1}^{-1} B_{t+1} - B_t + R_{t+1}^{m-1} \left( \frac{M_{t+1}}{p_{t+1}} - \frac{M_t}{p_t} \right)$$

Real return on money balances =  $R_{t+1}^m = \frac{p_t}{p_{t+1}}$

Real money balance

## Fiscal-Monetary Policy Links

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- Prima-facie, even without taking into account other (allocative) costs of price instability, the above eq. shows that money financing can be costly.
- E.g. if there is deflation (i.e.  $p_t > p_{t+1}$ ),  $R_{t+1}^m = \frac{p_t}{p_{t+1}}$  the rate of return paid on money can be high.
- So, money financing is not so trivial on a purely accounting basis!

# Money, Deficits and Inflation in General Equilibrium

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- This raises the fundamental question of why people hold money.
- And another, no less tricky question, of what is “money”.
- In this lecture, we shall confine ourselves to the former question.
- Under complete markets, fiat money can only be a store of value that, in the limit (i.e.  $T \rightarrow \infty$ , imposing the transversality condition), is valueless.

# Money, Deficits and Inflation in General Equilibrium

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- So, motivating money holdings would require some “friction”.
- Here we will review a model in which holding money saves transactions costs – “shopping time”
- The model follows L-S, chapter 24.
- This basic set-up will be used to discuss various fiscal-monetary models of inflation.



# Money, Deficits and Inflation in General Equilibrium

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□ Utility:

$$\sum_{t=0}^{\infty} \beta^t u(c_t, l_t)$$

leisure

□ Constraints:

$$c_t + \frac{b_{t+1}}{R_t} + \frac{m_{t+1}}{p_t} = y_t - \tau_t + b_t + \frac{m_t}{p_t}$$

$$1 = l_t + \delta_t$$

where  $\delta$  is shopping time (“s” in L-S but we use little delta to avoid using “s” which we used before for fiscal surplus).

As before: endowment economy with no uncertainty.

# Money, Deficits and Inflation in General Equilibrium

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- Shopping-time transaction technology:

$$\delta_t = 1 - l_t = H\left(c_t, \frac{m_{t+1}}{p_t}\right) = \frac{c_t}{m_{t+1}/p_t} \varepsilon_t$$

- So, we can now set up the Lagrangian and solve it:

$$\sum_{t=0}^{\infty} \beta^t \left\{ u(c_t, l_t) + \lambda_t \left( y_t - \tau_t + b_t + \frac{m_t}{p_t} - c_t - \frac{b_{t+1}}{R_t} - \frac{m_{t+1}}{p_t} \right) + \mu_t \left[ 1 - l_t - H\left(c_t, \frac{m_{t+1}}{p_t}\right) \right] \right\}$$

# Money, Deficits and Inflation in General Equilibrium

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FOC with respect to  $c_t, l_t, b_{t+1}, m_{t+1}$  yield:

$$R_t = \frac{1}{\beta} \frac{u_c(t) - u_l(t)H_c(t)}{u_c(t+1) - u_l(t+1)H_c(t+1)} \quad (1.26)$$

$$\frac{R_t - R_{mt}}{R_t} \lambda_t = -\mu_t H_{m/p}(t) \quad (1.27)$$

$$\frac{R_t - R_{mt}}{R_t} \left[ \frac{u_c(t)}{u_l(t)} - H_c(t) \right] + H_{m/p}(t) = 0 \quad (1.28)$$

(Homework: Provide the intuition for all these expressions)

# Money, Deficits and Inflation in General Equilibrium

Applying the implicit function theorem to the above yields:

$$\frac{m_{t+1}}{p_t} = F(c_t, R_{m_t} / R_t)$$

Recalling that  $R_{m_t} = \frac{p_{t-1}}{p_t}$  and  $R_t = \frac{(1+i_t)}{p_t / p_{t-1}}$ , it thus follows that:

$$\frac{m_{t+1}}{p_t} = F(c_t, R_{m_t} / R_t) = F(c_t, i_t) \tag{1.29}$$

Where  $F_c > 0, F_i < 0$ .

Thus this micro founded model delivers the familiar money demand (“LM” curve) function.

## Money, Deficits and Inflation in General Equilibrium

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As we did in the discussion of the Ricardian equivalence in our first lecture, now introduce the government. Recall the budget constraint in (1.25):

$$G_t^P = T_t + R_{t+1}^{-1} B_{t+1} - B_t + (M_{t+1} - M_t) / p_t$$

Where  $M$  is money supply. Equating  $M$  to money demand  $m$  in (1.26) and assuming exogenous sequences for government spending and taxation, and initial asset holdings, we can solve the model.

# Money, Deficits and Inflation in General Equilibrium

Let's characterize the stationary equilibrium of this economy.

- Let  $\{G_t = g_t^P, T_t = \tau_t, B\}$  be set by the government,  $\{B_0, M_0\}$  inherited from the past (all small caps denote equilibria).
- Let the resource constraint be  $c_t + g_t = y_t$  ; and let  $R\beta=1$ .
- The equilibrium is given by a price system so that for  $\{c_t, M_t, B_t\}_{t=1}^{\infty}$  , the household optimal problem and the government budget constraint are satisfied.
- Equilibrium  $R_m$  (1-inflation rate) and  $p_0$  are then pinned-down.

# Money, Deficits and Inflation in General Equilibrium

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- We seek an equilibrium for which  $X_t = X$ , where  $X$  is any of exogenous or endogenous variables in equilibrium.
- As shown in L-S (eq. 24.2.22), this equilibrium delivers the following expression linking the fiscal position and the rate of inflation,  $p_{t+1}/p_t$  in stationary equilibrium:

$$g_t^P - \tau_t + B_t \frac{(R-1)}{R} = f(R_m)(1 - R_m) \quad (1.30)$$

Overall Government Deficit

Seignorage financing

# Money, Deficits and Inflation in General Equilibrium

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- Note that  $f(R_m) = \frac{m_{t+1}}{P_t}$  and that  $1 - R_m = \frac{P_{t+1} - P_t}{P_{t+1}}$
- We can thus decompose total seignorage financing as the product of the inflation tax base component and the inflation rate component.
- Important: note from above that the inflation tax base is dependent on the inflation rate: rising inflation lowers money demand  $m_{t+1}$ !
- Hence, there is potential for multiple equilibria!



# Money, Deficits and Inflation in General Equilibrium

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Illustration of the relationship between deficit and inflation, the **seignorage Laffer curve** (L-S, 24.2.7):

- i) Put functional forms in  $u$  and  $H$  to compute  $f(R_m) = F(c, R_m/R)$
- ii) Set  $\beta$  to pin-down  $R = 1/\beta$ .
- iii) Set  $c$  to pin down  $l = 1 - c$ .
- iv) Set coefficient of risk aversion  $\sigma$  and the (inverse of) the leisure elasticity coefficient ( $\alpha$ ).
- v) Then plot  $R_m = 1 - (\text{gross})\text{inflation rate}$  against the deficit.

Homework: do it for various  $s$ . Then, fix  $\sigma = 2$  and change  $\beta = 0.9$

# Money, Deficits and Inflation in General Equilibrium

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Using this model's stationary equilibrium solution we can now

- Effects of an increase in  $M_0$

To see this, consider the solution at  $t=0$ :

$$\frac{M_0}{P_0} = f(R_m) - (g^P - \tau_0 + B_0) + B / R$$

where  $f(R_m) = m_{t+1} / p_t$

# Money, Deficits and Inflation in General Equilibrium

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Using this model's stationary equilibrium solution we can now study the effect of various policy experiments

- Effects of an increase in  $M_0$ , all else constant

To see this, consider the solution at  $t=0$ :

$$\frac{M_0}{P_0} = f(R_m) - (g_t^P - \tau_t + B_0 \frac{(R-1)}{R}) \quad (1.31)$$

where  $f(R_m) = m_{t+1} / p_t$

## Money, Deficits and Inflation in General Equilibrium

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Since  $(g_t^P - \tau_t + B_0 \frac{(R-1)}{R})$  will not change, from (1.30) it must also be that  $R_m$  will not change.

Hence,  $M_0/P_0$  will not change  $\rightarrow DM_0=DP_0$ .

So, there is concomitant jump in the price level as  $M$  increases

# Money, Deficits and Inflation in General Equilibrium

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Using this model's stationary equilibrium solution we can now study the effect of various policy experiments

## □ Effects of a persistent fiscal deficit

From (1.30) and the seignorage Laffer curve, it is clear that a permanent increase in the fiscal deficit will increase  $(1 - R_m)$ , i.e. the steady-state inflation rate, **if one is on the right side of the Laffer curve.**

However, there may be an equilibrium that the tax base increases, so the bigger deficit is financed with higher  $M/P$ .

# Money, Deficits and Inflation in General Equilibrium

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## □ Fiscal Requirement for Price Stability

Setting  $1 - R_m = 0$  in (I.30), clearly requires the overall (**not the primary!**) fiscal deficit to be zero.

With  $R$  given, this of course has implications for the required primary deficit too:

$$g_t^P - \tau_t + B_t \frac{(R-1)}{R} = 0$$
$$\therefore (\tau - g^P) = \frac{R-1}{R} B = \frac{r}{(1+r)} B$$

# Money, Deficits and Inflation in General Equilibrium

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- Limits to what Monetary Policy Can Do (“Unpleasant Monetarist Arithmetics”)

Suppose that  $g_t^P - \tau_t$  rises. Then from (1.30) permanent inflation  $1 - R_m$  will rise.

The central bank then tries to mitigate the impact on  $P_0$ , engaging into open market operations: buy high-powered money (reducing  $M$  in  $t=1$ ) and selling bonds (increasing  $B$ ).

$$\frac{M_0}{P_0} = \frac{M_1}{P_0} - (g^P - \tau_0 + B_0) + B / R$$

Effect is ambiguous: at best lower  $p_0$  but higher  $B$  (due to interest payments on debt) increases  $1 - R_m$

# Money, Deficits and Inflation in General Equilibrium

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## □ Optimum Quantity of Money (“Friedman rule”)

The idea is that reducing shopping time increases welfare. Hence monetary policy should satiate households with money.

Since  $R_m \in (1, \beta^{-1})$ , the Friedman rule implies that the opportunity cost of holding money should be as low as possible.

Here it is therefore bound by the return on (safe) bonds. So,

$$R_m \equiv R.$$



# Money, Deficits and Inflation in General Equilibrium

To see what implications this has for nominal interest setting (e.g. the instrument controlled by central banks), recall:

$$R_{m_t} = p_t / p_{t+1}$$

$$R_t = 1 + r_t \equiv 1 + i_t - E_t(1 - R_{m_t}) = i_t + R_{m_t} \quad (1.32)$$

with  $R_{m_t} \equiv R_t$ , this implies that  $i_t \equiv 0$ .

This is the well-known “Friedman rule”.

# Money, Deficits and Inflation in General Equilibrium

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## □ The Fiscal Theory of the Price Level

Recall that in solving the model  $B$  (the real value of public debt) is determined by the government and, given  $g$ ,  $t$ ,  $R$ ,  $B_0$  and  $M_0$ , inflation  $(1 - R_m)$  and  $P_0$  are then determined.

Under the FTPL,  $B$  is **endogenous**: while the government can decide on nominal debt, the price *level* will adjust to as to make  $B$  consistent with the inter-temporal budget constraint.

Again, we can use eqs. (I.30) & (I.31) to see how it works.

# Money, Deficits and Inflation in General Equilibrium

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- Re-arrange (I.30) to write:

$$\frac{B}{R} = \frac{1}{R-1} [\tau - g^P + f(R_m)(1 - R_m)]$$

$$B = \frac{R}{R-1} [\tau - g^P] + \frac{R}{R-1} f(R_m)(1 - R_m)$$

$$B = \sum_{t=0}^{\infty} R^{-t} [\tau_t - g_t^P] + \frac{R}{R-1} f(R_m)(1 - R_m)$$

- So, given  $\{g_t, \tau_t\}_{t=0}^{\infty}, R, R_m$  one can pin down real public debt,  $B$ .

# Money, Deficits and Inflation in General Equilibrium

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- So, the extra requirement here is that policy can determine seignorage  $(1 - R_m)$  or, equivalently, given (1.32), to peg the nominal interest rate  $i$ .
- Once this is done and, with  $B_0$  and  $M_0$  given, the price level is pinned down by computing  $p_0$  from re-arranging (1.31):

$$\frac{M_0}{P_0} + B_0 = \sum_{t=0}^{\infty} R^{-t} (\tau_t - g_t) + \sum_{t=0}^{\infty} R^{-t} f(R_m)(1 - R_m)$$

# Money, Deficits and Inflation in General Equilibrium

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- Note also that the path of money supply also gets determined using:

$$\frac{M_1}{P_0} = f(R_m)$$

- Given  $M_0$ , then,  $M_0, M_1, \dots$  is now determined. So, once the price level is pinned down by the fiscal theory of the price level, the path of money supply is now also endogenously determined.
- A corollary is that one does not need money for the price level to be determined.

# Money, Deficits and Inflation in General Equilibrium

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- So, we currently have two different fiscal theories of inflation!
- The earlier Sargent and Wallace one shows that the ***inflation rate*** adjusts to the overall fiscal deficit ( $g-t+rB$ ) in stationary equilibrium. So, fiscal policy is dominant.
- The price level ( $p_0, p_1, \dots$ ) is pinned down by money supply: as we saw, this is the so-called “Ricardian regime”.
- Monetary policy can only influence the *timing* of inflation (now vs. future), but not long-run inflation.
- So, no “true” inflation targeting under fiscal dominance.

# Money, Deficits and Inflation in General Equilibrium

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- New Fiscal Theory of the Price Level (Cochrane, Sims, Woodford), the steady-state inflation rate is chosen by policy (e.g. by nominal interest rate pegging); for a given nominal debt, inflation will then increase or reduce real debt.
- Then with inflation and real debt determined,  $p_0$  is pinned-down.
- Under FTPL, the inter-temporal budget constraint holds only at the *equilibrium* value of the price level.
- Under traditional Sargent-Wallace theory, it holds for all  $P_t$ .

# Money, Deficits and Inflation in General Equilibrium

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- Since we only observe equilibrium outcomes, it is virtually impossible to distinguish empirically the two theories.
- One advantage of the new fiscal theory of the price level is to rule out multiple equilibria in the traditional theory arising from the right hand side of (1.30):  $f(R_m)(1-R_m)$ .
- The extra restriction that seignorage (or its inverse  $1-R_m$ ) is set by policy (i.e. nominal interest peg) takes care of multiplicity:  $P_0, P_1$ , etc. can be uniquely obtained.



## Lecture III

# Monetary Policy Foundations in Closed and Open Economies

# Monetary Policy Discretion, Commitment, and Targeting Rules

- We shall now turn to a more standard “Ricardian” economy where fiscal policy is less dominant and the central bank/monetary authority has considerable leeway in setting the inflation rate.
- We shall also break away from the old-fashion (Keynesian) assumption that expectations are adaptative.
- Instead, there is “learning” by the public: expectations about the central bank behavior are forward-looking, so that they are less easily “fooled” by a surprise rise in inflation.

# Monetary Policy Discretion, Commitment, and Targeting Rules

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- Starting with Kydland and Prescott (1977), many studies have modeled the incentives for central banks to behave in different ways.
- Key question: whether it optimal for central banks to commit to a policy objective (e.g. target a certain inflation rate) in the form of strict rule (no matter what) vs. use (and perhaps abuse!) discretion in setting monetary policy.
- Key concept: whether a policy is “time-consistent” vs. “time-inconsistent”

# Monetary Policy Discretion, Commitment, and Targeting Rules

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- A policy is time consistent when it is optimal to adopt at  $t$  and remains optimal to adopt it in  $t+1$
- E.g. Policy towards hostage ransom
- In many practical situations, time-consistent policies are hard to implement: the incentive for discretion is non-trivial and (almost always) there.
- What we will discuss: How this affects average inflation?

# Monetary Policy Discretion, Commitment, and Targeting Rules

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- The underlying motivation/assumption is, of course, that **inflation is costly**.
- That said, there is considerable disagreement on the threshold above which inflation becomes really costly...
- Despite many studies on the relationship between inflation & growth
- Yet, there is considerable agreement also that inflation should not be optimally zero (risk of falling on a liquidity trap).
- For now, we skip this threshold debate and simply assume, for the sake of model exposition, that inflation is costly.

# Monetary Policy Discretion, Commitment, and Targeting Rules

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## The Barro and Gordon Model

[We shall closely follow Wash (2010, ch.7)]

The central bank objective is to max the expected value of:

$$U = \lambda(y - y_n) - \frac{1}{2}\pi^2 \quad (1.44)$$

[But shortly we'll see a variant where the CB loss depends on output variability around natural output]

[We will also discuss more of what actually enters and what *should* enter the central bank utility (or its converse, the central bank loss function "V") later].

# Monetary Policy Discretion, Commitment, and Targeting Rules

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with output no longer given (as in the endowment economy of previous models), but determined by a Lucas-type supply function:

$$y = y_n + \alpha(\pi - \pi^e) + e \quad (1.45)$$

One rationale is that wages are “sticky” in the short-run so inflation “surprises” increase output above “natural”.

And the central bank controlling inflation through money supply as the policy instrument (today’s equivalent being the interest rate):

$$\pi = \Delta m + v \quad (1.46)$$

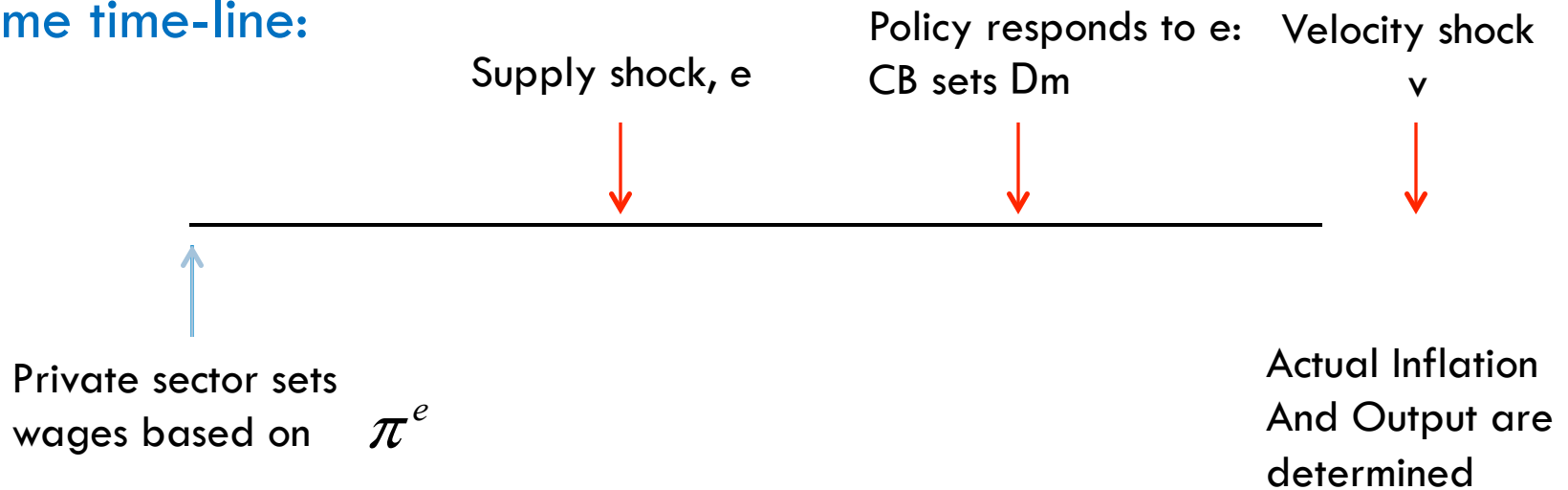
# Monetary Policy Discretion, Commitment, and Targeting Rules

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where  $e$  and  $v$  are uncorrelated shocks:

$$\text{cor}(e, v) \approx 0$$

Game time-line:





# Monetary Policy Discretion, Commitment, and Targeting Rules

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The critical point is that the central bank can observe the supply shock,  $e$ , ahead of any reaction by the private sector.

One rationale is that the CB has an informational advantage over the private sector in observing “supply shocks” (e.g., output statistics are known to policy makers before made public, at least in some cases).

Another rationale for this sequencing is that it is much less costly for the CB to react (e.g. more frequent monetary policy meetings) than for the private sector to reset contracts based on the post  $e$ -shock inflation expectations.

# Monetary Policy Discretion, Commitment, and Targeting Rules

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## Model's Solution:

Substituting (I.45) and (I.46) into (I.44) yields:

$$U = \lambda(\alpha(\Delta m + v - \pi^e) + e - \frac{1}{2}(\Delta m + v)^2)$$

FOC wrt  $\Delta m$  (recall: taking  $\pi^e$  as given) yield:

$$\Delta m = \alpha\lambda > 0$$

# Monetary Policy Discretion, Commitment, and Targeting Rules

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This implies from (1.46) that actual inflation will be:

$$\pi = \alpha\lambda + v$$

But now agents are forward-looking: Unlike in adaptive expectation models, they anticipate the incentives of the central bank in setting inflation expectations. Hence:

$$\pi^e = E(\Delta m) = \alpha\lambda > 0$$

# Monetary Policy Discretion, Commitment, and Targeting Rules

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So, average inflation is positive and fully anticipated!

But how about output? Do we gain anything from higher inflation?

From (1.45), we have:

$$y - y^n = av + e$$

So, CB policy does not improve output! In fact with  $v$  and  $e$  being  $N(0, s^2)$ , on average actual output = natural output!

# Monetary Policy Discretion, Commitment, and Targeting Rules

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## Summing up:

- Central bank discretion makes the economy “suffer” from a positive inflation bias with no permanent gains in output.
- This is, of course, only so as long as  $\lambda > 0$  .
- Later we will see that the so-called “strict” inflation targeting postulates this output “weight” factor =0 in CB objective function.
- The inflation bias rises with nominal rigidity, i.e.,  $\alpha$  higher.

# Monetary Policy Discretion, Commitment, and Targeting Rules

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- So, why any CB would undertake this policy?
- With  $\alpha > 0$ , and with the central banker caring about output and employment (i.e.  $\lambda > 0$ ), it is easy to see that its marginal benefit = marginal cost when  $\pi^* > 0$ . So, there is an incentive “in the margin”.
- To see what happens to the central bank utility (and hence social utility if the latter is fully benevolent), compute the CB expected utility using (1.44):

# Monetary Policy Discretion, Commitment, and Targeting Rules

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- Expected Utility under discretion:

$$E(U^d) = E \left[ \lambda(av + e) - \frac{1}{2}(a\lambda + v)^2 \right] = -\frac{1}{2} [a^2\lambda^2 + \sigma_v^2] \quad (1.48)$$

- It is easy to see that utility would be higher if the central bank could commit to a zero inflation policy, i.e., if would not care about output. In this case  $\pi = v$  and, using 1.48 expected utility would be higher:

$$E(U^c) = -\sigma_v^2 > E(U^d)$$

## Solutions to the Inflation Bias

- A large literature followed the Barro-Gordon set up.
- Partly was to show what happens to the inflation bias incentive in a repeated game (recall Barro-Gordon was a one-shot game) framework.
- Another, influential strand consisted of asking the kind of preferences should feature in optimal central bank design so that the incentive to deviating from low inflation commitment is mitigated.



# Monetary Policy Discretion, Commitment, and Targeting Rules

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- Perhaps the most influential idea there is that of a “conservative central bank” due to K. Rogoff (1985)
- This means a central bank having a more “conservative” stance than society regarding inflation, i.e., that puts a higher weight on the inflation component of central bank (dis)utility.
- To formalize this in the context more akin to that of Rogoff’s and the later literature, consider the converse of central bank utility – namely, its loss of function of the form:

## Monetary Policy Discretion, Commitment, and Targeting Rules

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- Central Bank Loss Function:

$$V = \frac{1}{2} \lambda (y - y_n - k)^2 - \frac{1}{2} \pi^2 \quad (1.49)$$

which differs from (1.42) for the quadratic term in the output gap ( $y - y_n$ ), meaning that output *volatility*, not just output *levels* matter.

As shown in Wash ([homework: do work out the full derivations](#)), inflation under discretion is given by:

## Monetary Policy Discretion, Commitment, and Targeting Rules

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- Inflation under discretion with quadratic CB loss:

$$\pi^d = \Delta m + v = a\lambda k - \left(\frac{a\lambda}{1+a^2\lambda}\right)e + v \quad (1.50)$$

- What Rogoff suggests is a central bank that places a weight  $1 + \delta > 1$  in  $V$  so that:

$$\pi^d = \frac{a\lambda k}{1+\delta} - \left(\frac{a\lambda}{1+\delta+a^2\lambda}\right)e + v \quad (1.51)$$

“distortion” in CB response to supply shocks

## Monetary Policy Discretion, Commitment, and Targeting Rules

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- So, the key parameter to be determined is  $\delta$ .
- To find that out compute the central bank  $E(V)$ , similar to done for  $E(U)$ ; then min wrt to  $\delta$  to obtain:

$$\delta = \frac{k^2}{\sigma_e^2} - \left( \frac{1 + \delta + a^2 \lambda}{1 + \delta} \right)^3 \quad (1.52)$$

- Since  $g(0) > 0$  and  $\lim_{\delta \rightarrow \infty} g(\delta) = \frac{k^2}{\sigma_e^2}$ , there will always be a Solution where  $\delta > 0$  and finite.

## Monetary Policy Discretion, Commitment, and Targeting Rules

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- Yet, the down-side is that inflation response to output shocks is now also distorted by  $\delta$ .
- Further, the higher  $\delta$ , the greater the variance of output to the shock  $e$ :

$$\text{var}(y) = a^2 \sigma_v^2 + \left( \frac{1 + \delta}{1 + \delta + a^2 \lambda} \right)^2 \sigma_e^2$$

- This fleshes out a perennial trade-off in monetary policy: you reduce the inflation bias and the inflation variability at the cost of higher output variability.

## Monetary Policy Discretion, Commitment, and Targeting Rules

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- Within this trade-off an important practical question is how a government commits to  $d > 0$ .
- After all, one could always hire a “conservative” central bank with  $d > 0$ , and then fire her/him, i.e., still maintain a time-inconsistent policy.
- Central bank independence has been one solution.
- But quite aside from the different forms of central bank independence (full vs. operational), the trade-off between inflation and output stabilization remains a crucial issue.

## Monetary Policy Discretion, Commitment, and Targeting Rules

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- This trade-off can be exacerbated by many things, including economic structure, current politics, as well as history of credible/non-credible policies.
- This suggests that  $d$  can (optimally) vary significantly across countries and time, so no “one-size-fits-all”.
- Other issues also arise. E.g. why would a government have an incentive to keep someone in a key public institution that does not share society’s average preferences?
- A potentially more fruitful approach is to think of a contract which is “incentive-compatible”.

## Monetary Policy Discretion, Commitment, and Targeting Rules

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- A key trade-off in practice is between flexibility and credibility.
- No one (or few) would deny that some flexibility is good, specially if  $\sigma_e$  is high.
- But this too much flexibility may seriously impair credibility.
- Hence the basis for the “contracting approach”: once the incentives are correct to attain a clear pre-specified target.



## Monetary Policy Discretion, Commitment, and Targeting Rules

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- Assuming that the chosen target reflects societal preferences for an inflation rate  $\pi$  (which is not necessarily zero), then (1.49) becomes:

$$V = \frac{1}{2} \lambda (y - y_n - k)^2 - (1 + h) \frac{1}{2} E(\pi^2 - \pi^*)$$

where  $h$  is analogous to Rogoff's conservative central banker parameter  $\delta > 1$ .

- Both approaches clearly dominate discretion and still allow for some flexibility through  $\lambda$  and  $k$ .

## Monetary Policy Discretion, Commitment, and Targeting Rules

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- Strict inflation targeting is then nested in the general targeting rule, by setting  $\lambda=0$  (or  $h \rightarrow \infty$ ) .
- Yet, in general, the optimality of such strict rules impose stringent restriction on  $\sigma_e$  not being too large (see discussion in Wash, 2010, pp.313-16).
- As we shall see, these trade-offs get more complex in the open economy, with the degree of exchange rate flexibility being another concern, but the underlying trade-offs remain of a similar nature.

# Monetary Policy Issues in the Open Economy

# The Nominal and the Real Exchange Rate

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We shall stick to most conventions and define the **Nominal Exchange Rate** as

$$E = \text{units of domestic currency} / 1\$ \text{ of foreign currency}$$

This means that a rise in  $E$  implies a nominal currency *depreciation*. And conversely for a fall in  $E$ .

This can be confusing, so some authors and institutions (like the IMF) define  $E$  in terms of e.g. dollar per euros so a rise in  $E$  means an appreciation.

Here we stick to the tradition as define  $E$  as above.

## The Nominal and the Real Exchange Rate

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The **Real** Exchange Rate is (as other real metrics) corrects for differences in price levels so is defined as:

$$RER = \frac{P}{\varepsilon P^*} \quad (6.1)$$

where  $P$  is the domestic consumer price level and  $P^*$  is the foreign consumer price level.

Now, here a rise in RER means an *appreciation*, i.e., the home country is becoming more **expensive** viz the foreign country.

## The Nominal and the Real Exchange Rate

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Going back to the work of English philosopher David Hume, the foundation of the RER concept is that countries' price levels, once measured on the same currency, should equalize:

$$P = \varepsilon P^* \quad (6.2)$$

Otherwise, it would be just cheaper to buy one good in the US and sell, say, in Portugal for a profit. As more and more people do this, then this would eliminate this “arbitrage opportunity”.

This is the famous “purchasing power theory” (PPP), which implies *in absolute terms* that  $RER=1$ !

## The Nominal and the Real Exchange Rate

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As you may have already experience with your international shopping experiences, and we will see in the data, equation (6.2) does not hold well in practice.

So, it is become usual to define PPP in relative terms ( $\varepsilon$  again = euros/dollar):

$$\begin{aligned}\Delta \ln P &= \Delta \ln \varepsilon + \Delta \ln P^* \\ \therefore \pi &= \Delta e + \pi^*\end{aligned}\tag{6.3}$$

Relative PPP thus says that inflation in the home country is given by the nominal exchange depreciation plus world inflation.

## Uncovered Interest Rate Parity

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As with PPP, a key arbitrage conditions in international macroeconomics is the *uncovered interest rate parity (UIP)* condition:

$$(1+i_t) = (1+i_t^*)E_t\left(\frac{\varepsilon_{t+1}}{\varepsilon_t}\right) \quad (7.1)$$

where as before  $\varepsilon_t$  is **spot** exchange rate.

Think of it as follows. The home country has an interest rate of e.g.  $i=4\%$  a year in reais, whereas the foreign country has  $i^*=1\%$  a year in US\$. So, if the exchange rate is expected to stay constant, it becomes profitable to borrow in US\$ at 1% and lend at home at 4%, yielding an arbitrage gain of 3%.



## Uncovered Interest Rate Parity

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But this cannot be an “equilibrium” condition when international capital markets are free of restrictions, in the same way that the same good cannot have perpetually a different price from the same good abroad when goods can move freely across borders.

So, either the  $i-i^*$  will adjust or the exchange rate will depreciate. E.g. the exchange rate first appreciates as dollars flow in people convert dollars into reais to buy the domestic bond and then depreciates when people pay back their dollar debts by selling the reais accruing at the maturity of the domestic bond.

## Uncovered Interest Rate Parity

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It is common to write (7.1) in linear form applying the log transformation and using the approximation  $\ln(1+x) \sim x$ :

$$i_t \simeq i_t^* + [E_t(e_{t+1}) - e_t] = i_t^* + E_t \Delta e_{t+1} \quad (7.1)$$

which clearly indicates that in countries where the nominal interest rate is higher, the currency is expect to eventually *depreciate*.

In practice, however, this relationship does not hold too well in the data (see, e.g., Frankel and Rose, 1995)

# Uncovered Interest Rate Parity

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The reasons can be various, pertaining to the way expectations are formed, the existence of a risk premium associated with nominal exchange rate volatility, default risk, and capital controls.

So, in more general terms (7.1) is written as:

$$i_t \simeq i_t^* + E_t \Delta e_{t+1} + \zeta_t \quad (7.2)$$

where  $\zeta_t$  is meant to capture a risk premium which can be positive or negative, and possibly time-varying.

# Flex-Price Monetary Model of the Exchange Rate

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- Armed with PPP and UIP, we can now readily develop a basic (but traditionally widely used) model of the *nominal* exchange rate.
- The first building block is a standard money demand function that we have seen in the first part of the course (the money demand function in the shopping time model of Ljungqvist & Sargent (2004) model:

$$m_t - p_t = -\eta i_t + \phi y_t \quad (7.15)$$

## Flex-price Monetary Model of the Exchange Rate

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Now substitute the log-linear PPP and UIP equations of (6.2) and (7.1) into (7.15) to substitute out  $i$  and  $p$  and obtain:

$$(m_t - \phi y_t + \eta i_t^* + p_t^*) - e_t = -\eta(E_t e_{t+1} - e_t) \quad (7.15)$$

$$\therefore \underline{\eta E_t e_{t+1}} - \underline{(1 + \eta)e_t} + (m_t - \phi y_t + \eta i_t^* - p_t^*) = 0$$

This is a stochastic difference equation in the (log of) nominal exchange rate, where  $m$ ,  $y$ ,  $i^*$  and  $p^*$  are the exogenous, forcing variables. These are the so-called “**fundamentals**”.

# Flex-price Monetary Model of the Exchange Rate

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To solve it, first ignore the stochastic part, assuming perfect forecast so that  $E(e_{t+1}) = e_{t+1}$ .

To simplify the algebra, call  $f_t = m_t - \phi y_t + \eta i_t^* + p_t^*$ .

Thus we have:

$$e_t = \frac{f_t}{1+\eta} + \frac{\eta}{1+\eta} e_{t+1}$$

Iterating forward yields:

# Flex-price Monetary Model of the Exchange Rate

$$e_t = \frac{1}{1+\eta} \sum_{s=t}^{\infty} \left( \frac{\eta}{1+\eta} \right)^{s-t} E_t(m_t - \phi y_t + \eta i_t^* - p_t^*) + \lim_{T \rightarrow \infty} \left( \frac{\eta}{1+\eta} \right)^T e_{t+T}$$

As usual, we rule out the presence of speculative bubbles (the equivalent of Ponzi games), by setting the last term to zero, so the nominal exchange rate is given by:

$$e_t = \frac{1}{1+\eta} \sum_{s=t}^{\infty} \left( \frac{\eta}{1+\eta} \right)^{s-t} E_t(m_t - \phi y_t + \eta i_t^* - p_t^*) \quad (7.16)$$

# Flex-price Monetary Model of the Exchange Rate

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Some important “take-home” points from this equation:

- The nominal exchange rate today reflects the future evolution of its “fundamentals” (in this case money supply, output, the foreign interest rate and foreign price level).
- That is, **the exchange rate is essentially a forward-looking variable.**
- As such, conditional on the model,  $e$  today should help predict  $f$ !
- Eq. (7.16) also tells us what to expect on the direction of the responses of  $e$  to changes in the various fundamentals.



# Flex-price Monetary Model of the Exchange Rate

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- A loosening of monetary policy, i.e., higher  $m$  in the future implies that the exchange rate should depreciate (i.e.,  $e$  rises).
- Conversely, a productivity improvement that raises  $y$  will tend to appreciate the nominal exchange rate (i.e.  $e$  falls).
- Consider now a rise in the foreign interest rate ( $i^*$ ) due to say the end of QE policies in the US. Assuming that  $p^*$  remains about stable, this implies a increase in US *real* interest rate.
- The model says that tends to depreciate the home exchange rate.

# Flex-price Monetary Model of the Exchange Rate

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Since we are particularly interested here in the effect of changes in monetary policy on the exchange rate, let's examine on what the model says on sensitivity of  $e$  to changes in money supply.

As usual in solving the models, we make progress by assuming an exogenous stochastic process for the respective "state" variable. As in O-R (section 8.2.7), assume:

$$m_t - m_{t-1} = \rho(m_{t-1} - m_{t-2}) + v_t \quad (7.17)$$

## Flex-price Monetary Model of the Exchange Rate

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As in O-R, assume to simplify that  $-\phi y_t + \eta i_t^* - p_t^* = 0$  so we plug (7.17) into (7.16) and take expected differences to obtain:

$$\begin{aligned}
 E_t e_{t+1} - e_t &= \frac{1}{1+\eta} \sum_{s=t}^{\infty} \left( \frac{\eta}{1+\eta} \right)^{s-t} E_t (E_t m_{t+1} - m_t) \\
 &= \frac{1}{1+\eta} \sum_{s=t}^{\infty} \left( \frac{\eta \rho}{1+\eta} \right)^{s-t} \rho (m_s - m_{s-1}) \\
 &= \frac{1}{1+\eta} \frac{(m_t - m_{t-1})}{1 - \frac{\eta \rho}{1+\eta}} = \frac{1}{1+\eta} (1+\eta) \rho \frac{(m_t - m_{t-1})}{1+\eta - \eta \rho}
 \end{aligned}$$

# Flex-price Monetary Model of the Exchange Rate

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We can then invoke (7.15) to yield:

$$\frac{e_t - m_t}{\eta} = E_t e_{t+1} - e_t \quad (7.18)$$

And then substitute out  $E(e_{t+1}) - e_t$ :

$$\frac{e_t - m_t}{\eta} = \frac{\rho(m_t - m_{t-1})}{1 + \eta - \eta\rho}$$

$$e_t = m_t + \frac{\rho\eta}{1 + \eta(1 - \rho)}(m_t - m_{t-1}) \quad (7.19)$$

# Flex-price Monetary Model of the Exchange Rate

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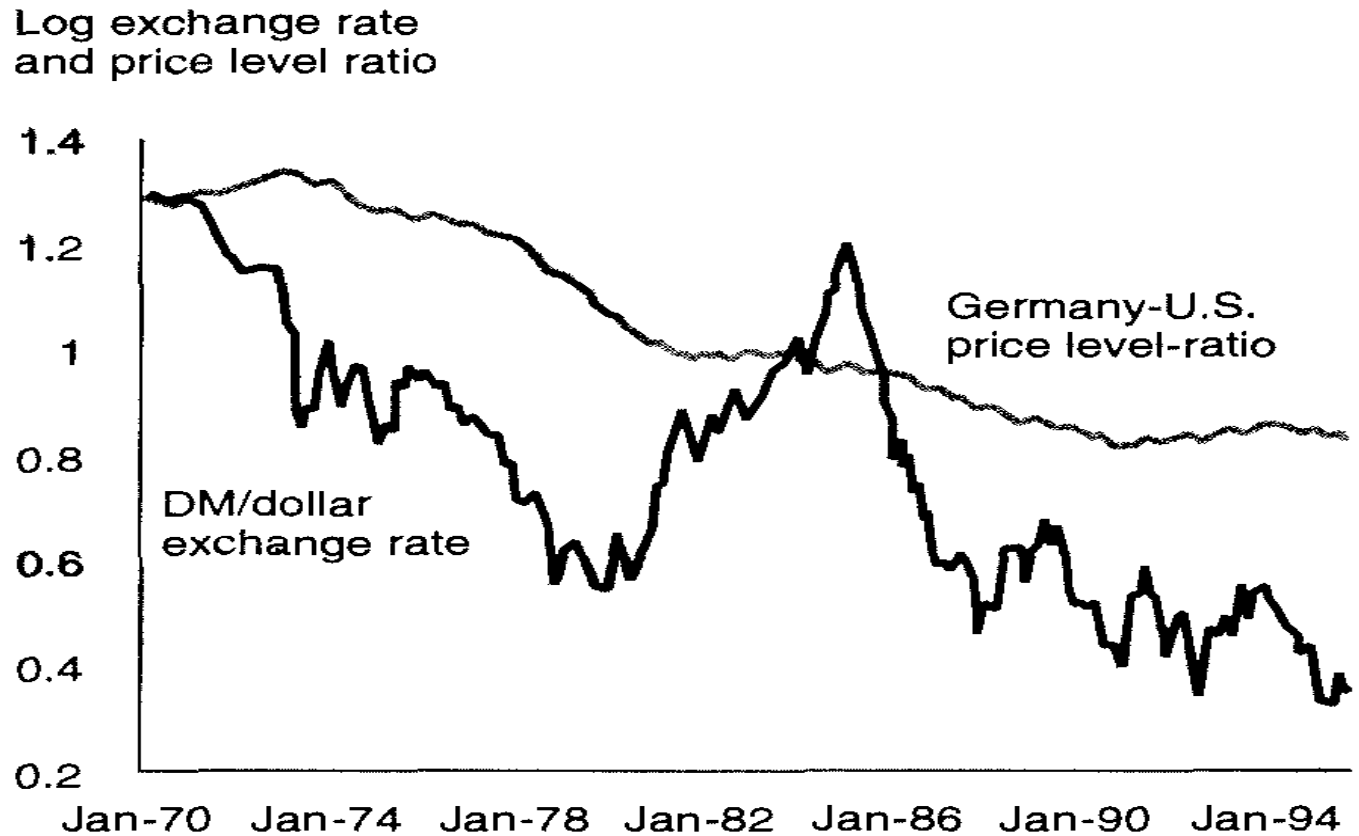
Equation (7.19) states that the impact of monetary shocks ( $v$ ) on the exchange rate will rise on

- The persistence of monetary shocks (higher  $\rho$ )
- On the semi-elasticity of money demand ( $\eta$ ).

Since the last term in (7.18) is positive, this means that shocks to money growth have a more than proportional effect on the nominal exchange rate.

# Sticky price Extensions

Motivation: Prices are far stickier than exchange rates so PPP does not hold



# Fix-price Monetary Models of the Exchange Rate

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So, because  $p-p^*$  are do not move in the short run, short-run movements in the real exchange rate will follow deviations in the **nominal** exchange rate from its expected path

- Hence, in contrast with the flexible price mode, output will also deviate from its “natural” or “potential” level ( $\bar{y}$ ) in tandem with shocks to the nominal and hence real exchange rate ( $q$ ):

$$y - \bar{y} = \Theta(q - \bar{q}) = f(e_{t+1} - e_t, \dots)$$

- Where the latter equation can be readily derived from sticky price models with micro-foundations (see e.g. Catão and Chang, JME, 2015)
- There will then be an extra term in equation 7.16 accounting for short-run deviations in the real exchange rate and given by the overshooting of the exchange to money shocks.

# Testing the Nominal Exchange Rate Model

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- Influential paper by Meese and Rogoff (1983) tests the flex- and sticky price monetary model based on out-of-sample performance.
- Because it is a bilateral relationship, what matters is the change of fundamentals in one countries vs. the other (denoted with \*)

Flex-price model: 
$$e_t = m_t - m_t^* - \gamma(y_t - y_t^*) + \lambda(i_t - i_t^*)$$

Sticky-price model: 
$$e_t = m_t - m_t^* - \gamma(y_t - y_t^*) + \lambda(i_t - i_t^*) + \theta(E_t e_{t+1} - e_t)$$

- Meese-Rogoff (1983) estimate these models for the DM-US\$ and Yen-US\$ over Mar73-Dec76 and compute the out of sample  $\hat{e}_t$  for 1-, 3-, 6-, 12-months ahead



# Testing the Nominal Exchange Rate Model

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- They do the same for the random walk model  $\hat{e}_{t+1} = e_t$
- They then compute the mean-squared error  $\frac{1}{k} \sum_{j=1}^k (e_j - \hat{e}_j)^2$
- They then find that those monetary models cannot beat the random walk
- Others (Mark, 1995; Mark & Sul, 2001) have found, however, that a longer horizons and over a longer sample (in Mark 1973:II to 1991:IV), the flex-price monetary model tends to beat the random walk.

# Testing the Nominal Exchange Rate Model

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- Subsequent research indicates that both results are quite sample dependent.
- In general, it appears that the monetary model has an (small) edge out of sample, but only for longer periods.
- Yet cumulatively this gain can be non-trivial; and non-linearities appear to be important.
- Others (Engel and West, 2006) question the meaningfulness of out-of-sample tests in the style of Meese and Rogoff.
- In short, while the jury is out, the flex-price monetary model should **not** be easily dismissed, at least as conceptual starting point.

# Nominal Exchange Rate Regimes

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- Today we see sizeable fluctuations in nominal exchange rates.
- *Prima-facie*, this is consistent with the case made by Friedman (1953) and many others that, in a world where prices and wages are somewhat sticky,  $E$  should be highly flexible.
- However, governments are not always very fond of seeing their exchange rate fluctuate wildly – the so-called “fear of floating” (Calvo and Reinhart, 2002).
- Indeed, going back in history, there were long periods in which most exchange rates were virtually fixed.

# Nominal Exchange Rate Regimes

Now recall what (7.19) says: once you peg the exchange rate, money supply becomes an endogenous variable.

Obviously, a constant money supply is an extreme assumption arising from assuming  $y$ ,  $i^*$ , and  $p^*$  constant and normalized to zero.

Yet, the key point is that, once the government is committed to a policy of fixing the exchange rate, and capital is freely mobile, the government gives up control of the money supply or, equivalently, of setting the domestic interest rate!

# Nominal Exchange Rate Regimes

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- This dilemma can be seen clearly in the context of a small open economy that takes  $i^*$  as given by invoking the UIP of equation (7.2):

$$i_t \approx i_t^* + E_t \Delta e_{t+1} + \zeta_t$$

- Once the government credibly pegs the exchange rate,  $E_t \Delta e_{t+1} = 0$ . If there are no capital controls and default risk, then  $\zeta_t = 0$ , so  $i = i^*$ . Hence, the government surrenders the control of the domestic interest  $i$  to the rest of the world – typically to countries that issue a world reserve currency like the dollar or the euro.

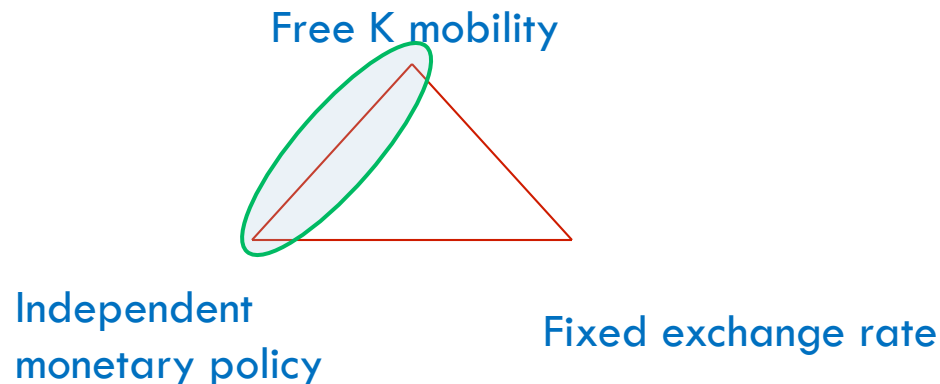
# Nominal Exchange Rate Regimes

- Yet, from (7.2), it is also clear that government can regain some control over  $i$  if it can control  $\xi_t$  somewhat.
- That is, if the government can put “sand in the wheels” of international capital mobility.
- The government has a variety of ways to impose such “capital controls”, notably via differential tax regulations that discriminate foreign investment viz investment by domestic residents.

# Mundel's Monetary Policy Trilemma

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- Thus, monetary policy faces not a dilemma but a **trilemma**.
- It can escape from the usual dilemma between fixing  $e$  and keep monetary policy sovereignty, but only at the cost of imposing capital controls!
- So, at any point in time policy choices lie at one the sides of the following triangle:



# The Mundelian Trillema

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- Using historical data, researchers have tested to what extent (if any) such a trillemma has been a bidding constraint on monetary policy of various countries.
- Obstfeld, Shambaugh and Taylor (2005) test the Trillemma by running the following regression:

$$\Delta i_{it} = \alpha + \beta \Delta i_{it}^* + u_{it} \quad (7.21)$$

for various sub-periods, i.e. those when countries floated vs. those when they fixed vs. those when they quasi-fixed.



# Mundel's Monetary Policy Trilemma

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- If a country has a credible pegged and capital is freely mobile, the trilemma implies that  $\beta=1$ .
- If  $\beta < 1$ , then the domestic monetary authority has some degree of monetary independence despite the pegged exchange rate and free capital mobility, i.e., the Trilemma is less binding.
- They find  $\beta=0.52$  to be the highest for countries under the classical gold standard. For the post-Bretton Woods  $\beta=0.46$  for pegged and 0.26 for non-pegged.
- For the Bretton-Woods,  $\beta=-0.2!$

# Mundel's Monetary Policy Trilemma

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- One take-home from these results is that the monetary policy trilemma is not that overwhelming as in theory but it is nevertheless strong
- A  $\beta=0.52$  indicates that once you peg the nominal exchange rate, your domestic interest is significantly affected by the foreign monetary policy (as measured by  $i^*$ ).
- A  $\beta=0.26$  for non-pegged regimes in the post-Bretton woods indicates that once you float the exchange rate you reduce that influence.
- Also consistent with the Trilemma, a  $\beta=-0.2$  for the capital control era of Bretton-Woods indicates in turn that capital controls can greatly help in reducing the  $i-i^*$  link.

## Lecture 4:

# International Risk Sharing and Sovereign Risk

# Objectives of this Class

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- Understand the concept of international risk sharing and why it is typically imperfect
- Relate imperfect international risk sharing to the risk of sovereign default
- Learn how to price a sovereign bond, or equivalently determine the pricing of sovereign risk
- Understand the main determinants of sovereign default

# Literature

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## Introductory Read

Feenstra and Taylor, *International Economics*, ch. 12

## Main Reading:

Obstfeld and Rogoff, *Foundations of ..*, chapters 5 and 6

Catão and Kapur, “Volatility and the Debt Intolerance Paradox”. In:

[https://papers.ssrn.com/sol3/papers.cfm?abstract\\_id=878874](https://papers.ssrn.com/sol3/papers.cfm?abstract_id=878874)

Catão, Fostel, and Kapur, “Persistent Gaps and Default Traps”, In:

[https://papers.ssrn.com/sol3/papers.cfm?abstract\\_id=1328571](https://papers.ssrn.com/sol3/papers.cfm?abstract_id=1328571)

Catão and Mano, “Default Premium”, In:

[https://econpapers.repec.org/article/eeeinecon/v\\_3a107\\_3ay\\_3a2017\\_3ai\\_3ac\\_3ap\\_3a91-110.htm](https://econpapers.repec.org/article/eeeinecon/v_3a107_3ay_3a2017_3ai_3ac_3ap_3a91-110.htm)

# International Risk Sharing

- If international financial markets were perfect, domestic residents can be insured against all types of risk that are particular to the country they live.
- This is possible because they could engage into the inter-temporal trade transactions we discussed in lecture 6.
- In other words, when a bad income shock hits one country but not others, that country could borrow so to prevent the consumption of its citizens to fall, and then repay when times are again good.
- The country who was hit by a bad shock would then run a current account deficit until the shock evaporates, and then repay back by running a current account surplus, as in the model of lecture 6.

# International Risk Sharing

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- Insert here the chart draw in class showing alternance of good and bad output realizations and how consumption is smoothed through international borrowing and lending (i.e. inter-temporal trade)
- But these require some assumptions about the behavior of borrowers and lenders which we will discuss here

# International Risk Sharing

- A key theoretical implication of perfect financial markets at the international level is that all individuals in home and foreign countries can equate their marginal rates of substitution between current consumption and (state-contingent) future consumption to the same state-contingent security prices.
- Start with the domestic resident having access to a full set of securities with the price  $p$  so that:

$$\underbrace{p_t(s_{t+1}) \frac{u'(C_t)}{P_t}}_{\text{Loss of utility of buying one unit of the security}} = \underbrace{\frac{\pi(s_{t+1}) \beta u'(C_{t+1})}{P_{t+1}}}_{\text{Marginal utility pay-off upon realization of } s(t+1)} \tag{8.1}$$

Loss of utility of buying one unit of the security

Marginal utility pay-off upon realization of  $s(t+1)$



# International Risk Sharing

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- Call  $Q_{t,t+1} = p_t(s_{t+1}) / \pi(s_{t+1})$  the stochastic discount factor, then:

$$\beta \frac{u'(C_{t+1})}{u'(C_t)} \frac{P_t}{P_{t+1}} = \frac{p_t(s_{t+1})}{\pi(s_{t+1})} = Q_{t,t+1}$$

Under CARA utility, it becomes:

$$\beta \left( \frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{P_t}{P_{t+1}} = Q_{t,t+1} \quad (8.2)$$

As the foreigner has access to the same security with the same pay-off in domestic currency, the analogous condition will hold:

# International Risk Sharing

- the stochastic discount factor, then:

$$\beta \left( \frac{C_{t+1}^*}{C_t^*} \right)^{-\sigma} \frac{P_t^*}{P_{t+1}^*} \frac{\varepsilon_t}{\varepsilon_{t+1}} = Q_{t,t+1} \quad (8.3)$$

where the exchange rate term converts the price index of the foreign basket to that of the home country unit.

Combining (8.2) with (8.3) yields

$$\left( \frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{P_t}{P_{t+1}} = \left( \frac{C_{t+1}^*}{C_t^*} \right)^{-\sigma} \frac{P_t^*}{P_{t+1}^*} \frac{\varepsilon_t}{\varepsilon_{t+1}}$$

# International Risk Sharing

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Re-arranging yields:

$$\left(\frac{C_{t+1}}{C_t}\right)^{-\sigma} = \left(\frac{C_{t+1}^*}{C_t^*}\right)^{-\sigma} \frac{\varepsilon_t P_t^* / P_t}{\varepsilon_{t+1} P_{t+1}^* / P_{t+1}} = \left(\frac{C_{t+1}^*}{C_t^*}\right)^{-\sigma} \frac{RER_t}{RER_{t+1}} \quad (8.4)$$

Taking logs and first differencing then yields:

$$\begin{aligned} -\sigma[\ln c_{t+1} - \ln c_t] &= -\sigma\Delta c_t + \Delta rer = -\sigma\Delta c_t^* \\ \therefore \Delta c_t &= \Delta c_t^* + \frac{1}{\sigma}\Delta rer \end{aligned} \quad (8.5)$$

which can also be written in level form:

$$C_t = \vartheta_{t-1} C_t^* RER_t^{1/\sigma} \quad (8.6)$$

# International Risk Sharing

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where  $v_{t-1} = C_{t-1} / (C_{t-1}^* RER_{t-1}^{1/\sigma})$  represents initial conditions.

Equation (8.5) advances two startling propositions:

- Consumption growth in any given country should be perfectly correlated with world consumption growth, once we adjust for fluctuations in the real exchange rate.
- Holding world consumption ( $C^*$ ) constant, consumption growth should rise with a real depreciation of the home currency, and more so the smaller risk aversion is.

# International Risk Sharing

- A lot of work has gone to test the growth correlations in (8.5) or the corresponding level relationship (8.6).
- A summary of the evidence for advanced economies is that international correlations in consumption are non-trivial for advanced country but quite low for emerging and developing economies (EMDEs)
- Even for advanced countries, correlations have been going up only after the 1990s as international financial integration increased

# International Risk Sharing

**Correlations between Domestic and World consumption Growth**  
(medians for each country group)

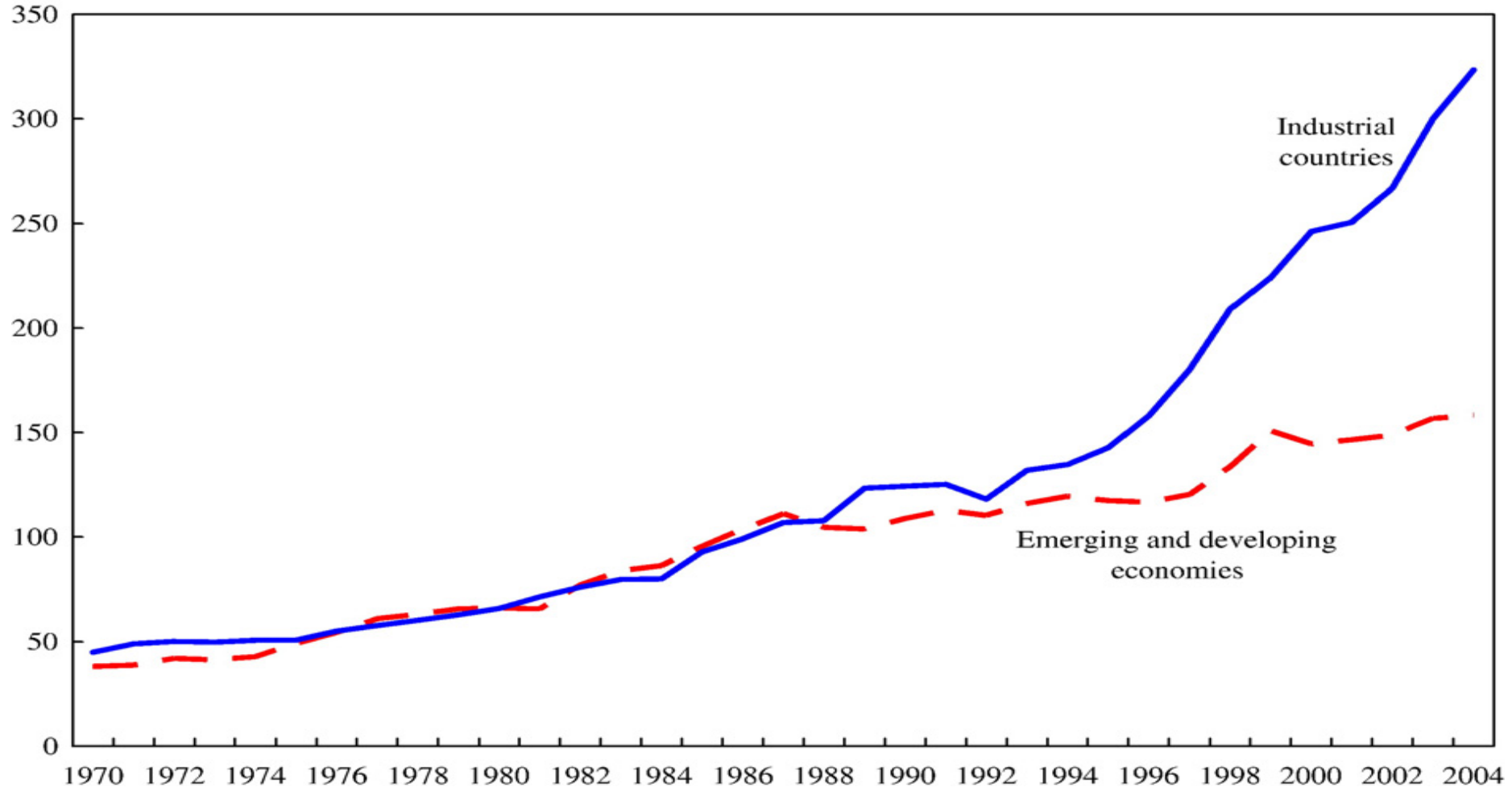
	<b>1961-2004</b>	<b>Bretton- Woods</b>	<b>Common Shocks</b>	<b>Globalization</b>
All Countries	<b>0.14</b> [0.04]***	<b>0.07</b> [0.05]	<b>0.2</b> [0.05]***	<b>0.07</b> [0.03]**
Industrial Countries	<b>0.5</b> [0.05]***	<b>0.22</b> [0.14]	<b>0.47</b> [0.11]***	<b>0.52</b> [0.10]***
Developing Countries	<b>0.03</b> [0.03]	<b>0.03</b> [0.05]	<b>0.04</b> [0.07]	<b>-0.03</b> [0.04]
Emerging Countries	<b>0.09</b> [0.04]*	<b>0.05</b> [0.09]	<b>0.02</b> [0.09]	<b>-0.11</b> [0.06]

From Kose, Prasad, and Terrones, 2009.

# International Risk Sharing

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## International Financial Integration $[(A+L)/GDP]$



From Lane and Milesi-Ferretti (2007)

# International Risk Sharing

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- The main problem is more the much more limited degree of risk sharing across emerging markets: this is not only much lower than across advanced countries but also shows no sign of increasing on average
- One explanation as we will see is higher default risk in EMDEs
- Another explanation is that, over and above default risk, cross-border financial flows are subject to high transactions costs (“financial frictions”) and those tend to be higher in EMDEs
- As shown in Catão and Chang (unpublished manuscript), these financial frictions can modeled as financial wedge in the Euler equation, making consumption more dependent on domestic income.
- In theory the coefficient of  $c-c^*$  on  $y-y^*$  should be zero on average but in fact it is something like 0.7-0.4 even for advanced countries only!



# Regression Results

VARIABLES	(1) FE d_c_pc	(2) FE+TE d_c_pc_rel	(3) IV (G, TFP) d_c_pc_rel	(4) CCE d_c_pc_rel	(5) CCE with IV1 d_c_pc_rel	(6) CCE with IV2 d_c_pc_rel
d_yr_pc_rel		0.704*** (0.0940)	0.615*** (0.144)	0.534*** (0.062)	0.513*** (0.076)	0.438*** (0.076)
d_tot	0.0776*** (0.0226)	0.075*** (0.0232)	0.081*** (0.026)	0.076** (0.039)	0.093** (0.035)	0.055 (0.052)
d_c_pc_wo	1.008*** (0.209)					
d_yr_pc	0.712*** (0.0873)					
d_yr_pc_wo	-0.656***					
Country Effect	Yes	Yes	Yes	Yes	Yes	Yes
Time Effect	No	Yes	Yes	No	No	No
Observations	2,232	2,232	2153	2,232	2,232	2,232
R-squared	0.635	0.550	0.547	--	--	--
Number of Countries	31	31	31	31	31	31
Cross-Sectional Independence (p-value)	0.019**	0.000***	0.000***	0.765	0.45	0.000*

# International Risk Sharing

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- Another problem with EMDEs is also in the composition of external liabilities: a lot of it is still in debt rather than equity instruments.
- And is clear from the figure below that the cross-sectional dispersion in consumption growths take place around financial/debt crises.
- The bottom-line is that EMs have not yet benefitted more fully from the risk sharing benefits of financial globalization

# International Risk Sharing

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## Cross-Country Dispersion of Real Consumption growths



# International Risk Sharing

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This takes us straight into issue of sovereign risk and external debt crises

To cement basic concepts let's first look at the simple two-period sovereign risk model. The references are O-R's (ch.6) and Catão and Kapur (IMF staff papers, 2005)

We will conclude with a discussion of the empirical determinants of external debt crises.

# Sovereign Defaults

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- Countries may not honor debts contracted by their sovereign government.
- This maybe because of *inability to pay* (GDP suddenly drops to too low levels) or because of **unwillingness to pay** (meaning strategically or opportunistic behavior).
- Naturally, investors take the default risk into account when lending to country, i.e., when buying the bonds of a certain government

# Sovereign Defaults

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- **General question:** as an investor, how much interest would you charge a borrower that may default on you?
- Remember: in economic equilibrium, it is assumed that investors **break even**, i.e., there can be no extra profit from a borrowing-lending or purchase-selling arbitrage (we have seen that this is the principle of PPP and UIP conditions)
- Thus, we will have here a simple equation that will rule out the possibility of excess returns (or profits) by the investor that lends to a sovereign government

# Sovereign Defaults

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- The equation assumes that a **risk-neutral** investor has two investment possibilities: one is to invest a given amount  $D$  into a risk-free bond that yields an interest rate  $r^*$ . So, at the end of the investment period, it will get:

$$D (1 + r_f)$$

- The alternative is to lend to a sovereign country (e.g., to buy a sovereign bond), which promises to pay her/him a better interest rate (call it  $r_L$ ), but subject to the risk of default. In this case, the **expected** return of the investor will be:

$$((1 - \pi) (1 + r_L) + \pi c (1 + r_L)) D$$

# Sovereign Defaults

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Where  $\pi$  is the probability of default and  $c$  is the rate of recovery (with  $1 - c$  being the so-called hair cut, i.e., the share of the debt that the investor will not be able to recover should default occur).

In equilibrium, i.e., in the absence of extra-profits due to the choice of investing on a risk free rate or on a government bond, we will equalize the two equations:

$$D (1 + r_f) = ((1 - \pi) (1 + r_L) + \pi c (1 + r_L)) D$$



# Sovereign Defaults

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You can now solve for the interest rate that investor will charge to the government:

$$rL = 1 + rf / (1 - \pi(1 - c)) - 1$$

As this formula demonstrates, the rate of interest paid by the government to investors will be rising on the risk free interest rate and the probability of default and the size of the so-called hair-cut.

# Sovereign Defaults

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From the preceding formula, you can also derive a classical measure of “country” or sovereign risk, the so-called sovereign bond spread:

$$spread = rL - rf = \pi(1-c)(1+rf) / 1 - \pi(1-c)$$

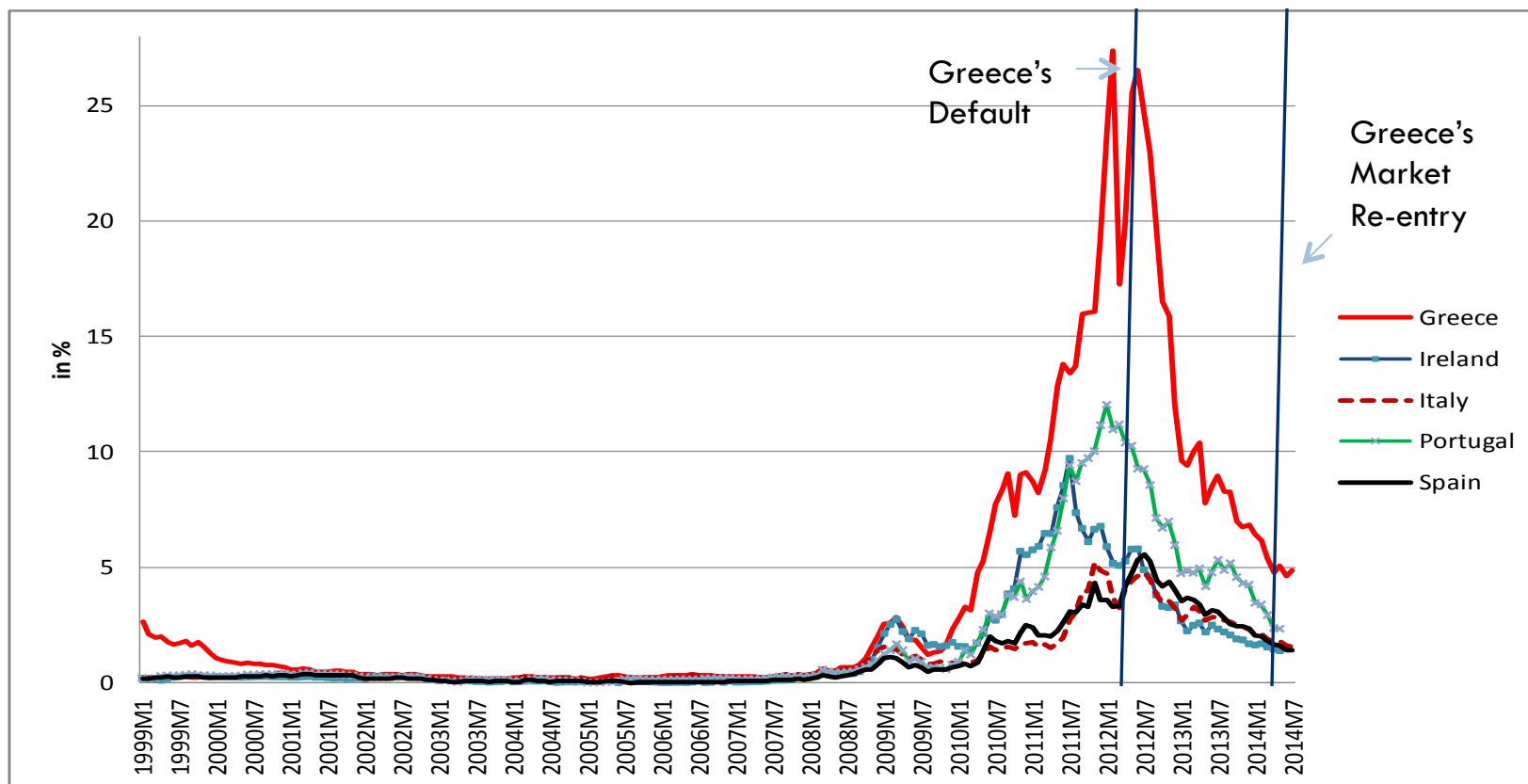
As this formula demonstrates, the spread will be rising on the risk free interest rate and the probability of default multiplied by the size of the so-called hair-cut, where the hair cut is  $(1-c)$ .

# Sovereign Default Facts

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**Figure 1. Eurozone Spreads Before and After Default**

(Yields on 10-year sovereign bond relative to Germany)



- Steep decline from default year but spreads still much higher than pre-crisis
- Spread much higher for the only country that defaulted vs. other debt crises

## Sample of Defaults & Settlements

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country	1870-1938		country	1970-2011	
	default	settle		default	settle
Argentina	1890	1893	Argentina	1982	1992
Austria	1914	1915	Argentina	2001	?
Austria	1932	1933	Brazil	1983	1992
Austria	1938	1938	Bulgaria	1990	1994
Brazil	1898	1901	Chile	1971	1975
Brazil	1914	1919	Chile	1983	1990
Brazil	1931	1933	Costa Rica	1981	1990
Brazil	1937	1943	Croatia	1992	1996
Bulgaria	1916	1925	Dominican Rep.	1982	1994
Bulgaria	1932	1932	Dominican Rep.	2003	2004
Chile	1880	1883	Ecuador	1983	1995
Chile	1931	1947	Ecuador	1999	2000
Colombia	1900	1904	Ecuador	2008	2014
Colombia	1932	1944	Indonesia	1998	2002
Czechoslovakia	1938	1946	Jamaica	1978	1979
Egypt	1876	1880	Jamaica	1981	1985
El Salvador	1921	1922	Jamaica	1987	1993
El Salvador	1932	1935	Jamaica	2010	?
El Salvador	1938	1946	Mexico	1982	1990
Germany	1932	1949	Morocco	1983	1983

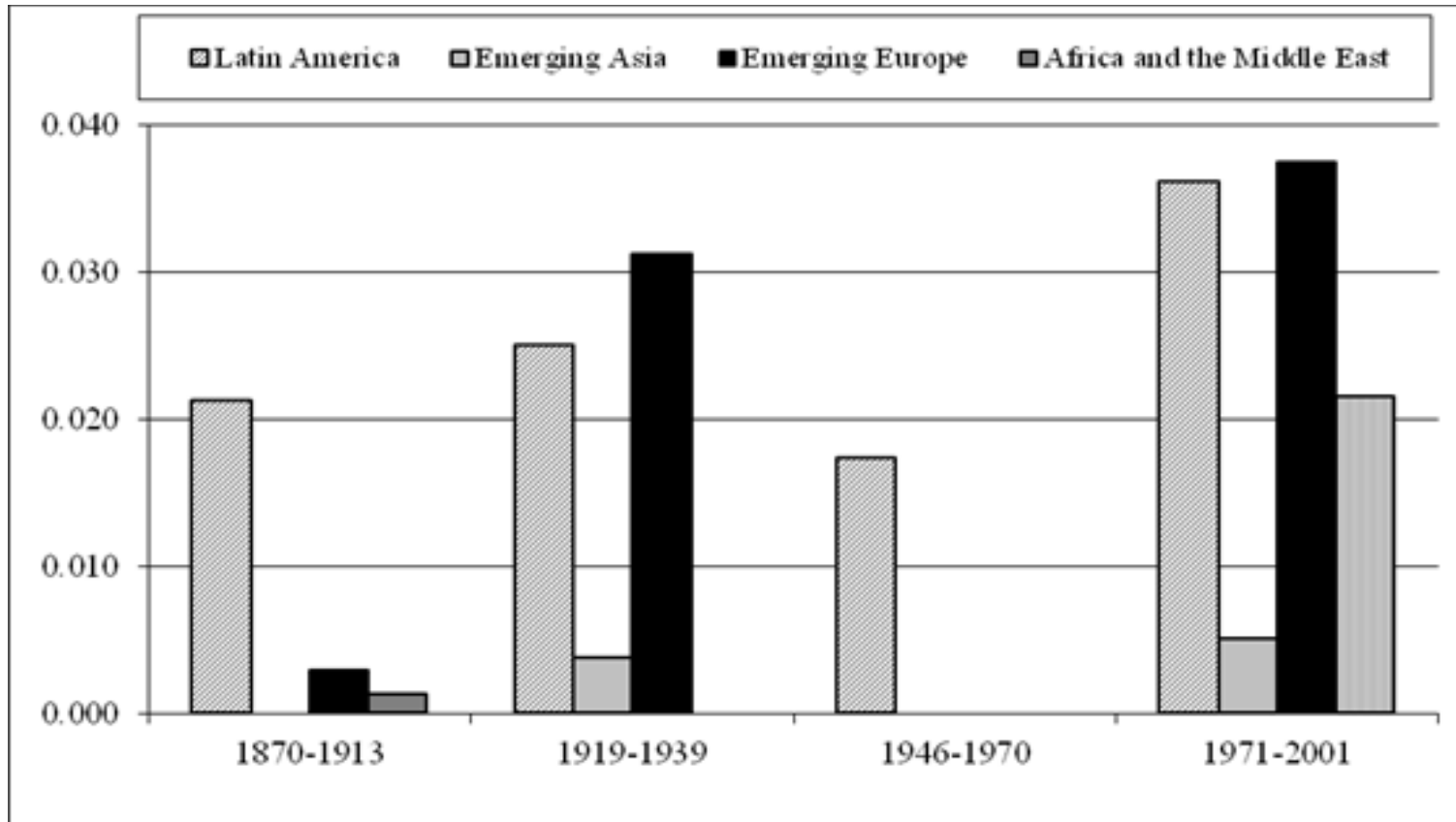
# Sample of Defaults & Settlements (II)

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1870-1938			1970-2011		
country	default	settle	country	default	settle
Greece	1894	1897	Morocco	1986	1990
Greece	1932	1964	Pakistan	1998	2000
Hungary	1932	1937	Panama	1983	1996
Mexico	1866	1885	Peru	1976	1976
Mexico	1914	1922	Peru	1978	1992
Mexico	1928	1942	Philippines	1983	1992
Peru	1931	1951	Romania	1982	1994
Poland	1936	1937	Russia	1917	1995
Portugal	1892	1901	Russia	1998	2000
Russia	1918	1995	Serbia	2000	2004
Spain	1837	1867	Slovenia	1992	1996
Spain	1873	1882	South Africa	1985	1993
Turkey	1876	1881	South Korea	1982	1986
Turkey	1915	1928	Turkey	1978	1982
Uruguay	1876	1878	Uruguay	1983	1991
Uruguay	1891	1891	Uruguay	2003	2003
Uruguay	1915	1921	Ukraine	1998	2000
Uruguay	1933	1938	Venezuela	1983	1997
Venezuela	1865	1881			
Venezuela	1892	1895			
Venezuela	1898	1904			

# Sovereign Risk: Facts

- One reason as to why risk sharing is impaired: Countries sometimes default on their commitments to pay back. As will show, in equilibrium, this reduces their capacity to borrow



# Sovereign Risk: The Canonical Two-period Model

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- Single good, two periods
- Sovereign country contracts  $P$  or borrows  $D$  in  $t=1$  and repays or defaults on this contract in  $t=2$ , when the world ends.
- To simplify, it cares only about period-2 utility:

$$U_1 = Eu(C_2) \quad (8.7)$$

- Output in  $t=2$  is stochastic and the country's total income will be output (GDP) plus any interest income from borrowing and saving the borrowing proceeds in  $t=1$ :

# Sovereign Risk: The Canonical Two-period Model

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$$Y_2(D) = \bar{Y} + \varepsilon + RD \quad (8.8)$$

where  $\varepsilon$  has zero mean.

- In the case of an **equity-type contract** (as in O-R, ch. 6),  $D=0$  so

$$Y_2(D) = \bar{Y} + \varepsilon \quad (8.8a)$$

- Lenders/insurers operate in a competitive market and are risk neutral so:

$$\sum_{i=1}^N \pi(\varepsilon_i) P(\varepsilon_i) = 0 \quad (8.9)$$



# Sovereign Risk: The Canonical Two-period Model

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Full Insurance Case: The country can commit and pay any  $P \leq Y_2$  as required by the equity type contract in  $t=1$ .

$$\text{With } P(\varepsilon) = \varepsilon: \quad C_2(\varepsilon) = Y_2 - P(\varepsilon) = Y_2 - \varepsilon = \bar{Y}$$

So, so consumption is fully smoothed at the level of the country's mean income.

But when  $\varepsilon > 0$ , the country has to make a payment to foreigners and can thus be tempted to renege on that.

In other words, the above contract needs to be made **incentive-compatible**.

## Sovereign Risk: The Canonical Two-period Model

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To facilitate this to be case, in sovereign risk models, it is common to assume that there is a penalty for defaulting on a contract.

In finite horizon models (as well as some infinite horizon ones), the penalty is an output loss  $= \eta Y_2$ . Thus:

$$P(\varepsilon_i) \leq \eta(\bar{Y} + \varepsilon_i) \quad (8.10)$$

If so, the incentive compatible contract can be solved as follows:

# Sovereign Risk: Canonical Two-period Model

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$$\max \sum_{i=1}^N \pi(\varepsilon_i) u[C_2(\varepsilon_i)] = \sum_{i=1}^N \pi(\varepsilon_i) u[\bar{Y} + \varepsilon_i - P(\varepsilon_i)] \quad (8.11)$$

st. (8.9) and (8.10). Then set-up the Lagrangian:

$$\begin{aligned} L = & \sum_{i=1}^N \pi(\varepsilon_i) u[\bar{Y} + \varepsilon_i - P(\varepsilon_i)] - \sum_{i=1}^N \lambda(\varepsilon_i) [P(\varepsilon_i) - \eta(\bar{Y} + \varepsilon_i)] \\ & + \mu \sum_{i=1}^N \pi(\varepsilon_i) P(\varepsilon_i) \end{aligned}$$

## Sovereign Risk: The Canonical Two-period Model

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Differentiate wrt the decision to pay the amount  $p$ :

$$\pi(\varepsilon)u'[C_2(\varepsilon)] + \lambda(\varepsilon) = \mu\pi(\varepsilon) \quad (8.12)$$

$$\lambda(\varepsilon)[\eta(\bar{Y} + \varepsilon) - P(\varepsilon)] = 0 \quad (8.13)$$

If  $\lambda(\varepsilon) = 0$ , the constraint is never binding, so the country can ensure smooth consumption. If not, a positive  $\lambda$  multiplier may imply uneven consumption across realizations of the output shock as in this case  $u'(C)$  is not equal to the constraint  $\mu$ .

# Sovereign Risk: The Canonical Two-period Model

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- Clearly, for low values of  $\varepsilon$ , **with such an equity contract (but not with a debt contract as we shall see)**, repayment is not an issue so the constraint never binds and  $P(\varepsilon) = P_0 + \varepsilon$ , and  $u'(Y - P_0) = \mu$ .
- The critical step is to compute a threshold value  $\varepsilon = e$  above which the constraint starts binding. That is for  $\varepsilon$  above  $e$ ,  $\lambda(\varepsilon) > 0$ .

This definition of  $e$  implies:

$$\begin{aligned} Y - P_0 &= \bar{Y} + e - \eta(\bar{Y} + e) = (1 - \eta)(\bar{Y} + e) \\ \therefore P_0 &= \eta\bar{Y} - (1 - \eta)e \end{aligned} \tag{8.14}$$

# Sovereign Risk: The Canonical Two-period Model

We can now draw the repayment curve:

$$P(\varepsilon) = \begin{cases} \eta(\bar{Y} + e) + \varepsilon - e & \text{for } \varepsilon \leq e \\ \eta(\bar{Y} + \varepsilon) & \text{for } \varepsilon \geq e \end{cases} \quad (8.15)$$

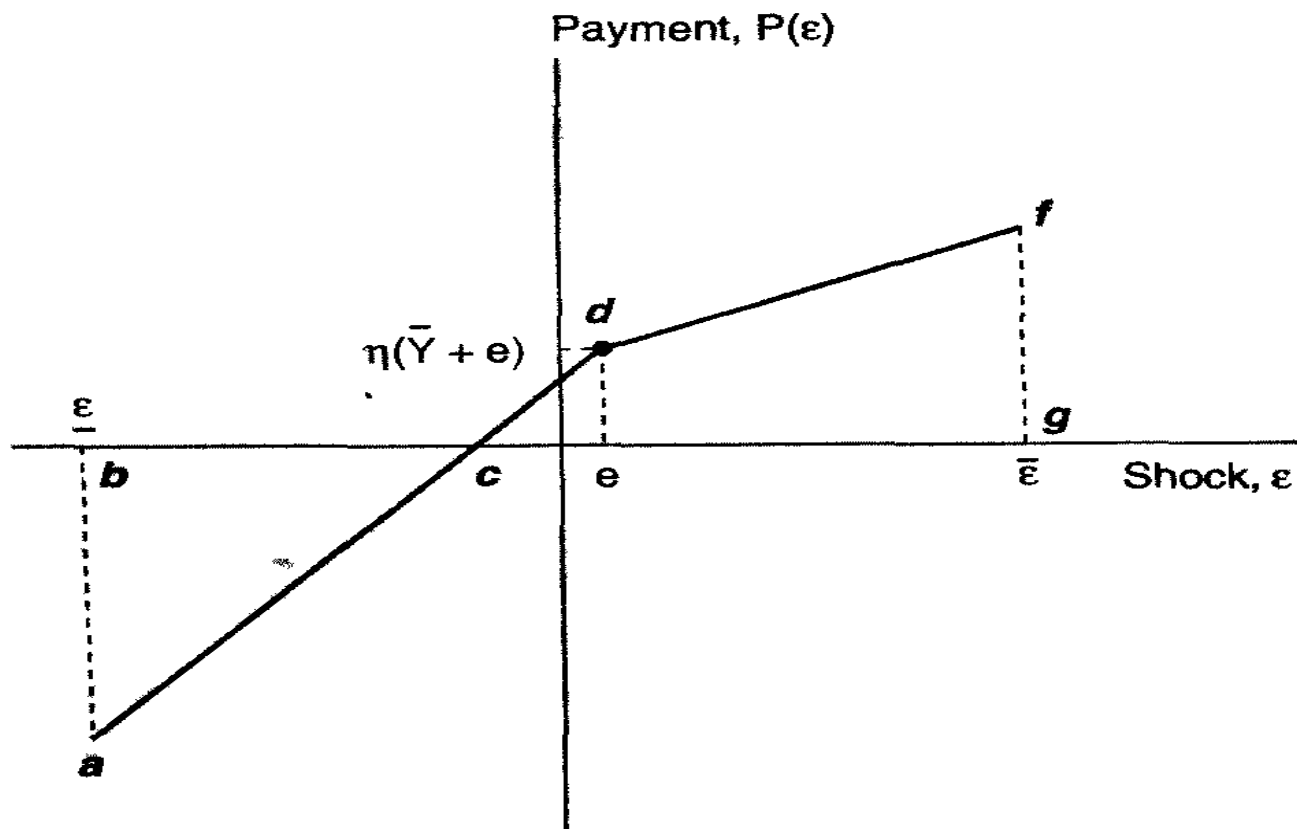
Clearly, the repayment curve will be 45 degree sloped until  $e$  and then its slope =  $\eta$ .

Consumption will be flat until  $e$  and then will rise proportionally to  $(1-\eta) \varepsilon$ .

This repayment schedule is plotted in O-R, page 358.

# Sovereign Risk: The Canonical Two-period Model

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**Figure 6.1**  
The optimal incentive-compatible contract

## Sovereign Risk: The Canonical Two period Model

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What remains to be done is to pin-down  $e$ . This is done by assuming a distribution for  $\varepsilon$  and using the lender's break-even condition (8.9).

We are going to see how this is done shortly in the context of a debt (rather than equity contract) but see example in O-R 6.1.1.4 for how  $e$  is calculated.

A key point: Default in this model, with an equity-type of contract, takes place during “good times”, i.e.,  $\varepsilon > e$ . However, we shall see that this is not typically the case! In the model that follows, we shall see a different prediction.



# Sovereign Risk: The Canonical Two-period Model

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- Now consider a model with a **debt contract**.
- Debt rather than equity type of contracts can arise for different reasons, costly monitoring of  $\varepsilon_i$  being a chief reason.  
 $\varepsilon$
- We stick to eqs. (8.7), (8.8) and the recovery technology in eq. (8.10), except for a change in the latter to take into account the size of the default.
- The model sketched is fully developed in Catão and Kapur (2005).

# Sovereign Risk: The Canonical Two-period Model

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- Now the country borrows  $D$  and promises to repay  $R_L D$ .
- The commitment problem now arises over lower realizations of  $\varepsilon$ . That is when the country has a problem to come with  $R_L D$ .
- So, payment takes the form of:

$$P(\varepsilon, R_L, D) = \text{Min}[R_L D, \eta Y_2(D)]$$

# Sovereign Risk: The Basic Model

So, we have:

$$P(\varepsilon, R_L, D) = \begin{cases} R_L D & \text{for } e \leq \varepsilon \leq \varepsilon_m \\ \eta[\bar{Y} + \varepsilon + RD] & \text{for } -\varepsilon_m \leq \varepsilon < e \end{cases} \quad (8.16)$$

where, as before  $e$  is the critical threshold between default and full repayment of contractual obligations:

$$e(R_L, D) \equiv \frac{[R_L - \eta R]D}{\eta} - \bar{Y} \quad (8.17)$$

$R$  being the risk-free interest rate.

# Sovereign Risk: The Canonical Two-period Model

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- While the borrower loses a fraction  $\eta$  of its income upon defaulting, this doesn't mean that the lender will fully capture it.
- In earlier models (Cohen and Sachs, 1986), it was assumed that this was lost (the so-called deadweight losses of default).
- It is reasonable to assume that some of it is recovered by lenders (e.g., through gunboats or vulture funds)
- Here we assume a default of size  $S$  imposes a cost  $(1+q)S$  on the lender to recover the  $\eta$  income share.

# Sovereign Risk: The Canonical Two-period Model

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- Hence, in case of default the net return to lenders will be:

$$P^*(\varepsilon, R_L, D) = R_L D - (1+q)S(\varepsilon, D). \quad (8.18)$$

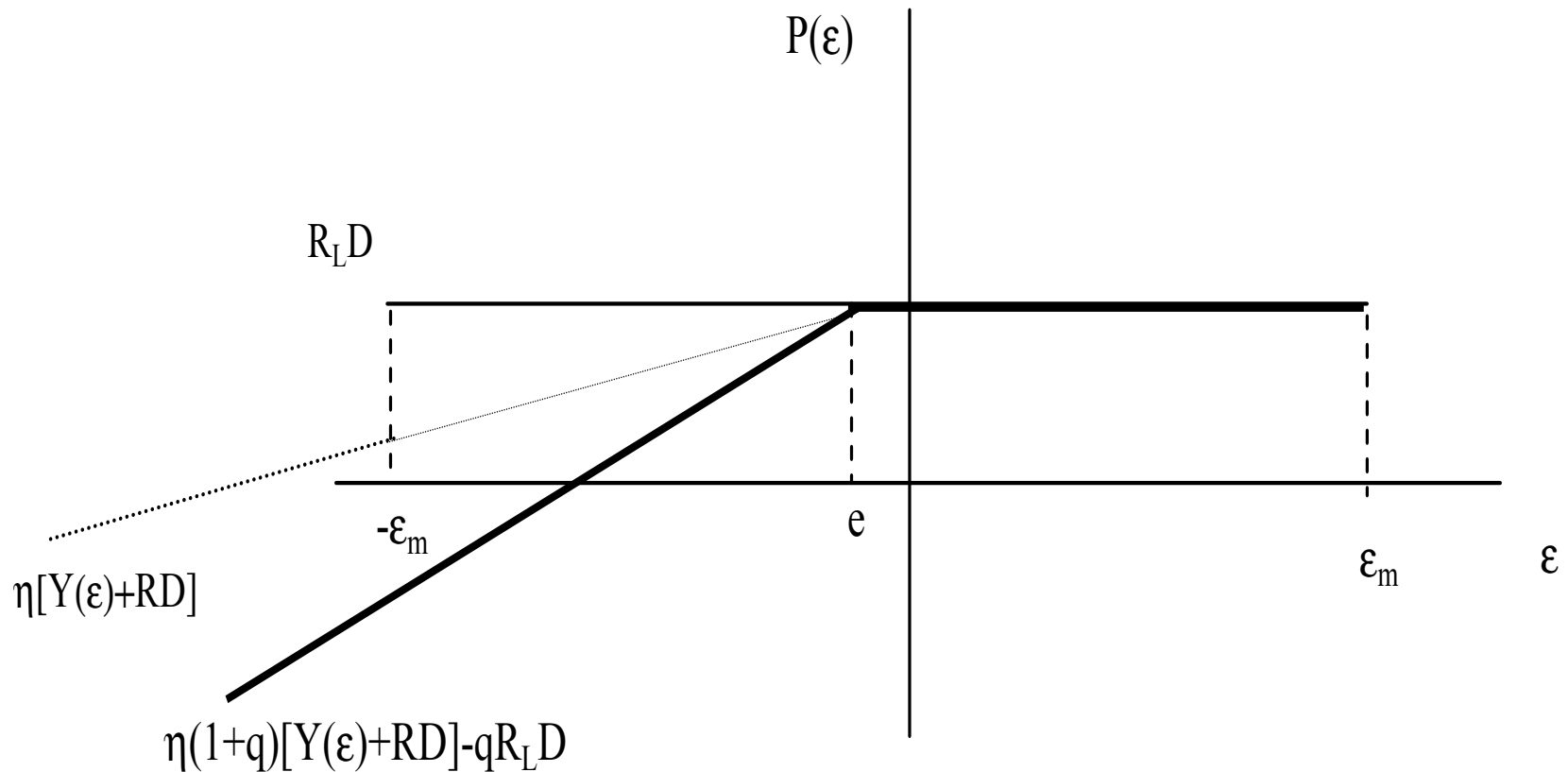
where  $q$  is a parameter that captures bargaining power between borrowers and lenders over the post-default income.

So, the payment schedule to lender will look like this:

# Sovereign Risk: The Canonical Two-period Model

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Return to Lenders



# Sovereign Risk: The Canonical Two-period Model

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For a continuous distribution, and sticking to the assumption that competitive lenders are risk neutral and break-even:

$$\int_{-\varepsilon_m}^{\varepsilon_m} P^*(\varepsilon, R_L, D) \pi(\varepsilon) d\varepsilon = RD \quad (8.19)$$

where  $(-\varepsilon_m, \varepsilon_m)$  is the support of the distribution.

Note that

$$\int_{-\varepsilon_m}^{\varepsilon_m} \pi(\varepsilon) d\varepsilon = 1 \quad (8.20)$$

# Sovereign Risk: The Canonical Two-period Model

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For a continuous distribution, and sticking to the assumption that competitive lenders are risk neutral and break-even:

$$\int_e^{\varepsilon_m} R_L D - \int_{-\varepsilon_m}^e [\eta(1+q)(Y(\varepsilon, D) + RD) - qR_L D] \pi(\varepsilon) d\varepsilon = RD$$

where  $(-\varepsilon_m, \varepsilon_m)$  is the support of the distribution.

Using (8.20) in the above yields:

$$(R_L - R)D = \int_{-\varepsilon_m}^{e(R_L, D)} \eta(1+q)[e(R_L, D) - \varepsilon] \pi(\varepsilon) d\varepsilon$$



# Sovereign Risk: The Canonical Two-period Model

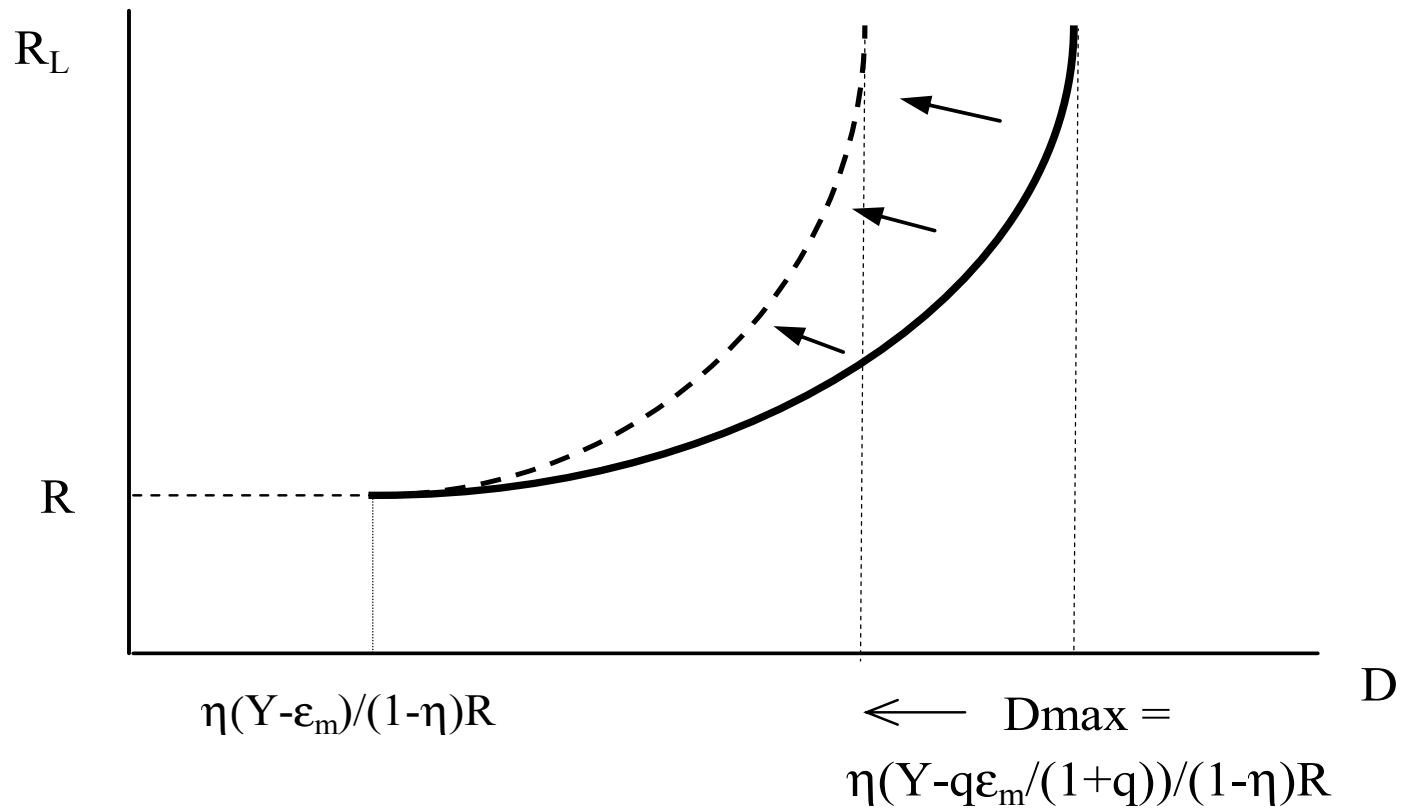
201

## Proposition 1

- (a)  $R_L(D)$  is well-defined for levels of debt in some bounded interval  $[0, D_{\max})$ , where  $D_{\max}$  depends, inter alia, on the probability distribution of shocks,  $\pi(\varepsilon)$ .
- (b)  $R_L(D) = R$  for  $D \in [0, \frac{\eta}{1-\eta} \frac{\bar{Y} - \varepsilon_m}{R}]$ . For higher values of  $D$ ,  $R_L(D)$  exceeds  $R$  and is strictly increasing in  $D$ .
- (c)  $R_L(D)$  is increasing in the variance of shocks.

# Sovereign Risk: The Canonical Two-period Model

## Effects of Volatility on Spreads and Borrowing Ceilings

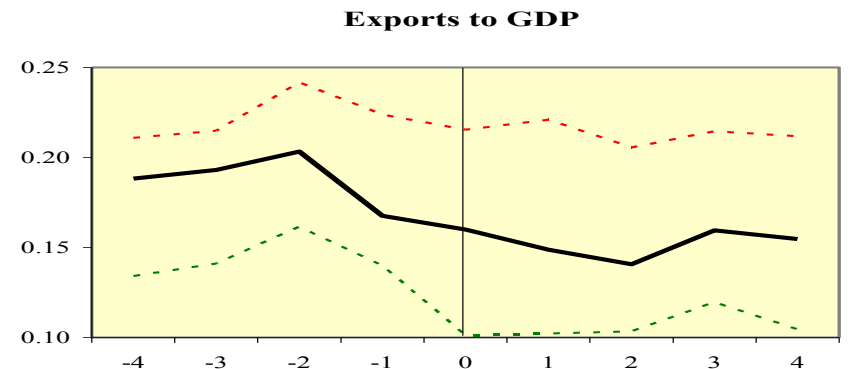
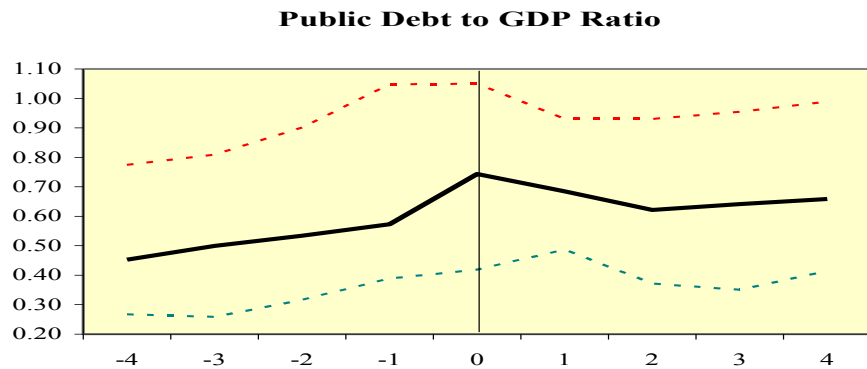
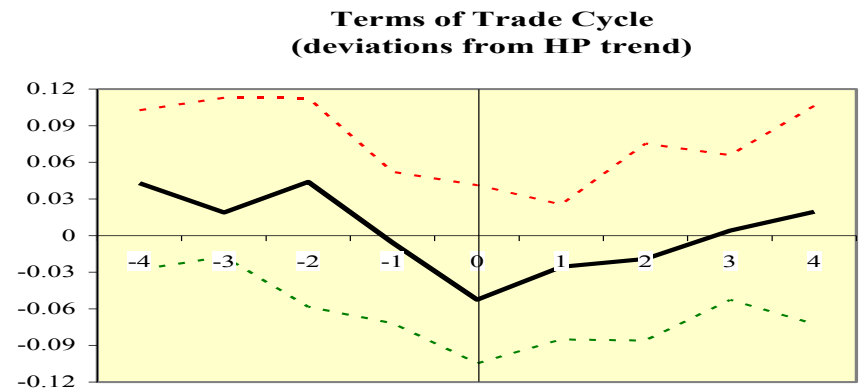
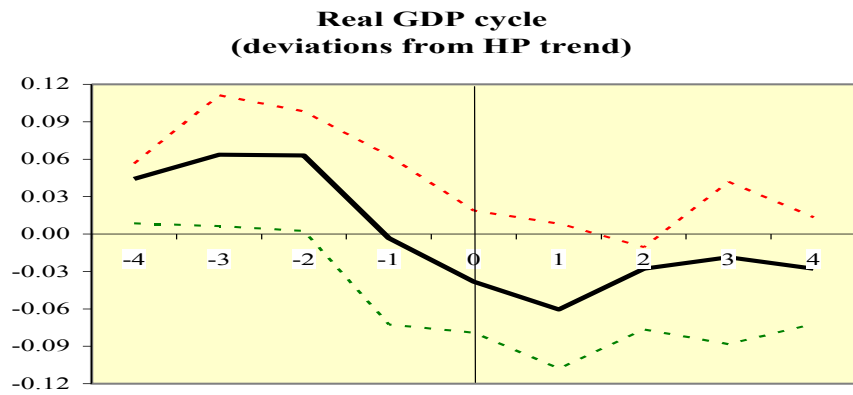
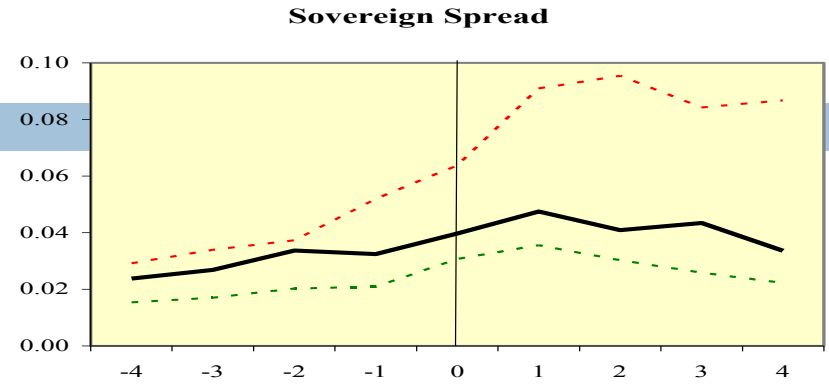
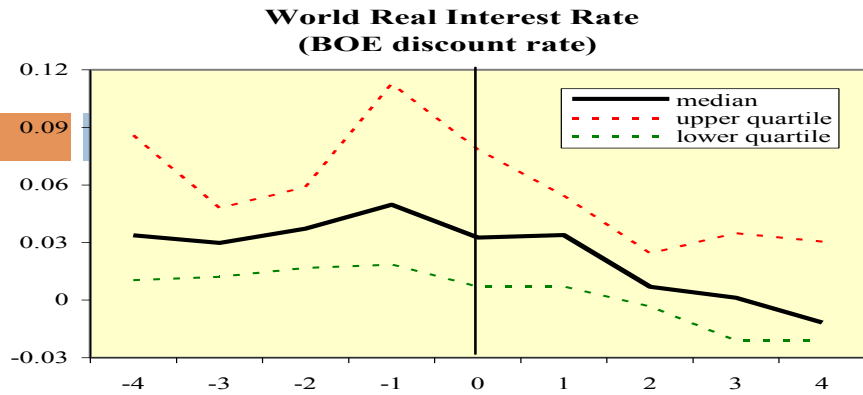


# Sovereign Risk: Stylized Facts

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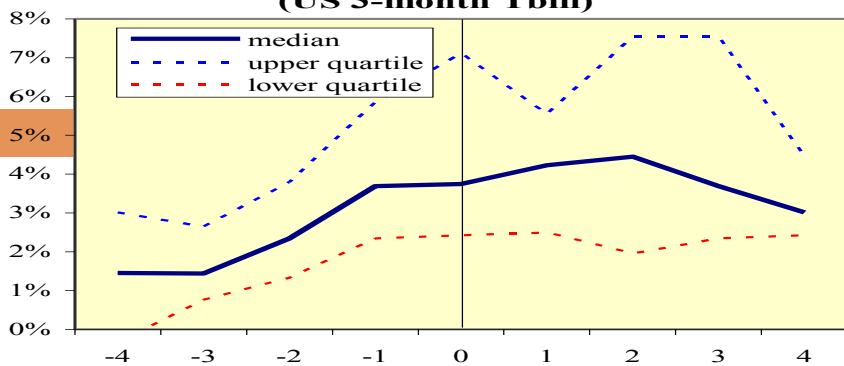
- Do defaults take place in good states (as in the OR equity contract 2-period model) or in bad states (as in Catão and Kapur, 2005)?
- How about the role of debt levels? Does higher debt/GDP increase significantly default risk?
- Does a higher (world) risk free rate increases default risk and sovereign spreads?

**Figure 3. Macroeconomic Developments around Sovereign Defaults, 1870-1939**

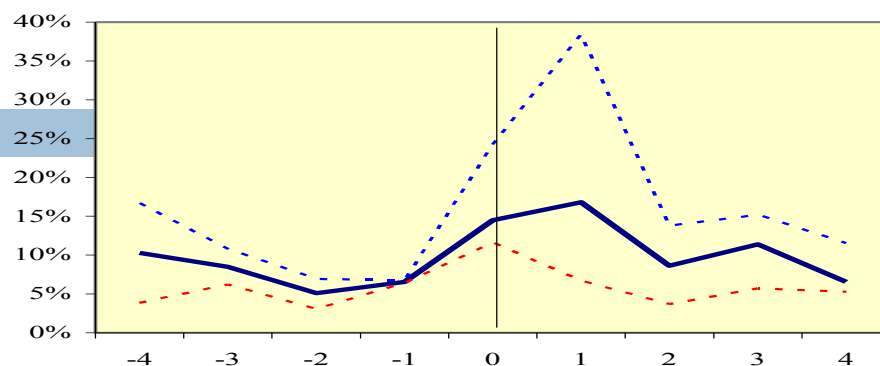


**Figure 4. Macroeconomic Developments around Debt Crises, 1960-2004**

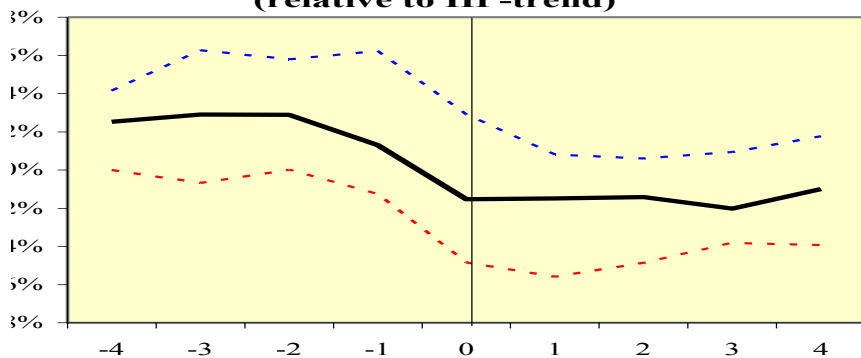
**World Real Interest Rate  
(US 3-month Tbill)**



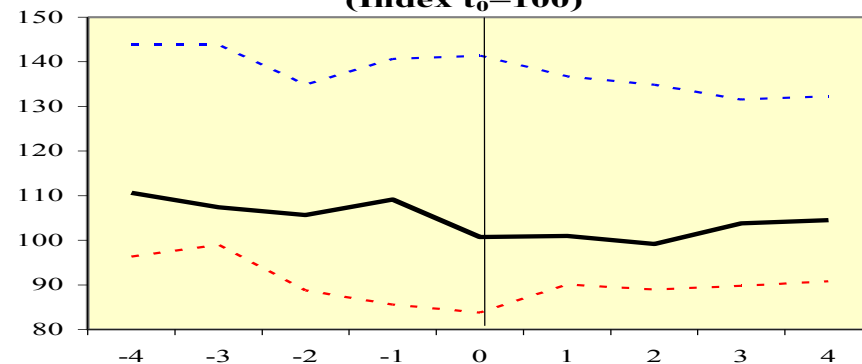
**Sovereign Spread**



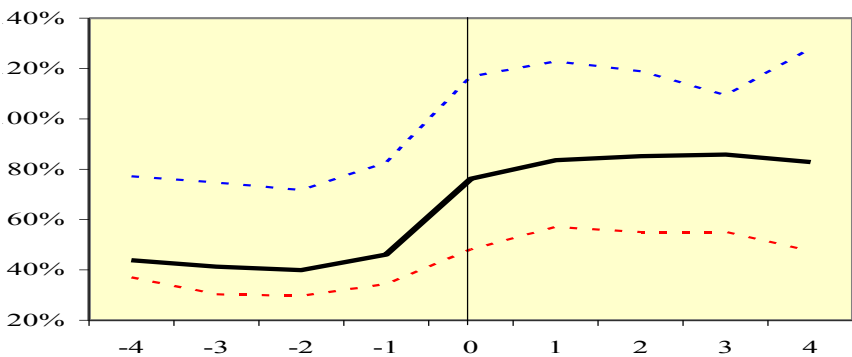
**Output Gap  
(relative to HP-trend)**



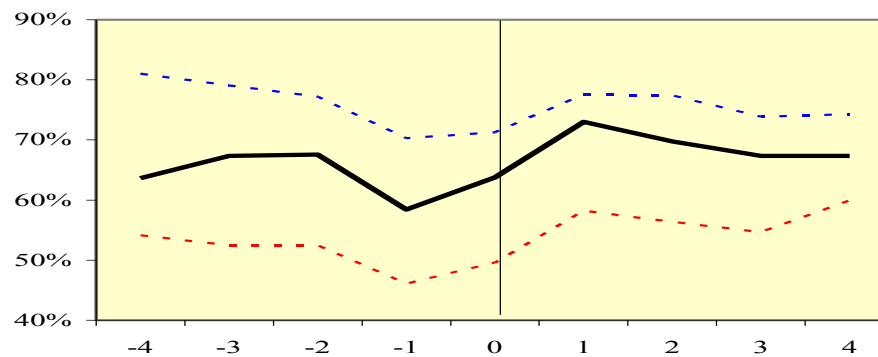
**Terms of Trade  
(Index  $t_0=100$ )**



**Public Debt to GDP**



**External to Total Public Debt**



# Sovereign Risk: Allowing for the Role of output persistence

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- In the two period model, the world “ends” after the output realization uncertainty is resolved
- But how about if the uncertain output shock is allowed to persist?
- Also in the canonical model, the shock is observed by everyone (borrowers and lenders), i.e., information on the shock is **symmetric**
- How about if borrowers observe directly the shock but lenders do not, i.e., information on the shock is asymmetric
- We will now turn to a model that provides theoretical predictions in a setting with persistent output shocks and asymmetric information on those shocks

## 3-period Model of Sovereign Default with Persistent Shocks and Asymmetric Information

### Sovereign:

Has borrowing needs  $I_0$  and  $I_1$  which are financed with one-period discount bonds at the beginning of each period so that  $D_1 = I_0/p_0$  and  $D_2 = I_1/p_1$ .

### Lenders:

Risk-neutral in a competitive bond market, seeking to break even period by period.

Punishment:  $t_1$ : lenders can enforce partial recovery  $cD_1$   
 $t_2$ : borrowers lose  $cD_2$  and  $sY_2$ .

→ Operate in an environment with two key features:

## 1) Stochastic Output Persistence.

2 sources of output uncertainty: *persistent* and *transitory*.

$$\tilde{Y}_1 = \bar{Y}_1 + \tilde{\epsilon}_1 + \tilde{\omega}_1$$

$$\tilde{Y}_2 = \bar{Y}_2 + \rho\tilde{\epsilon}_1 + \tilde{\omega}_2$$

$\tilde{\epsilon}_1 \sim N(0, \sigma_{\tilde{\epsilon}}^2)$  = permanent shock

$\tilde{\omega}_t \sim N(0, \sigma_{\tilde{\omega}}^2)$  = transitory shock

$\rho$  = persistence measure



## 2) Asymmetric Information.

Unlike borrowers, lenders **do not** observe the realization of  $\theta_1$ .

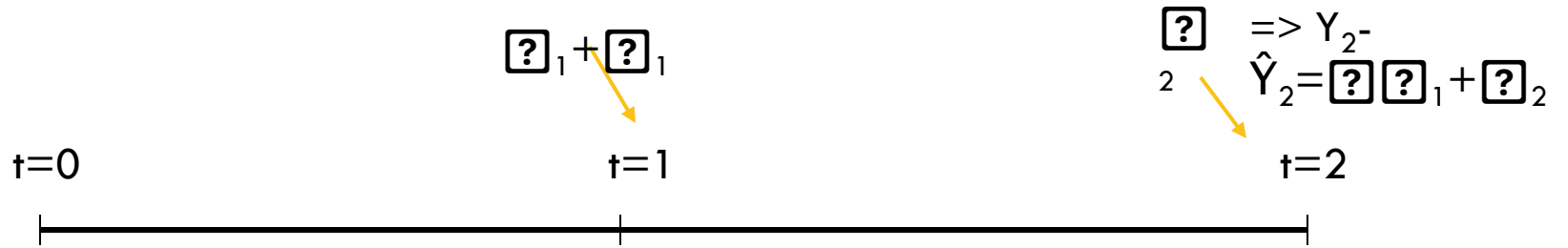


*They cannot distinguish the temporary  
from the persistent component of the shock*

Hence lenders will form *beliefs* on  $\theta_1$  based on borrower's actions at  $t_1$ :

**default vs. repayment.**

# The Game:

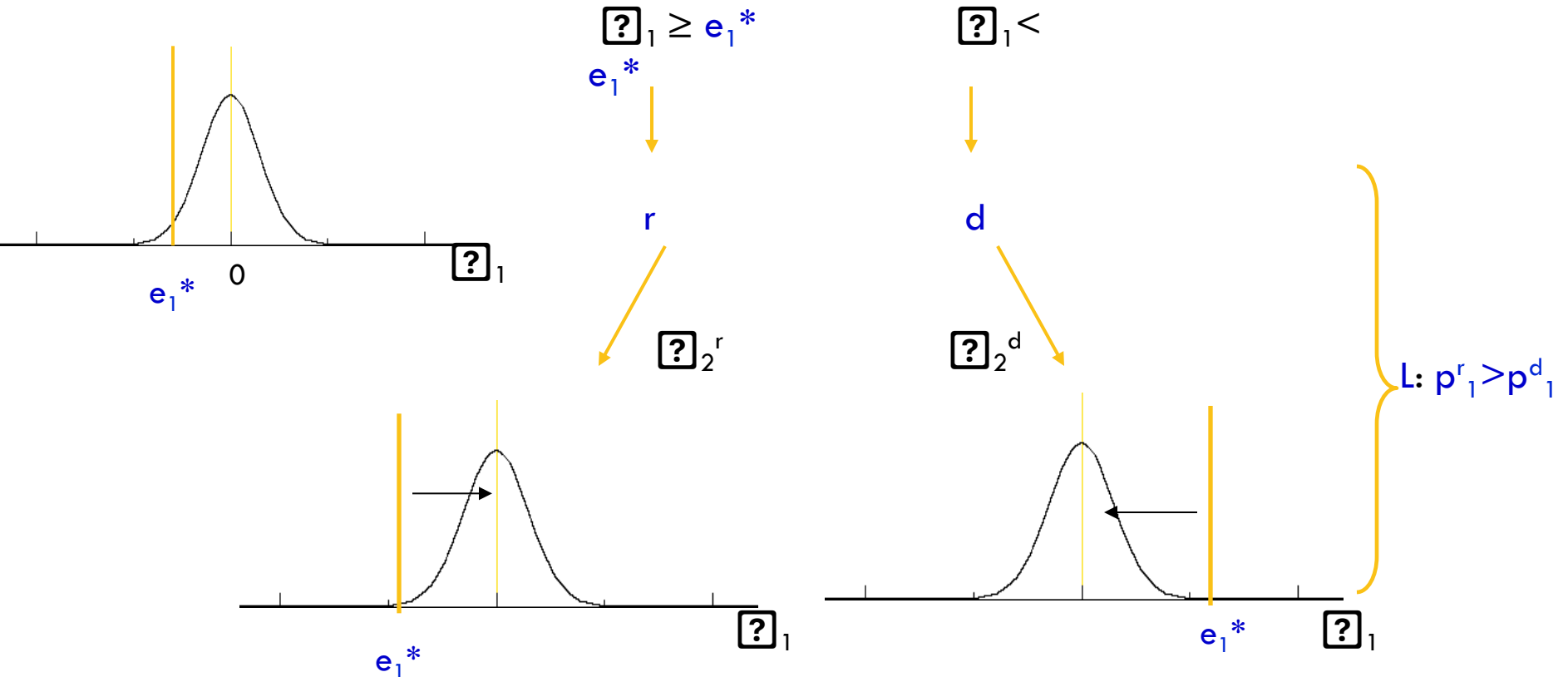


L:  $p_o$

$?'_1$  (given by prior):

B:  $r, d$

B:  $d: D_1^h/Y_2 > s/(1-c)$



In the paper we prove the existence of a *Perfect Bayesian Equilibrium* of this game and show that:

- Such asymmetry of information generates a positive *default premium* ( $p^r_1 - p^d_1 > 0$ ) that sustains positive levels of debt in the absence of reputational considerations or punishments by lenders!

Intuition: In equilibrium we have

$$\boxed{?}[V^r_2(e^*_1) - V^d_2(e^*_1)] = (1-c)D_1(e^*_1)$$

Gains from repayment  
(lower debt financing  
costs in future in PV terms)

Gain from default  
(higher present consumption)

- This PBE also has something to say about Stylized Facts 1 and 2:

### 1) Vicious circles or “Default Traps” (SF 1)



### 2) Borrower stays in the market but facing sharp correction in spreads (SF 2)

$$(R_i - R_f) \uparrow \uparrow$$

***Unlike other studies, default is informative so spreads can shoot right up!***

- Key point: The double role of the “default premium”

**Ex-ante**: the default premium ( $p^r_1 - p^d_1 > 0$ ) provides deterrence which can sustain positive debt.

**Ex-post**: but once a bad shock hits and the country defaults, this default premium hikes up spreads and hence the debt burden in the subsequent periods.

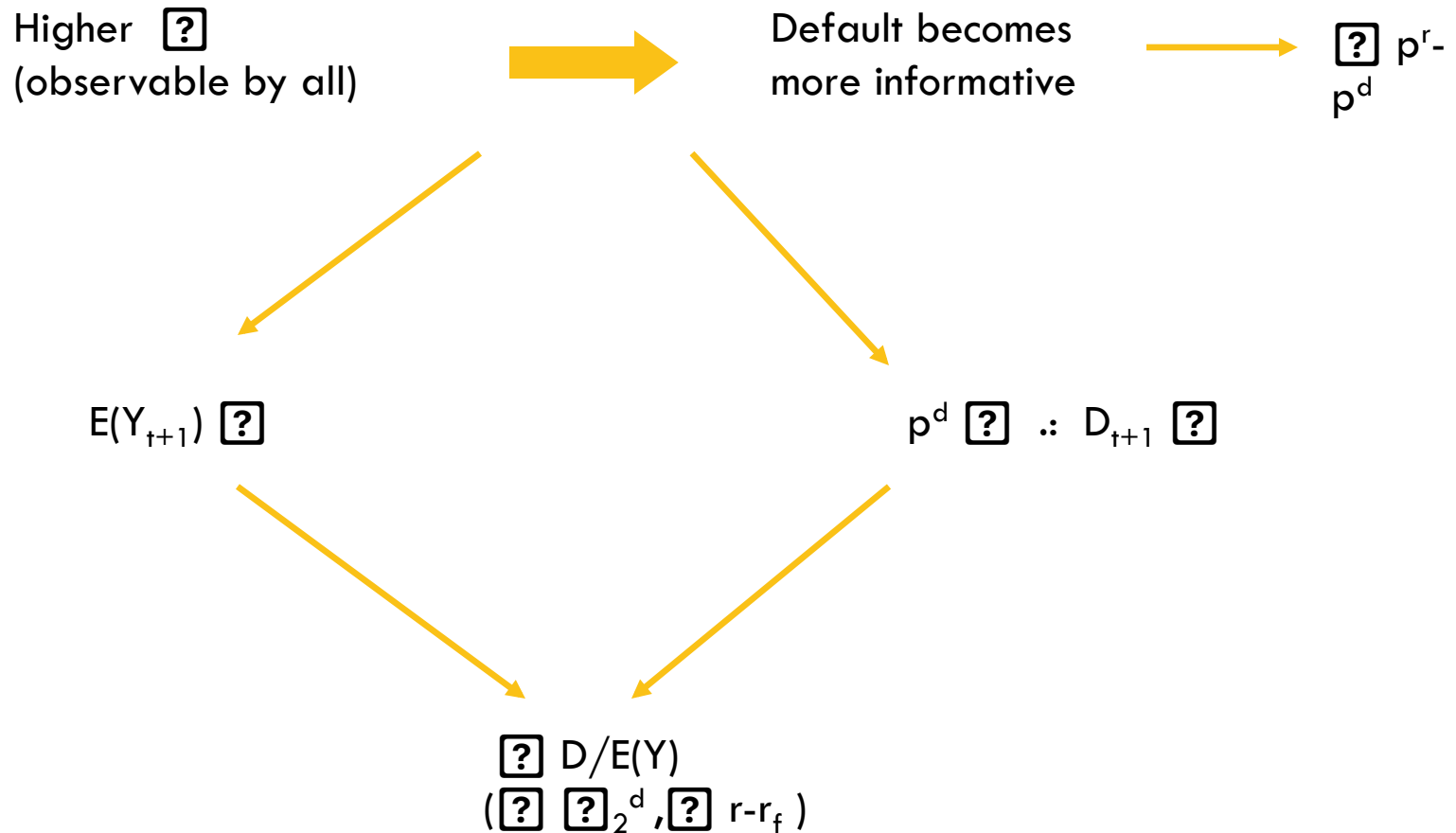
So, it makes future defaults less costly, creating **default traps**.

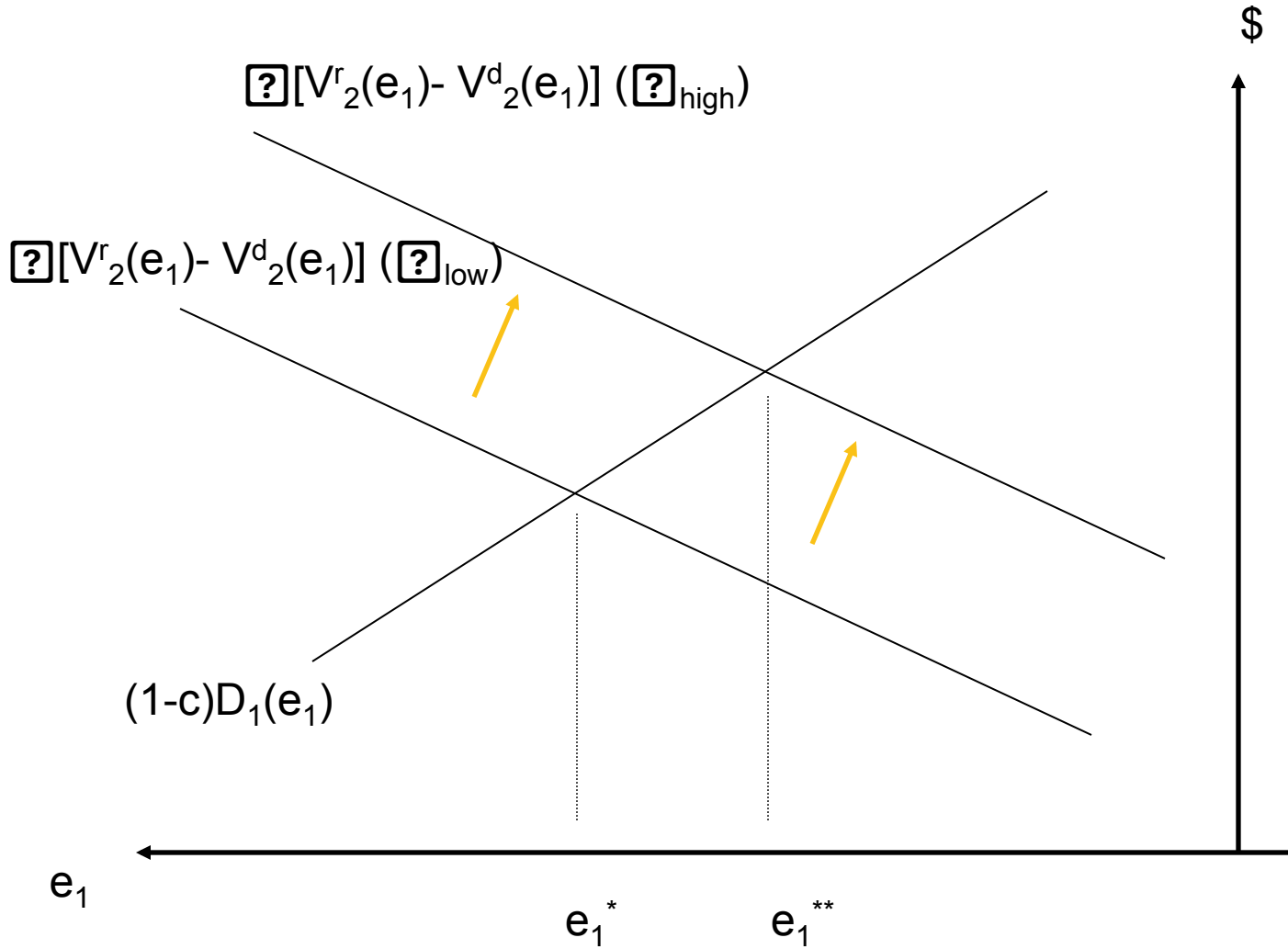
Note also that this mechanism is entirely symmetric, so it also helps explain “**virtuous paths**” of borrowing, repayment and declining spreads.

## Comparative Statics

Our model also shows that **higher output persistence** (higher  $\rho$ ) exacerbates this default trap mechanism, **increasing** the equilibrium probability of default, the default premium, and hence the country spread  $R_i - R_f$ .

### Basic Intuition:





**Table 1. Real GDP Volatility and Persistence and Countries' Repayment Records**  
 (in deviations from HP trend, excluding default periods)

	<b>1870-1913</b>		<b>1919-1939</b>		<b>1960-2005</b>	
	Std. Dev.	AR(1)	Std. Dev.	AR(1)	Std. Dev.	AR(1)
<b>Developing</b>	4.50%	0.44	7.53%	0.58	3.85%	0.65
<b>Developed</b>	4.12%	0.32	6.86%	0.53	2.07%	0.59
<b>Defaulters</b>	4.50%	0.44	5.74%	0.56	3.85%	0.62
<b>Serial Defaulters</b>	6.37%	0.53	6.54%	0.65	3.80%	0.67
<b>Non-defaulters</b>	3.72%	0.35	5.61%	0.57	2.41%	0.60

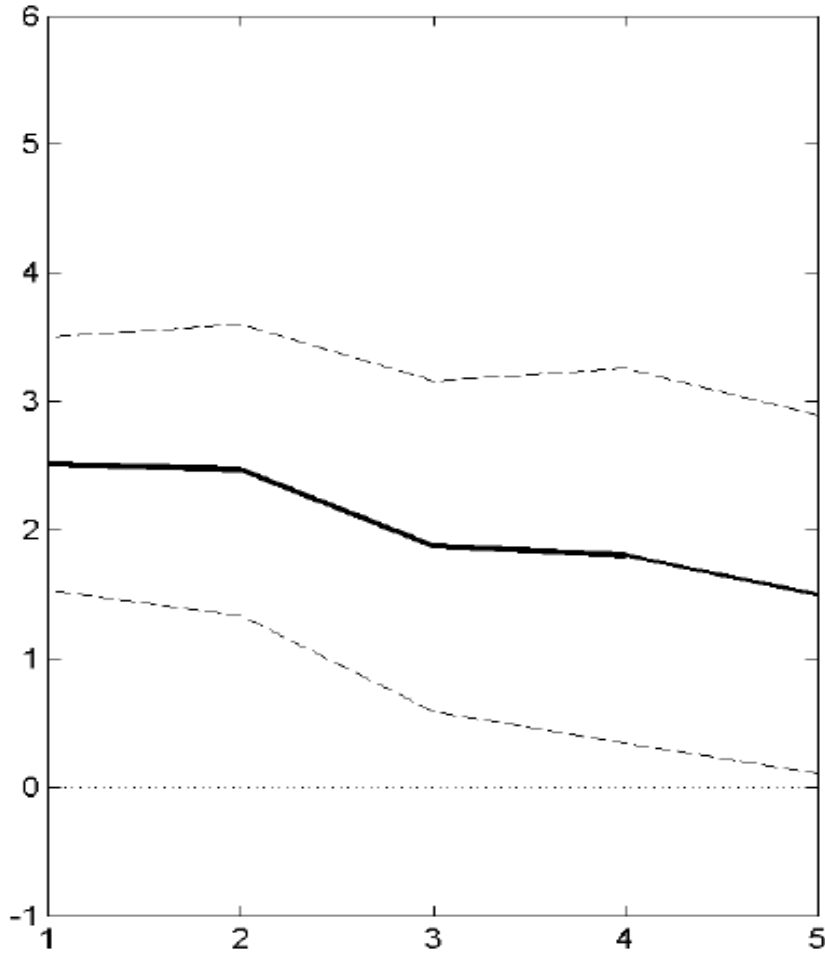




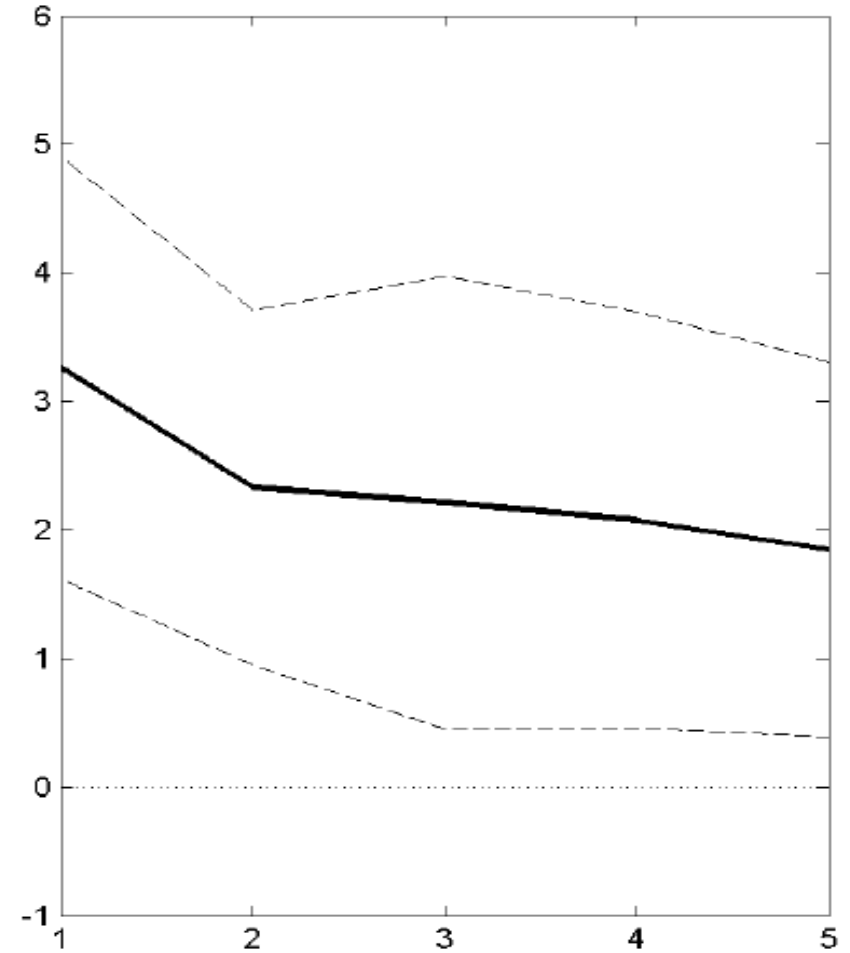
# Interest Premium Paid due to Default

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PRE-WWII SAMPLE

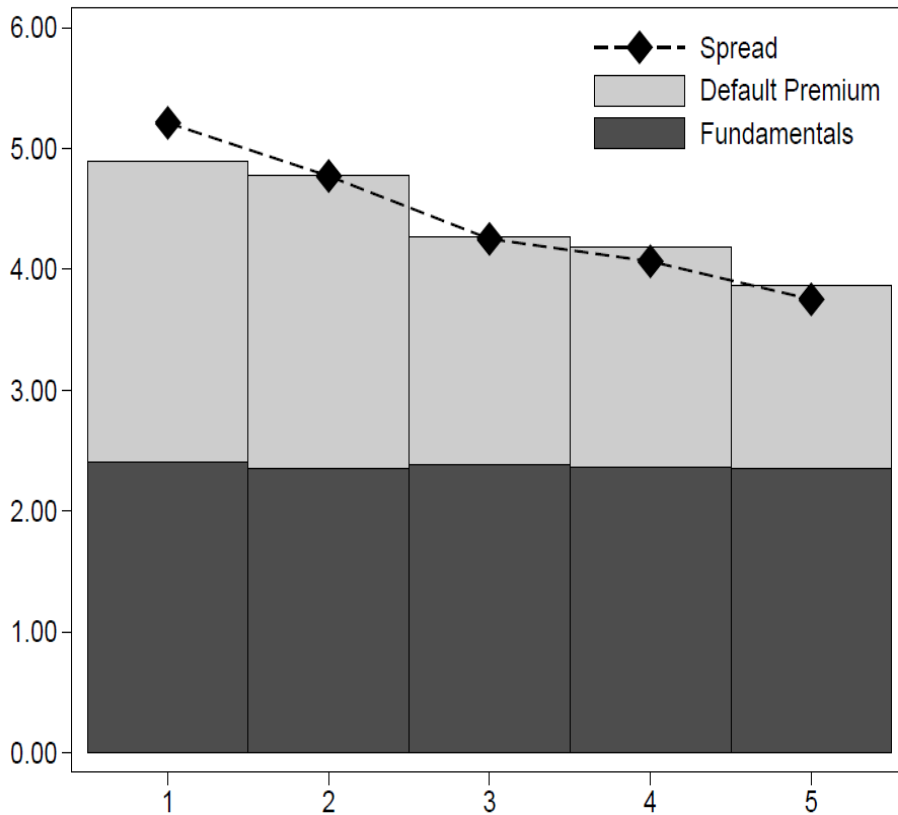


POST-WWII SAMPLE

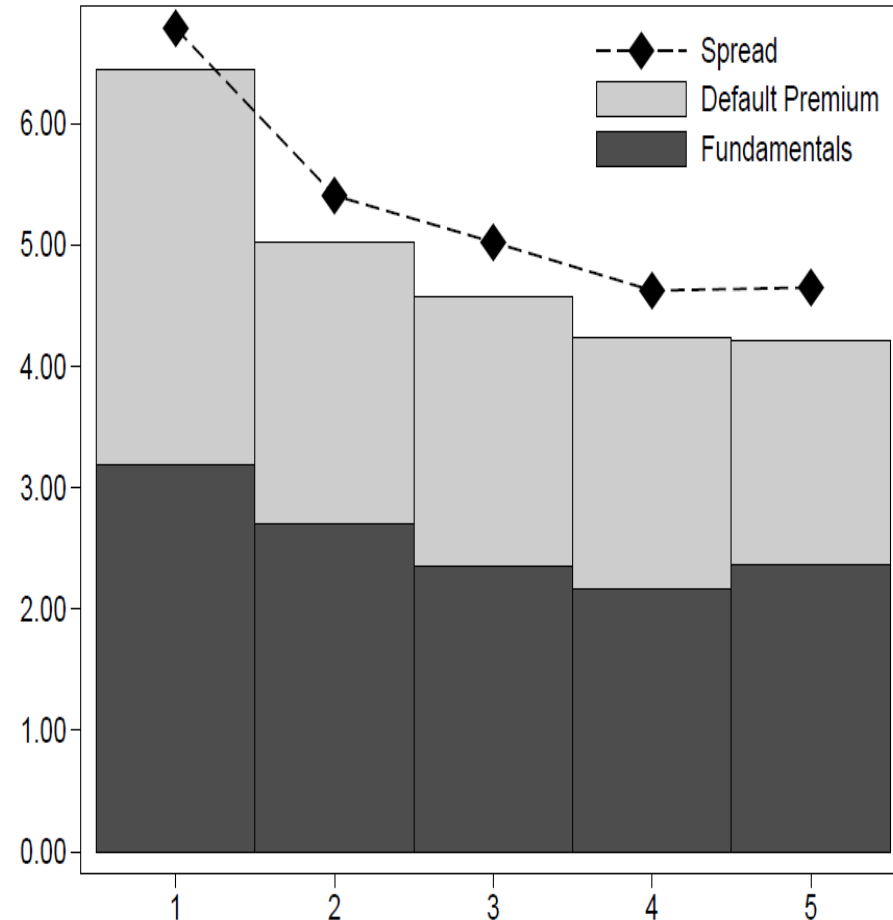


# Deconstructing the Spread: Fundamentals vs. DP

### Pre-WWII



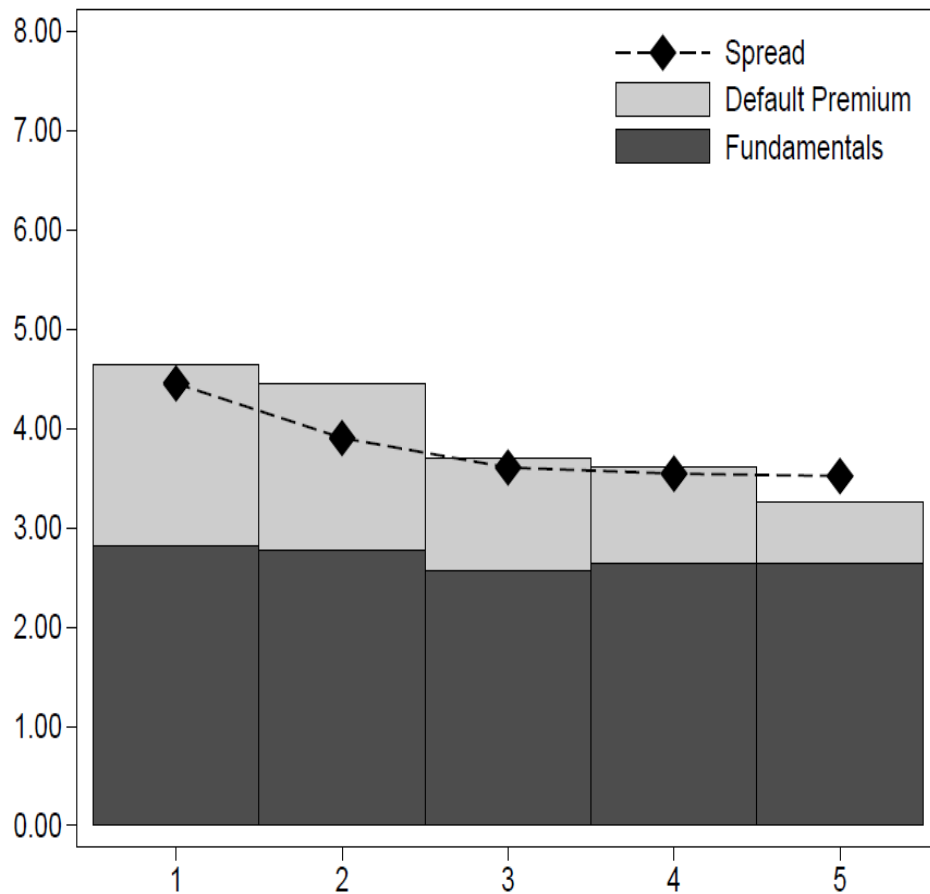
### Post-WWII



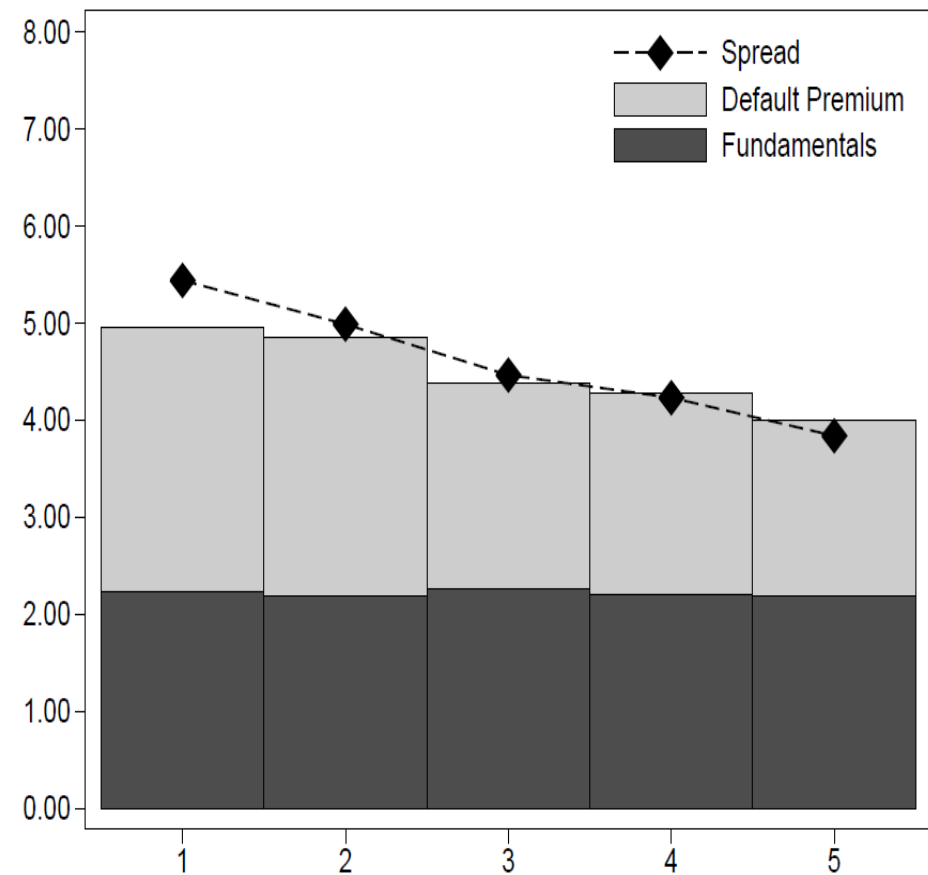
# Deconstructing the Spread: Mono vs. serial Defaulters (pre-war sample)

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## Mono defaulters



## Serial Defaulters



# International Risk Sharing and Risk Sharing: Main Take-ways

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- So, the more volatile the country and the smaller the cost of default (i.e. the smaller the recovery rate parameter  $c$  or higher the hair-cut,  $1-c$ ) the more risk it is.
- So, investor limit their exposure to the country, limiting the amount of maximum debt they lend → So, less scope for risk sharing
- Investors will also charge a higher interest rate, i.e., a higher spread over the risk free rate, specially after the country default (i.e. there is a positive **default premium**)
- 3-period model with asymmetric information: countries that have output shocks that are more persistent, tend to default more often, so face a higher spread on average and lower maximum debt limits
- To the extent that some countries are chronically more volatile, subject to more persistente shocks and investors have asymmetry of information about them, this helps explain why some countries are also persistente or “serial” defaulters