History of Economic thought 2019-2020

Seminar 2.4

Growth theories and models

Summary of the lecture

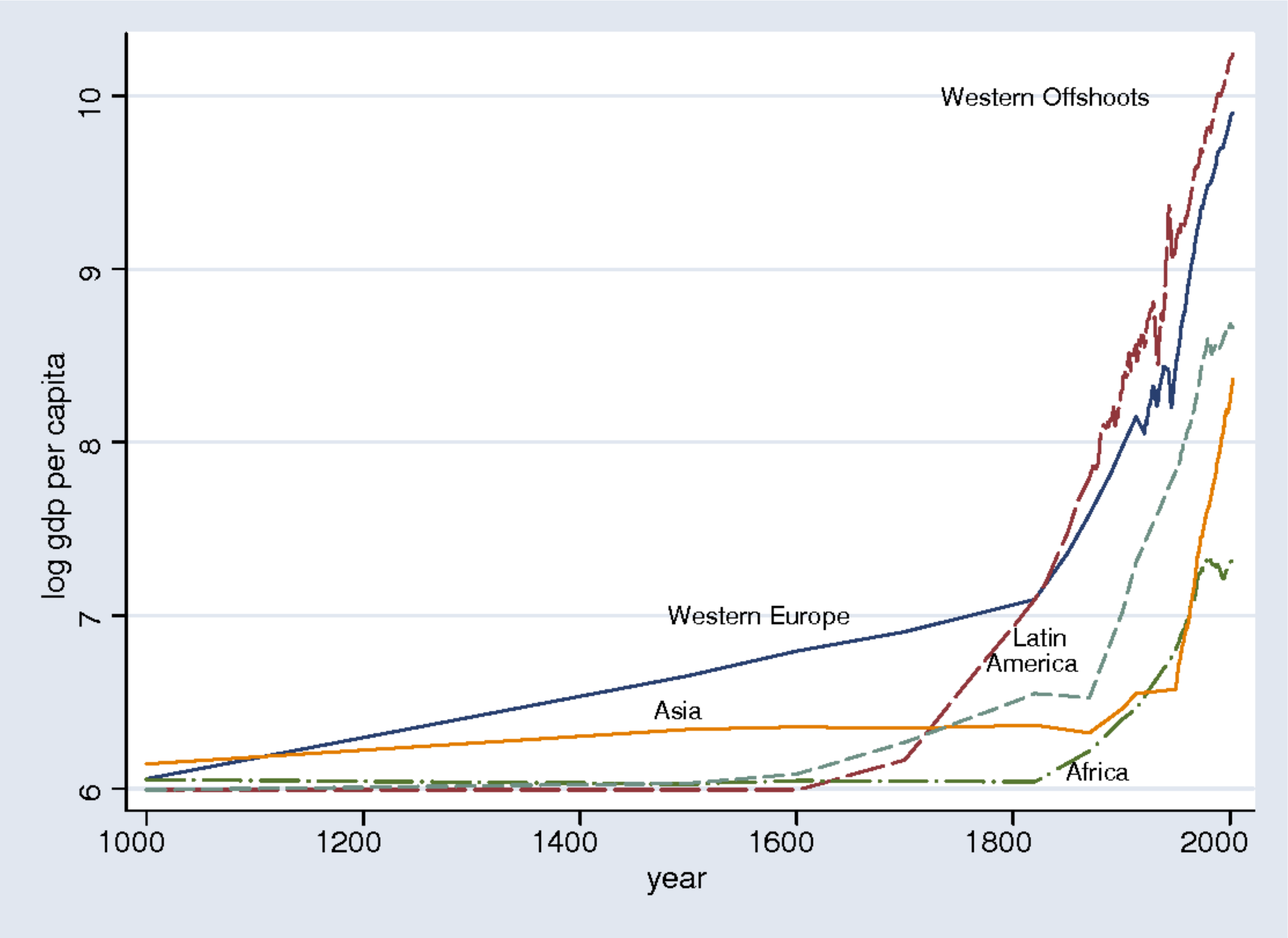
- Modern economic growth: facts and figures
- Theories and models:
  - Harrod-Domar
  - Solow-Swan
  - Endogenous growth theory
- Problems and limits to growth

Modern Economic Growth

Six characteristics of modern economic growth (Kuznets 1971)

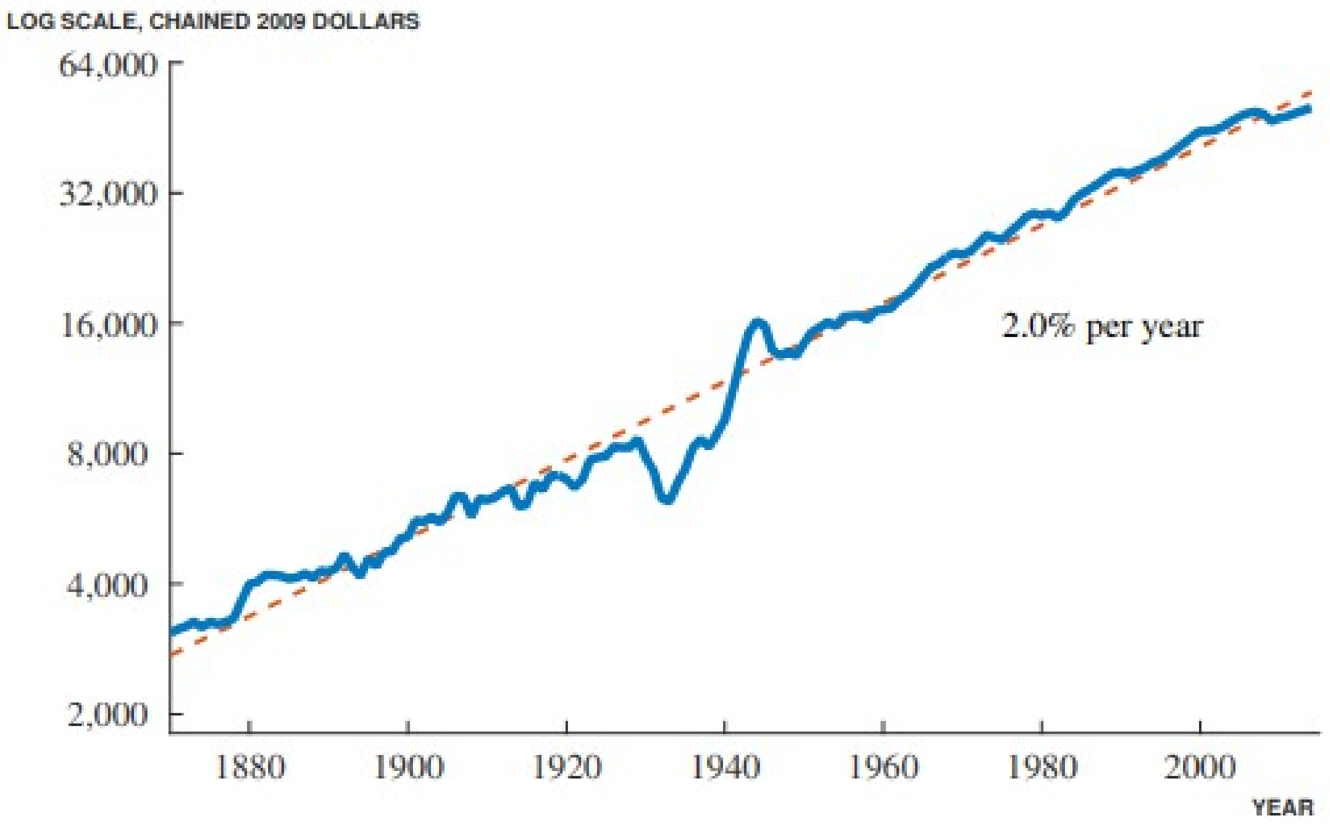
- High growth rates of per capita output and population
- Fast productivity growth, even controlling for factor accumulation (TFP)
- High rates of structural tranformation of the economy (sectoral, scale of productive units, etc)
- High rates of social and ideological change
- International economic outreach
- Limited spread of growth

Modern Economic Growth

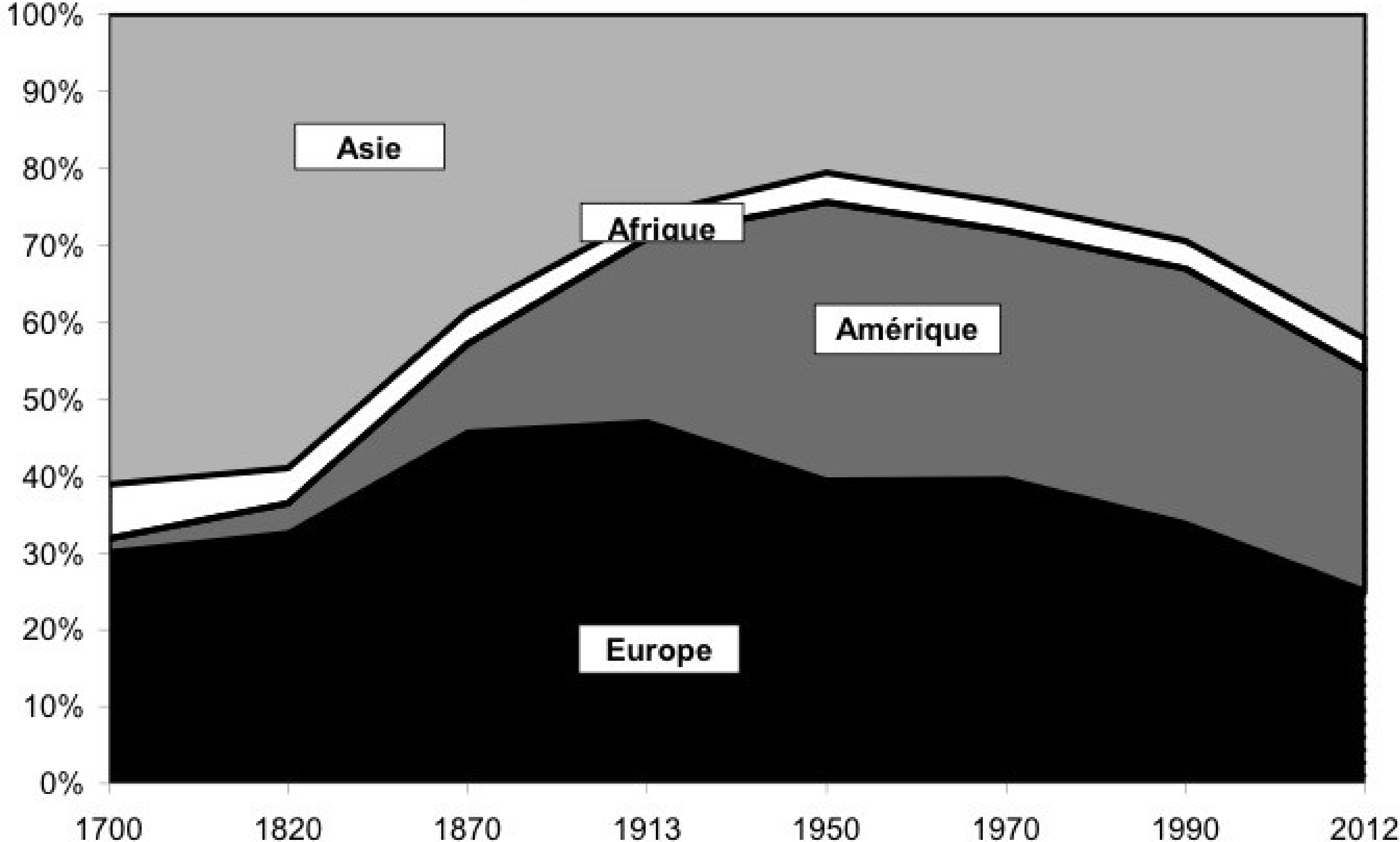


Modern Economic Growth

Figure 1: GDP per person in the United States



Graphique 1.1. La répartition de la production mondiale 1700-2012



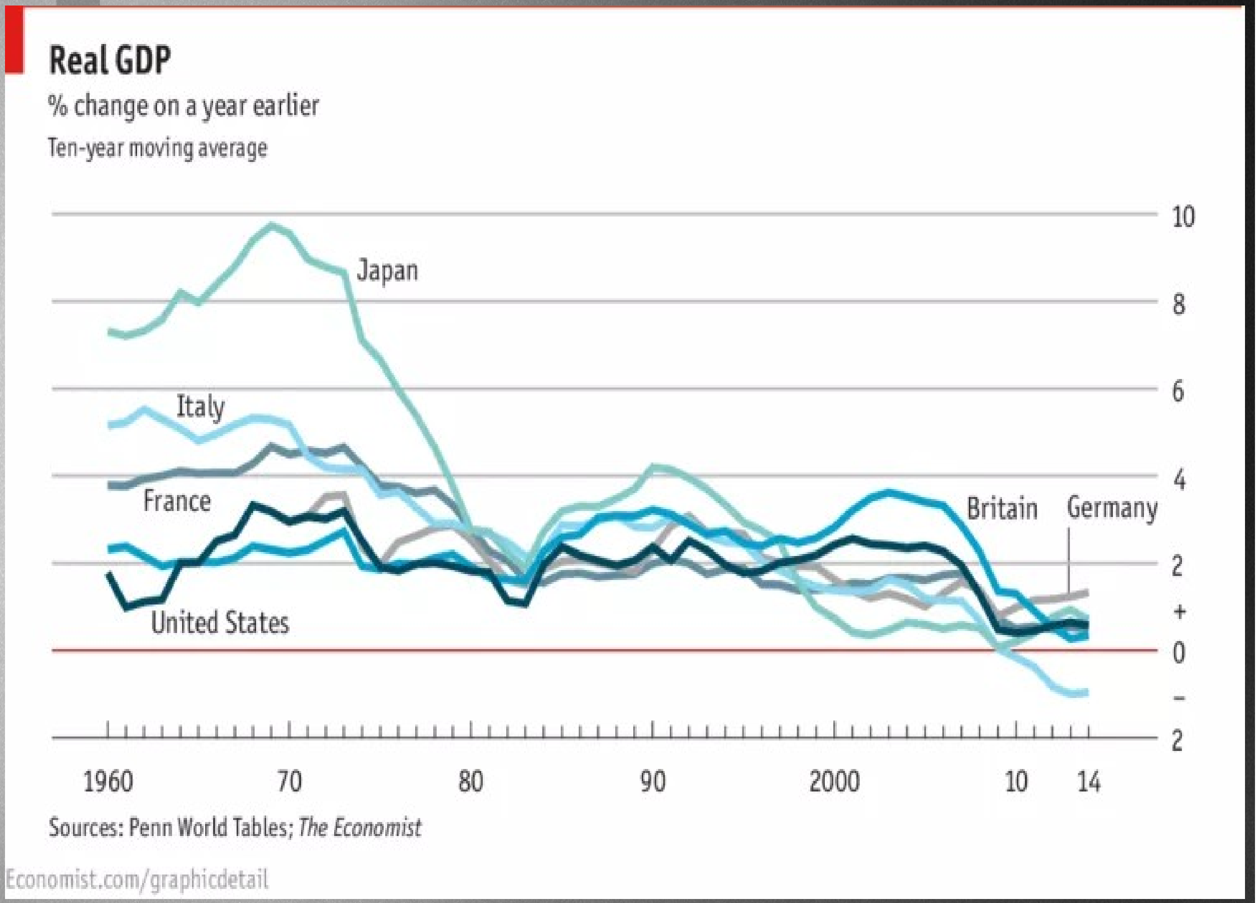
Lecture: le PIB européen représentait 47% du PIB mondial en 1913, et 25% en 2012.

Sources et séries: voir piketty.pse.ens.fr/capital21c.

Secular stagnation?

▶ Definition:

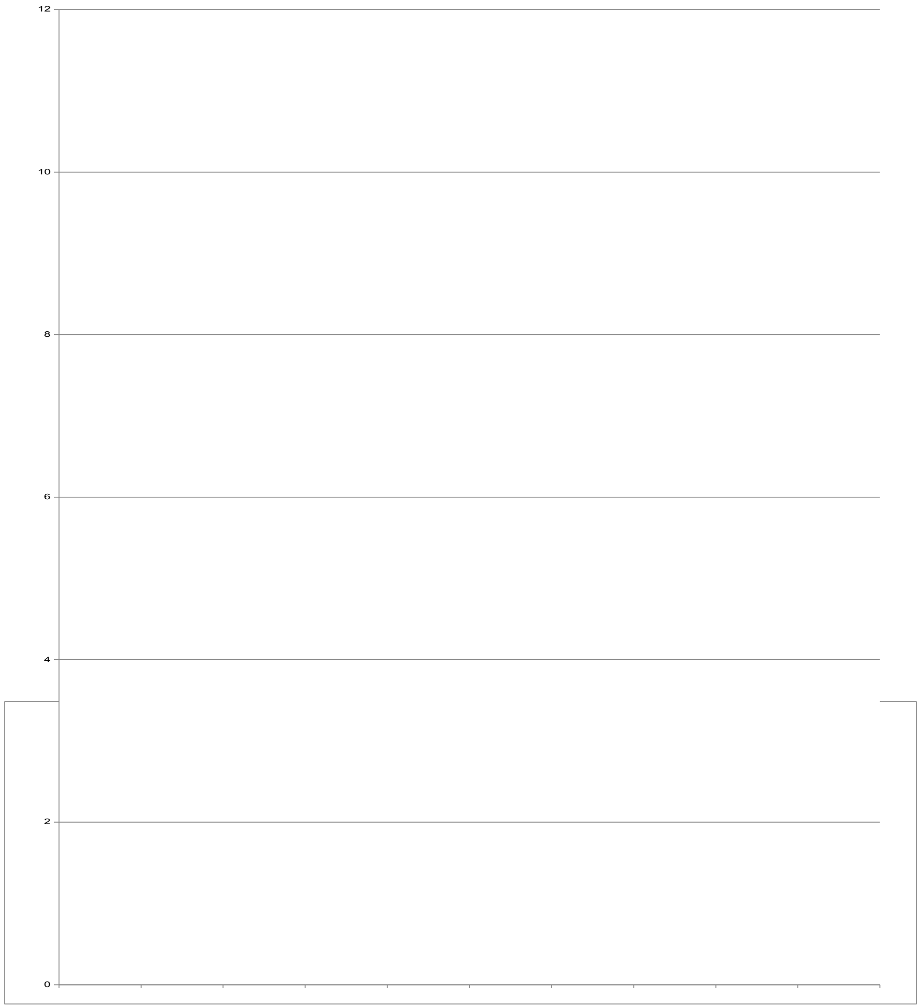
A long period of slow growth, deep recessions and chronic instability.



Secular stagnation?

GDP: Annual average growth rates per period

Periods: 1961-1974; 1975-1985; 1986-1998; 1999-2017



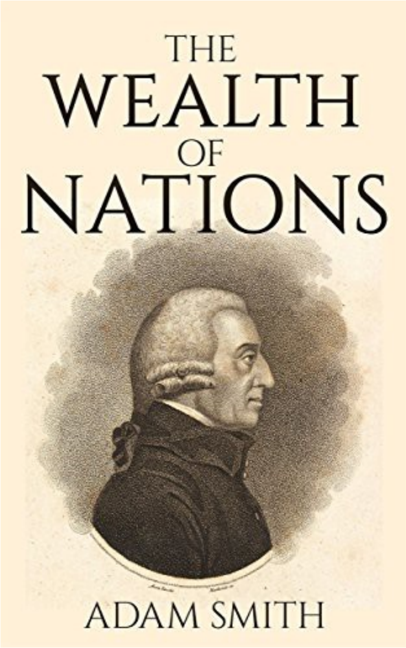
Theories and models

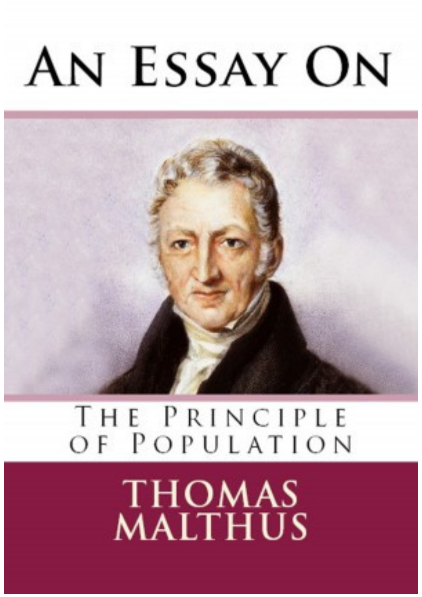
Solow (1994), *Perspectives on growth theory*

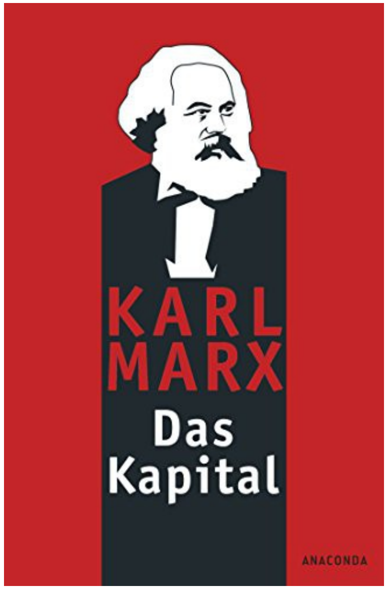
"There have been three waves of interest in growth theory during the past 50 years or so. The first was associated with the work of Harrod (1948) and Domar (1947) (…). The second wave was the development of the neoclassical model. (…) The third wave [*endogenous growth theory*] began as a reaction to omissions and deficiencies in the neoclassical model."

Theories and models

However, economic growth and dynamics have been matters of interest and concern from the very origins of classical political economy.







Roy **Harrod** (1900 – 1978)

Evsey **Domar** (1914-1997)

[redacted]
(1900-1978)

English economist, born in London. He studied at Oxford and, apart from a short break for service in World War II, and again as adviser to the International Monetary Fund in the early 1950s, he remained there, at Christ Church, throughout his career (1922-1967). He wrote the official biography of Keynes (1951), and wrote also on philosophy and logic as well as economics. He made several other discoveries more or less in parallel with others (the marginal revenue curve, the long-run envelope of short-run average cost curves, the theory of imperfect competition, the multiplier-accelerator model, and the IS-LM model). At last, another discovery brought him recognition: Harrod's "Essay in Dynamic Theory" (1939). The idea, which marked the beginning of the modern theory of growth, had also been developed by Domar, but at least Harrod got his name on the model this time. He was personal advisor to Winston Churchill during World War II.

[redacted]
(1914-1997)

American economist. Born in Lodz, Russia (now Poland), he was raised and educated in Harbin, Manchuria, but moved permanently to the United States in 1936 and completed his studies there at the UCLA, Michigan, and Harvard, where he got his Ph.D. in 1947. He taught at several universities, including Johns Hopkins, before moving to MIT in 1958. He made contributions in at least three main areas of economics: economic growth, comparative economics, and economic history. His work on economic growth began with his 1944 model on government debt, which considered how economic growth can lighten the burden of the government debt. His major claim to fame, however, was in developing, parallel to Roy Harrod, a dynamic-equilibrium growth model (1946) as a way of extending the Keynesian demand-determined equilibrium into the long run.

Keynesian analysis in the models of economic growth

Roy **Harrod** (1939) and Evsey **Domar** (1946)

Harrod-Domar model

"Harrod and Domar expressed the dynamic relationship (*the effect of capital accumulation on growth*) in a simple equation, which neatly formalized, simplified and summarized the essence of almost 200 years´ theorizing about economic growth" (Gylfason, 2003, p. 25)

Economic growth depends on three factors:

Savings rate of the households (S = s.Y)

Capital-output ratio (v = K/Y, or = ΔK/ΔY)

Depreciation rate (δ)

The hypotheses of the Harrod-Domar (H-D) model

1. The technology

production function with complimentary factors (i.e., non-substitutable), with two (2) production factors: capital and labour

technology with constant coefficients (Leontief production function)

Y = A.K = (1/v).K,   v = K/Y (capital-output ratio) **constant**; A = 1/v is capital productivity;

K (stock, €), Y (flow, €/year)

K/L constant (L not explicit in the model)
K and L grow at the same rate

Y/L constant (the growth rate of labour productivity is 0)
Y and L grow at the same rate

there is **no technological progress**

The hypotheses of the Harrod-Domar (H-D) model (cont.)

2. Full employment
productive capacity of the economy is fully used; for a given K,
Y = (1/v).K
is the maximum GDP that the economy may create, given the hypotheses (output equals potential output)

3. Closed economy

4. Savings
total savings is a constant proportion of GDP
S = s.Y

5. Equilibrium
I = S

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Harrod-Domar model: behaviour of the economic agents

households/**consumers**
(eq. 1) **S = sY,**
s is savings rate

firms/**producers**
(eq. 2) **K = vY**
v is the capital-output ratio

definition of investment

(eq. 3) **I = ΔK + δK**; δ being the depreciation rate, and I gross investment, ΔK net investment

Condition of **equilíbrium**
(eq. 4) **S = I**

Harrod-Domar model

4 equations

4 variables
K stock variable
Y, I, S flow variables
3 parameters
v, s, δ

Harrod-Domar model: the reduced-form equation

From $K = vY$ (eq. 2), we obtain $\Delta K = v \,.\, \Delta Y$

and then, considering that $S = sY$ (eq. 1) and $I = \Delta K + \delta K$ (eq. 3) and $S = I$ (eq. 4), we get:

$I - \delta K = v.\Delta Y$

$S - \delta K = v.\Delta Y$

$s.Y - \delta K = v.\Delta Y$

$s - \delta(K/Y) = v.\Delta Y/Y$

$s - \delta.v = v.\Delta Y/Y$

$$\Delta Y/Y = (s/v) - \delta$$

A summary of the logic of the model

A simple circular logic:

Δ+S → Δ+I → Δ+K →Δ+Y → Δ+S ...

Increase in savings – investment – capital stock – GDP – savings

And, **growth of Y may be increased by further savings and/or efficiency of capital**

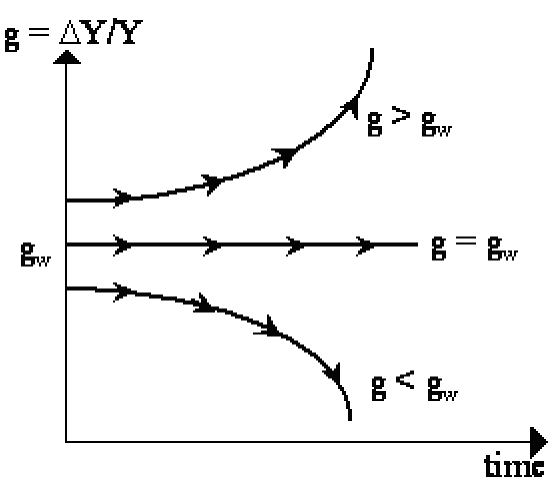
<u>Harrod-Domar model</u>:

$$rw(Y) = \Delta Y/Y = s/v - \delta$$

where **rw(Y)** is the ***<u>warranted</u>*** growth rate of GDP, that is, the highest growth rate that the economy may reach, given the hypotheses, namely given the amount of capital K.

<u>But: unstable equilibrium!</u>

- If labour grows at lower (or higher) rate than g**w**, widespread unemployment (or labour scarcity) follows, without any adjustment mechanism (fixed coefficients)
- <u>if</u> output growth deviates from g**w**, the economy shall <u>no longer be in equilibrium</u>, and <u>does not move back</u> to g**w**.



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The Neoclassical critique of the Keynesian models of growth

Robert **Solow** (1956) and Trevor **Swan** (1956)

Criticisms to Harrod-Domar model by **Robert Solow**

capital-output ratio as a constant is **not** a realistic hypothesis

relevance of labour: it is implicit (complementary factors), but not explicit in the model

The response by Robert **Solow**: **new hypotheses**

GDP depends on physical capital and labour, with decreasing returns

Production function with substitutable factors

v (capital-output ratio) becomes the adjustment variable

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[redacted]
(1924-)

American economist, born in Brooklyn, New York, educated at Harvard (B.A. 1947, M.A. 1949, Ph.D. 1951). He is best known for his path-breaking work on capital and growth. Since 1950, he has taught at MIT; he has never had or wanted any other job. He was president of the American Economic Association in 1979 and was awarded the Nobel Prize for his work on economic growth in 1987. Recently, he wrote: “Maybe the main function of economics in general is not, as we usually think, the systematic building of theories and models, or their empirical estimation. Maybe we are intellectual sanitation workers. The world is full of nonsense ... Maybe the higher function of economics is to hold out against nonsense, ... All those theories and models we invent and teach are just nature’s way of making people who know nonsense when they see it.”

Robert Solow, Nobel Prize Lecture, December 1987

"Growth theory did not begin with my articles of 1956 and 1957, and it certainly did not end there. Maybe it began with *The Wealth of Nations*; and probably even **Adam Smith** had predecessors. More to the point, in the 1950s I was following a trail that had been marked out by Roy **Harrod** and by Evsey **Domar**, and also by Arthur **Lewis** in a slightly different context. Actually I was trying to track down and relieve **a certain discomfort that I felt with their work**. (…) That was the spirit in which I began tinkering with the theory of economic growth, trying to improve on the Harrod-Domar model. I can not tell you why **I thought first about replacing the constant capital-output (and labour-output) ratio by a richer and more realistic representation of the technology**"

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The hypotheses of the Solow model

the economy is represented by a production function with two substitutable production factors (capital, labour);

the production function exhibits constant returns $Y = F(K, L) = K^{\alpha} L^{1-\alpha}$

the production factors have positive decreasing marginal productivities;

domestic savings is a fixed proportion of GDP;

the macroeconomic equilibrium in the closed economy is: I = S;

there is no unemployment;

the growth rate of active population is equal to the growth rate of total population;

the economy is closed.

The basic Solow model

The model is explained by two equations:
production function
capital accumulation equation

A. The production function

Cobb-Douglas (remind the properties: lecture 06)

$Y = F(K, L) = K\alpha \ L1-\alpha \ , \ 0 < \alpha < 1$

α is (i) the partial elasticity of output with respect to capital; and (ii) under neoclassical assumptions, the share of output that goes to capital

per capita production function

per capita means ***<u>per worker</u>***

Y/L = F(K/L)

since y = Y/L and k = K/L, the production function Cobb-Douglas *per capita* is:

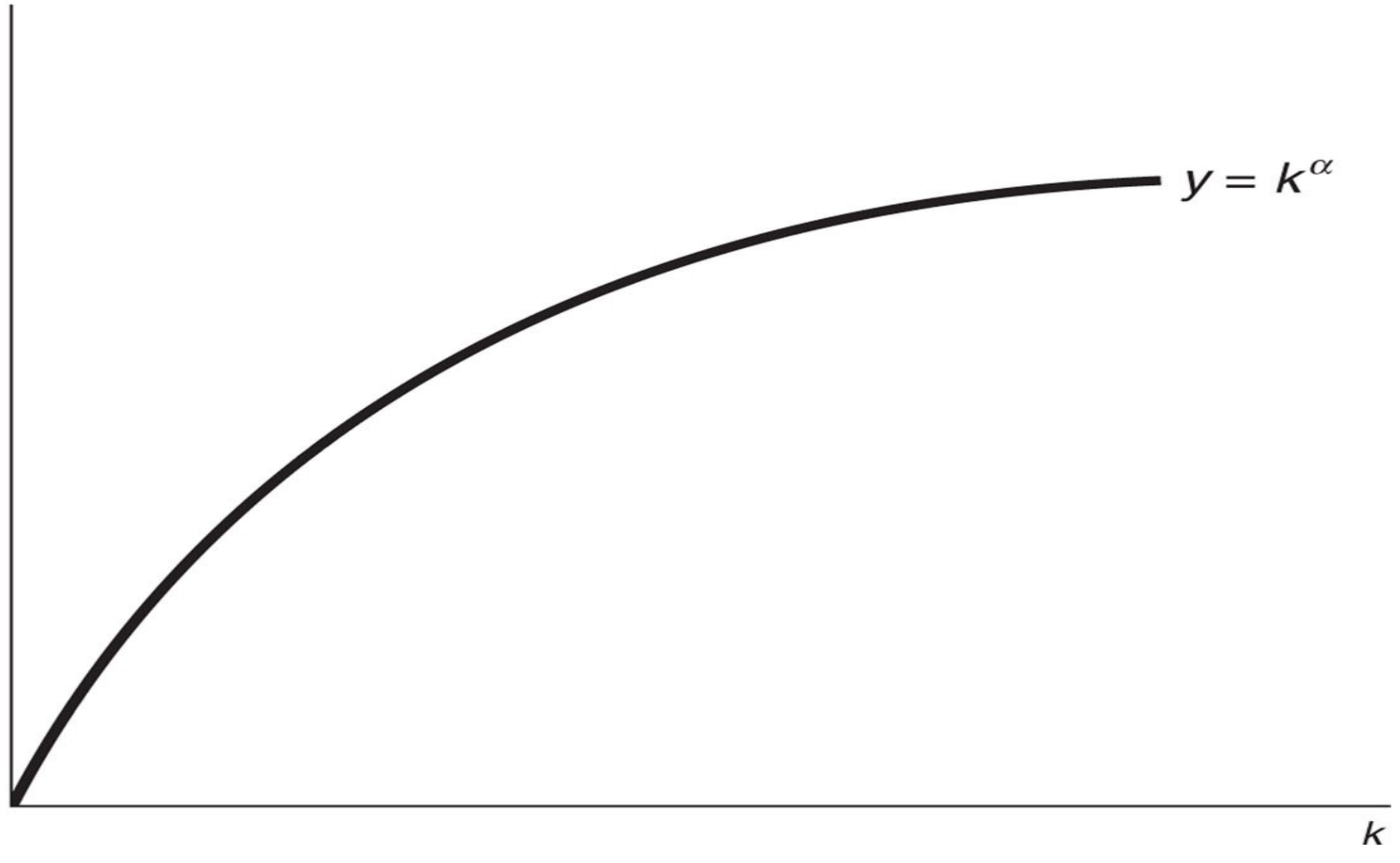
$y = k\alpha$, $0 < \alpha < 1$

<u>interpretation</u>:

if k (K/L) is higher, the firms produce higher y (Y/L);

for each additional unit of capital *per worker*, the <u>increase</u> of output *per worker* becomes lower;

FIGURE 2.1 **A COBB-DOUGLAS PRODUCTION FUNCTION**



B. the accumulation of capital equation

$$dK/dt = s.Y - \delta.K$$

$(dK/dt)/K = s.Y/K - \delta$ growth rate of the capital stock

the accumulation of capital equation *per worker*

$k = K/L$

$\ln k = \ln K - \ln L$

$(dk/dt)/k = (dK/dt)/K - (dL/dt)/L$ growth rate of the stock of capital *per worker*

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growth of the active population

The population (and the active population) grow at the rate *n*

L(t) = L0 ent

(dL/dt)/L = n

(dk/dt)/k = (dK/dt)/K – (dL/dt)/L

= s.Y/K – n – δ

= s. y/k – n – δ

dk/dt = s.y – (n + δ).k

Solow model

$y = k^{\alpha}$, $0 < \alpha < 1$ production function *per capita*

$dk/dt = s.y - (n + \delta).k$ stock of capital variation *per capita*

$$dk/dt = s.\ k^{\alpha} - (n + \delta).k$$

the model addresses the following question:

If an economy has, in some year, the stock of capital per worker **k0**, the growth rate of population is **n** and the stock of capital depreciates at the rate **δ**, ***what is the trend of output per worker in this economy***? How does this economy compare to another one with a different savings/investment rate?

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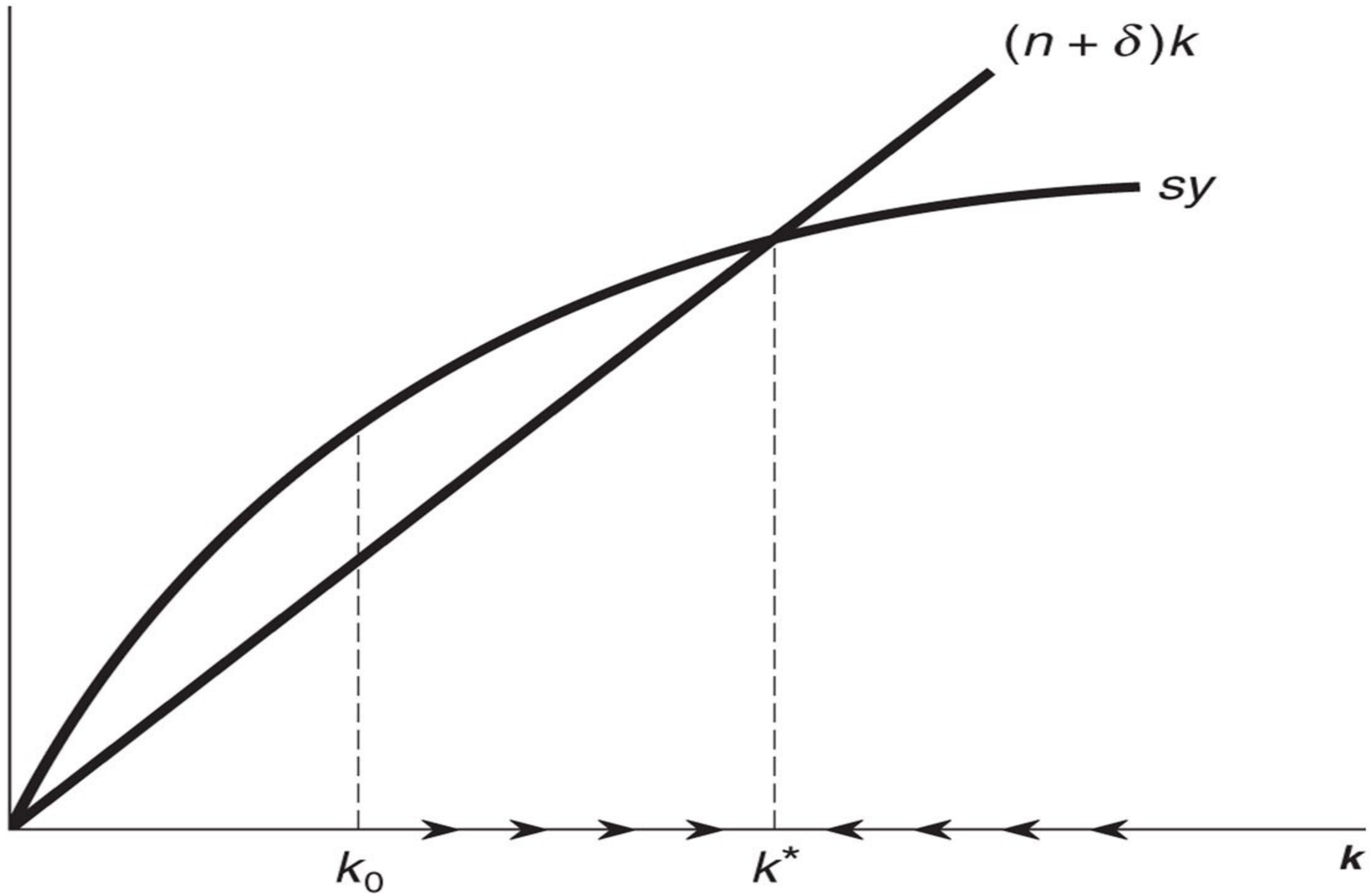
FIGURE 2.2 THE BASIC SOLOW DIAGRAM



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$s.k^{\alpha}$ *actual* investment per capita (per worker)

$(n + \delta).k$ investment per capita (per worker) that is *required* to keep the stock of capital per capita (per worker) constant

when $s.k^{\alpha} > (n + \delta).k$, the economy increases the stock of capital per capita (worker); ***capital deepening*** of the economy;

when $s.k^{\alpha} = (n + \delta).k$, when $k = k^*$, the stock of capital K is rising at the growth rate $n>0$ (the same as that of active population and that of population);
the stock of capital *per capita* (worker) is not growing (rate of growth = 0) → ***steady state (capital widening)***

The notion of *steady state*

The *steady state* occurs at the level k for which the depreciation of capital (δK) matches investment (sY):

$$\mathbf{dK = sY - \delta K = 0}$$

Therefore, the creation of capital per worker is 0 (or $dK = 0$)

The adjustment occurs through changes in the capital-product ratio.

the dynamic equilibrium in the Solow model (***steady-state***)

dk/dt = s.y – (n + δ).k = 0

in the steady state, k = K/L remains constant

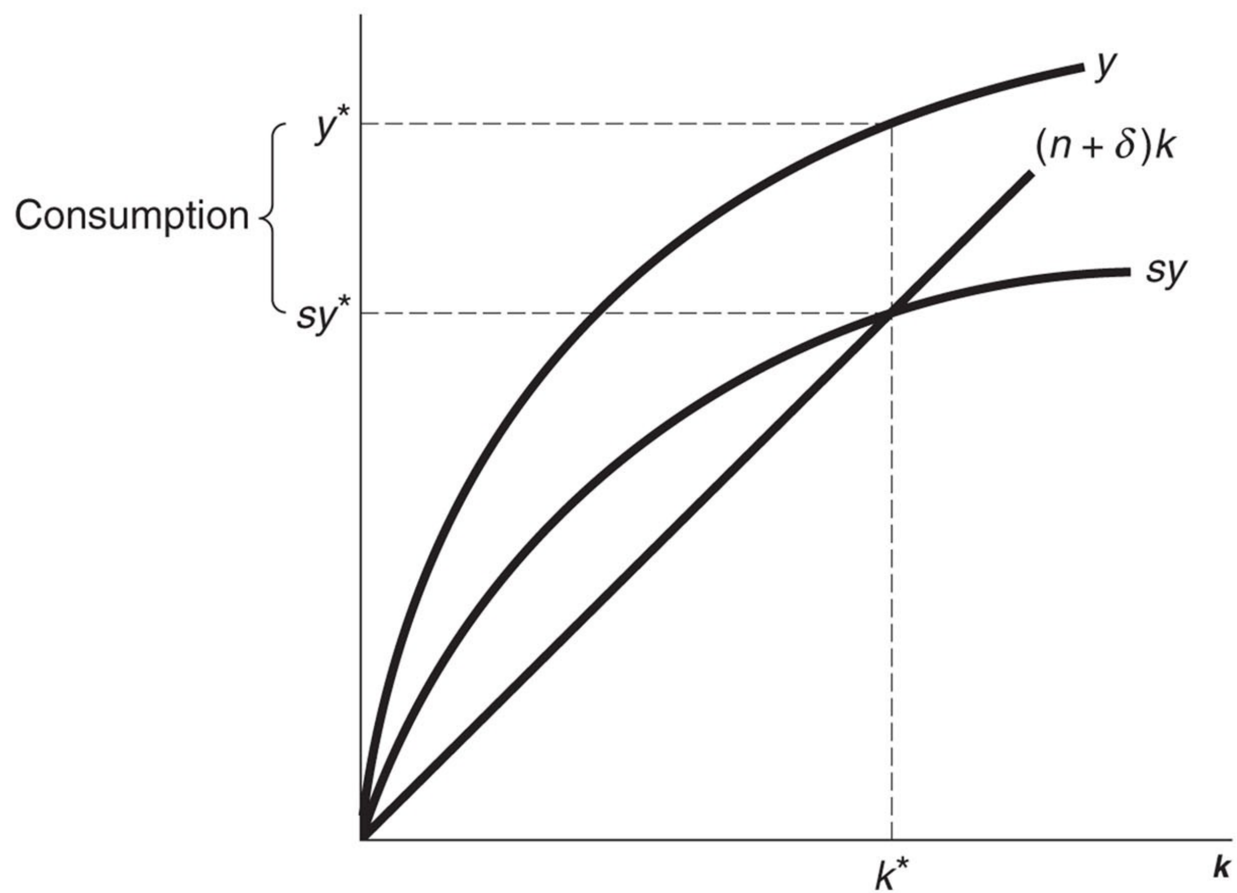
s.y = (n + δ).k

K grows at the same rate as L

K grows at the rate n

capital widening

FIGURE 2.3 THE SOLOW DIAGRAM AND THE PRODUCTION FUNCTION



main conclusions/contributions of Solow model

each economy has characteristics that determine its ***steady state*** (when the economy grows at a rate such that the amount of capital per worker will not change over time)

the main characteristics of the economy that determine its *steady state* are:
investment (= saving) rate, rate of population growth, rate of technological progress

Solow model is a theory of income differences among countries
countries may have different *steady states*;
countries (even with the same steady state) may be outside of their *steady state*: in this case, these economies may have different income levels;
if an economy has an income level different from its *steady state* income level, then it moves into its *steady state* income level;

Solow model is a theory of relative growth rates
two "similar" countries with the same rate of investment have the same *steady state* income level, but may have different income levels (if they are not at their *steady state*) ; in this case, the poorer income level country (that more distant to its *steady state*) grows faster into its *steady state* income level: conditional convergence

what is crucial in the Solow model

assumption of diminishing returns to capital: economies that have less capital per worker (relative to their long-run capital per worker) tend to have higher rates of return and, then, higher growth rates;

in the Solow model, economic policy may change the **growth rate** of GDP per capita but such change is **temporary** during the process to a new steady state; they **do not have long-run effect on economic growth**;

in the Solow model, economic policy may change the **long-run level of GDP *per capita***, **permanently**.

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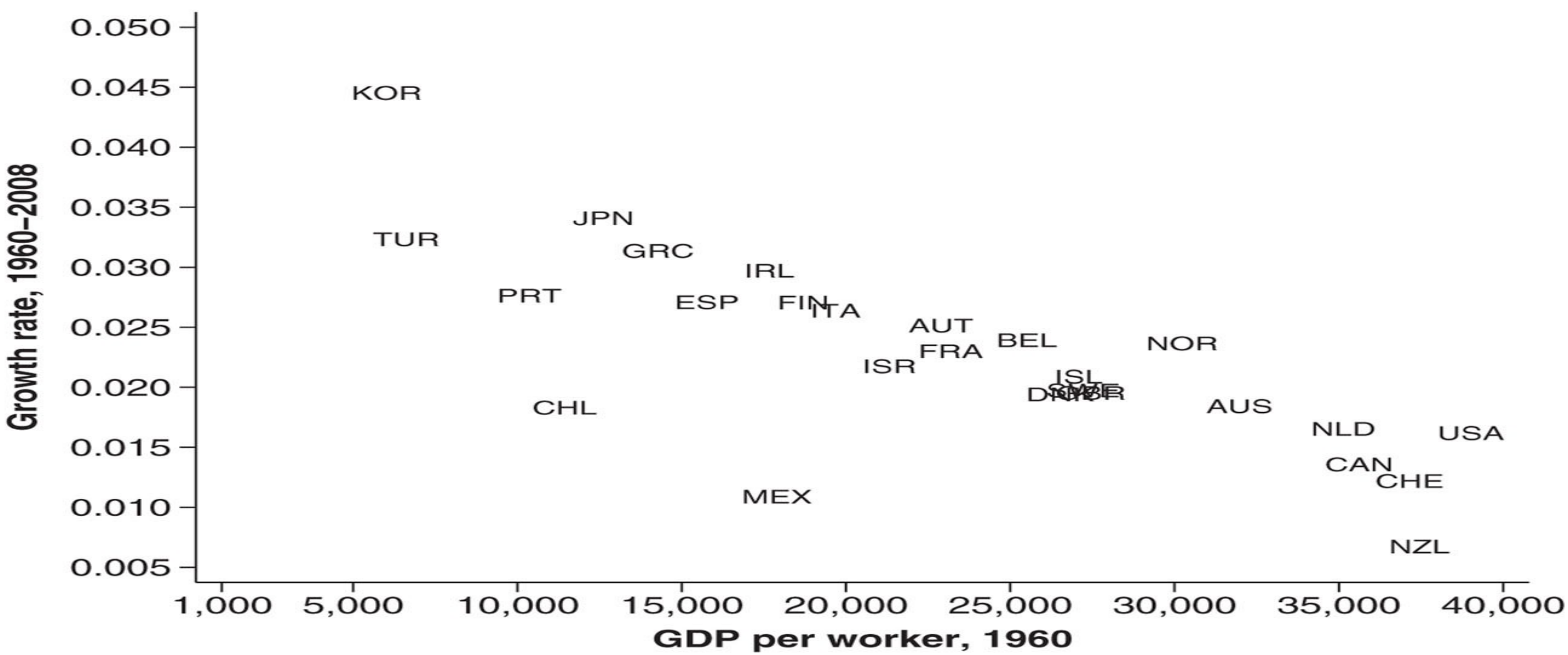
Comparison of Harrod-Domar with the Solow model

Harrod-Domar: output growth **adjusts** to the capital-output ratio (which is **exogenous**)

Solow: capital-output ratio is **endogenous**, and **adjusts** to the output growth, which is **exogenous** (determined by **exogenous** factors: population growth and technological progress)

Convergence?

FIGURE 3.5 CONVERGENCE IN THE OECD, 1960–2008



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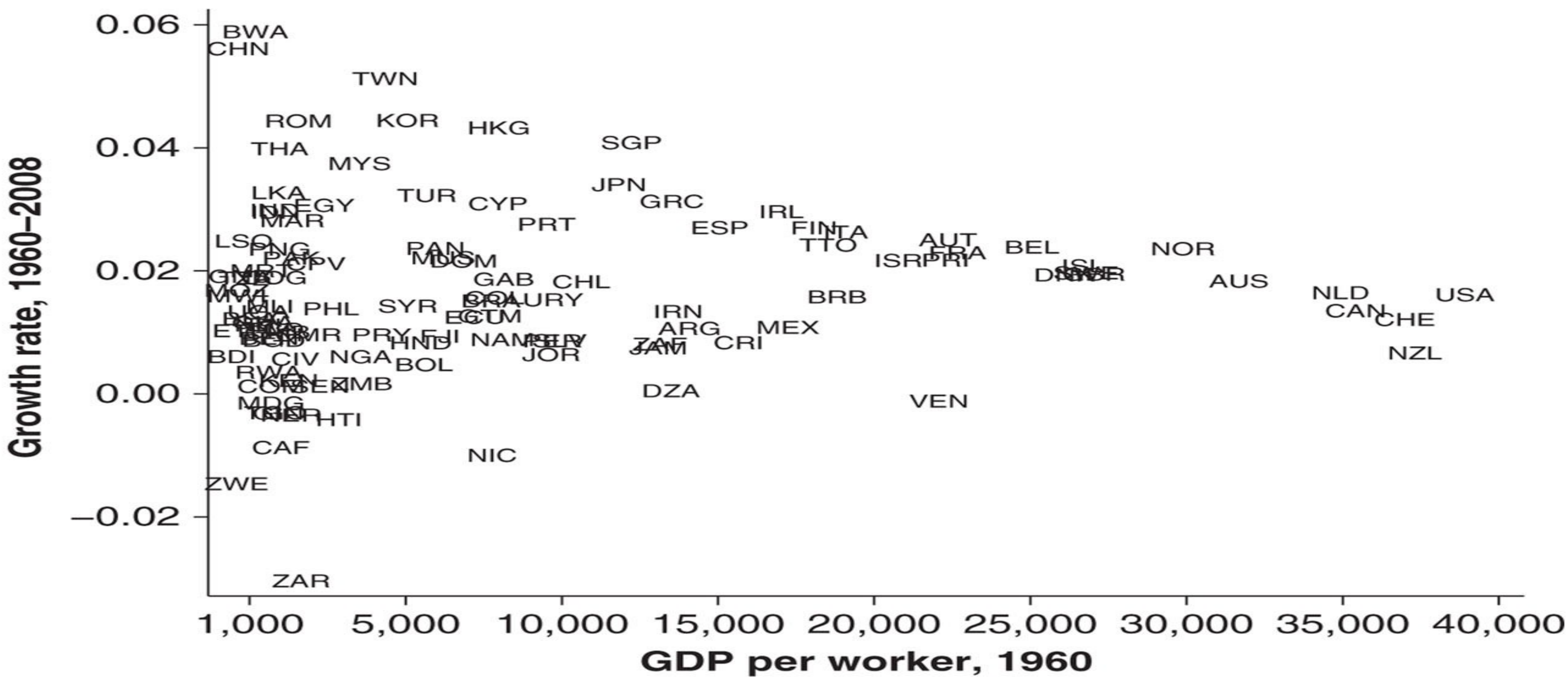
Convergence?

FIGURE 3.6 THE LACK OF CONVERGENCE FOR THE WORLD, 1960–2008



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The “Solow Residual”

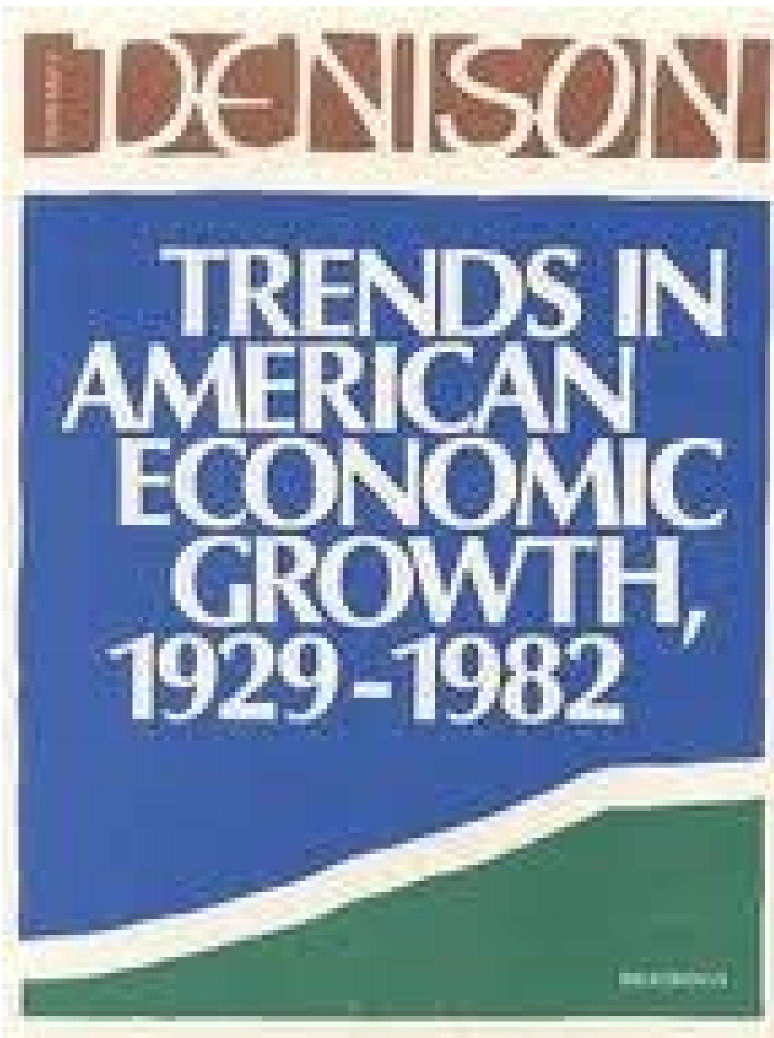
- The enigma of the “Solow residual” in the Cobb Douglas version:

$$\mathbf{Y = F\ (A,K,L) = A\ K\alpha\ L1\text{-}\alpha}$$

- “Total factor productivity” or “technical progress”: dA/A

So, dynamics and change are not explained by the model

Edward Denison (1915-1992) and the computation of the residual



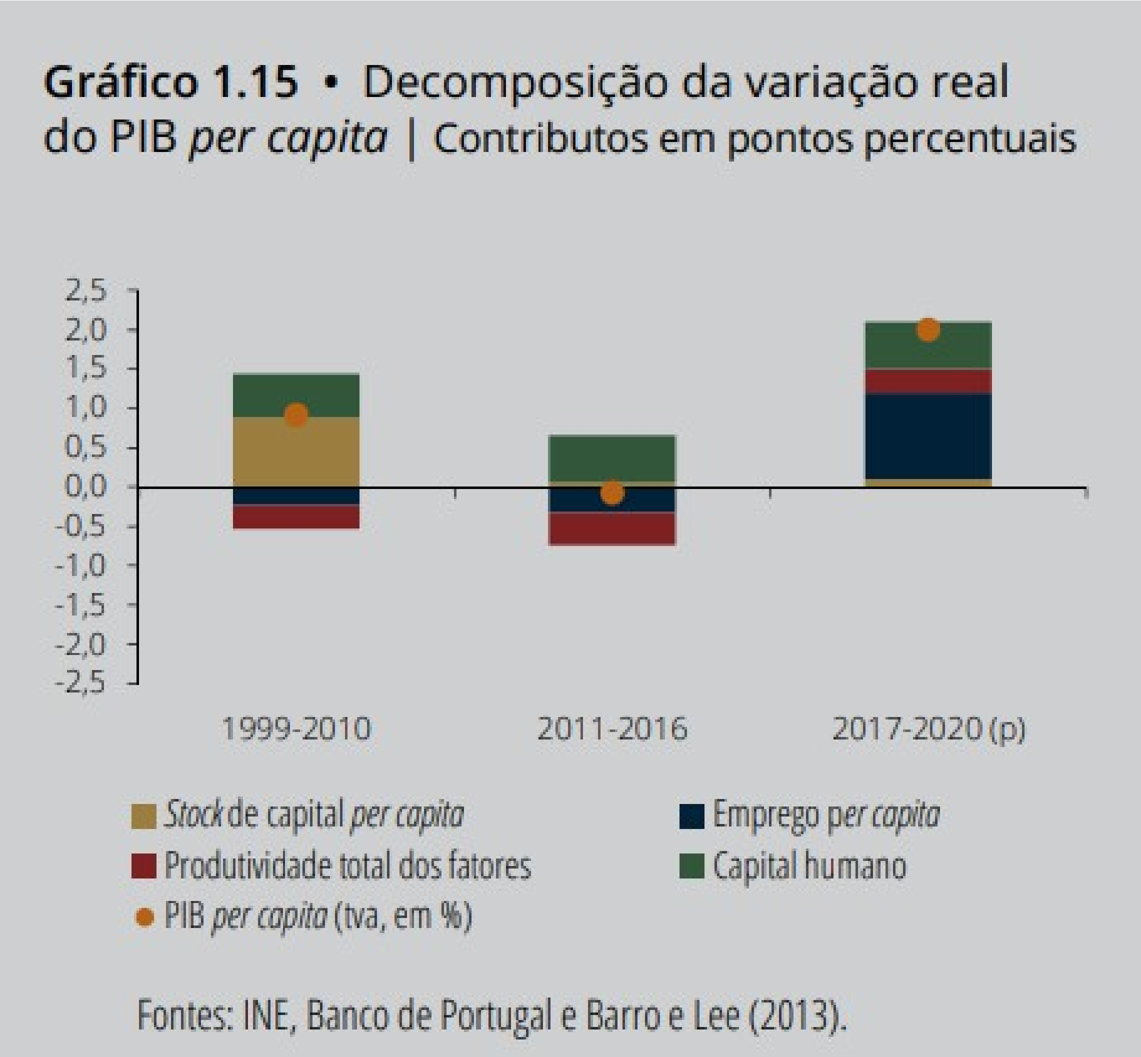
Mesuring GDP growth:

$$\dot{Y} / Y = \dot{A} / A + \in(\dot{K} / K) + (1 2 \in)(\dot{L} / L)$$

Surprisingly, dA/A explains more than half of US GDP growth

Or, recent computations, ~30%

Growth accounting - Portugal



https://www.bportugal.pt/sites/default/files/anexos/pdf-b

Endogenous growth

If the marginal productivity of capital is not decreasing, then the capital accumulation can be pursued and the economy does not converge to a *steady state.*

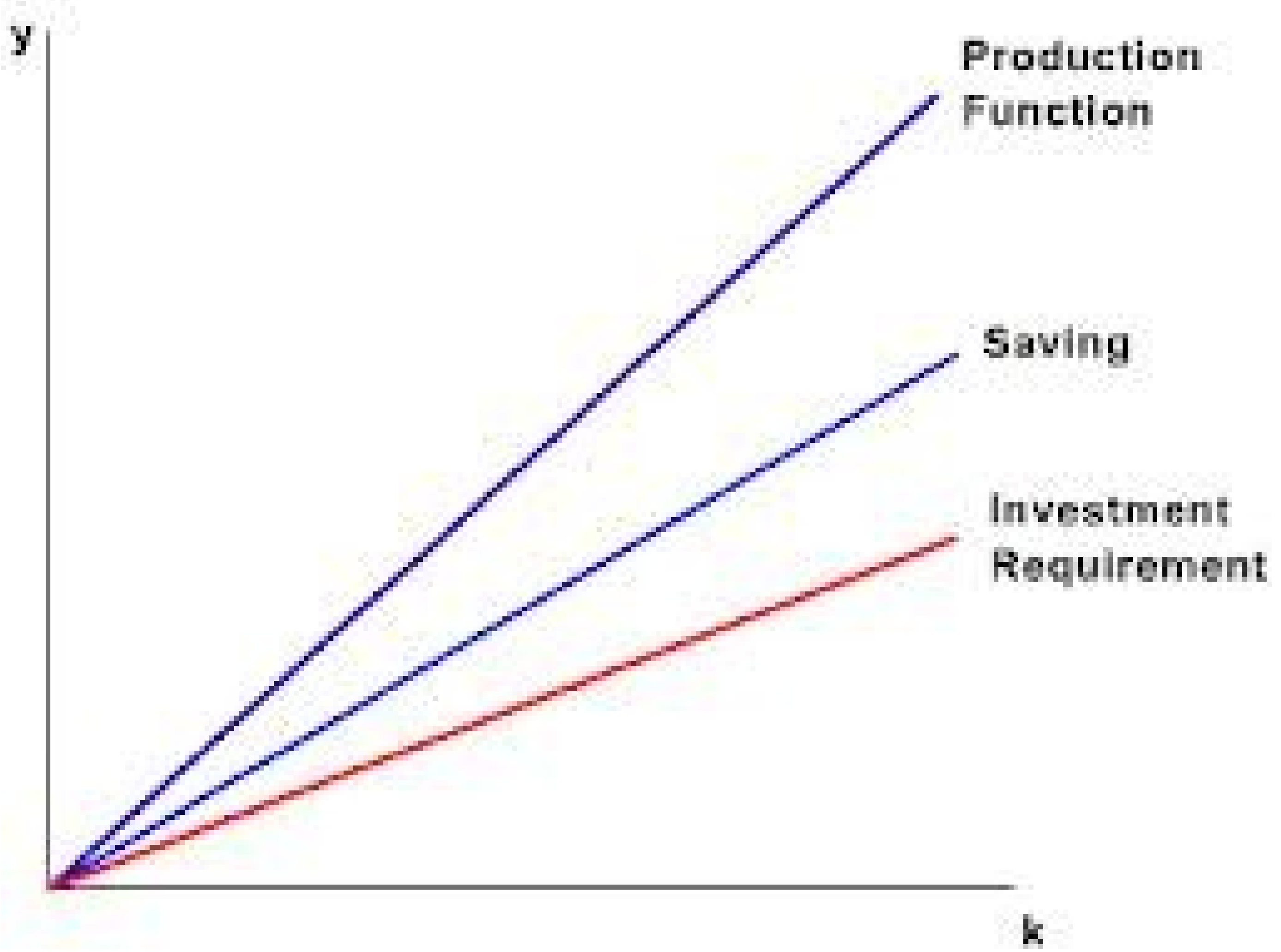
·Ex: AK model (Frankel, Rebelo):

Y = AK with A, technological level, constant, and K, level of capital (including "human capital")

Constant returns to scale make a return (instead of diminishing returns as in the Solow model). This is due to hypothesis that capital accumulation contributes to technological progress through "learning by doing" (so ΔK increases Y directly *and* indirectly through ΔA).

Romer's later endogenous growth model arrives at a similar result by assuming that population growth (ΔL) increases Y directly and indirectly through ΔA. In this case, the idea is that population growth increases the number of researchers, which leads to more 'ideas' (faster technological progress)

AK Model



Another view: instability is growth Schumpeter on capitalism

"**industrial mutation** – if I may use the biological term – that incessantly revolutionises the economic structure **from within, incessantly destroying the old one, incessantly creating a new one**. This process of **Creative Destruction is the essential fact about capitalism**."(CSD, 1942: 83)

Instead of equilibrium → innovation, evolution and change

Problems and limits to growth

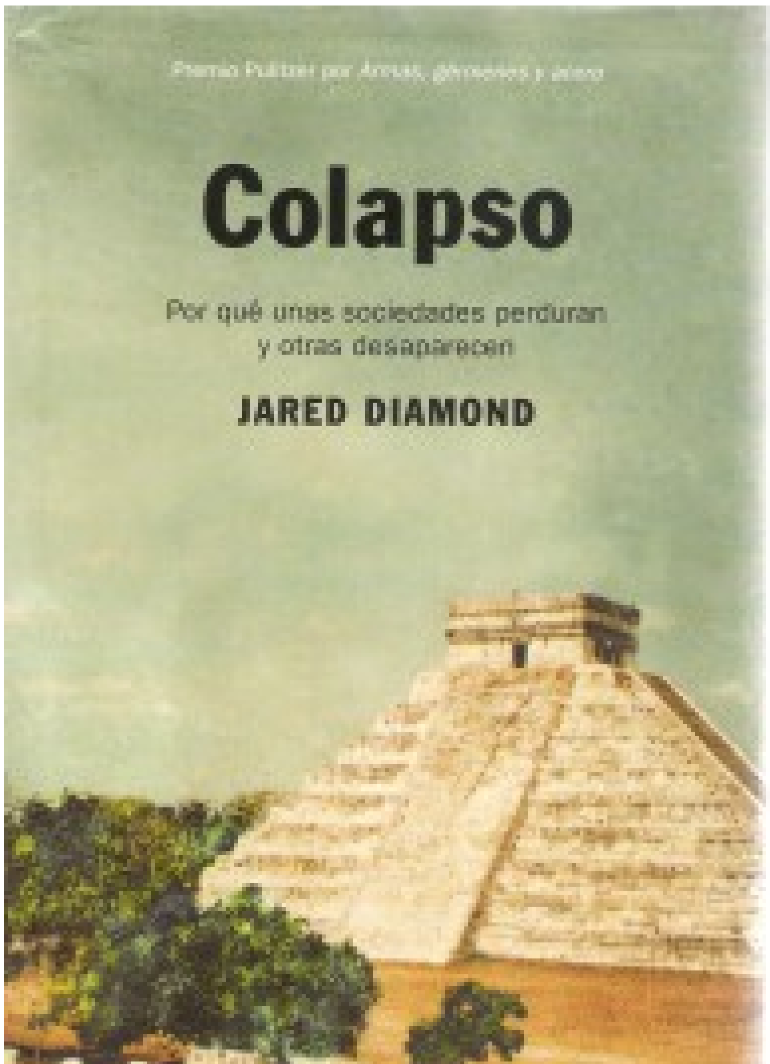
Problems

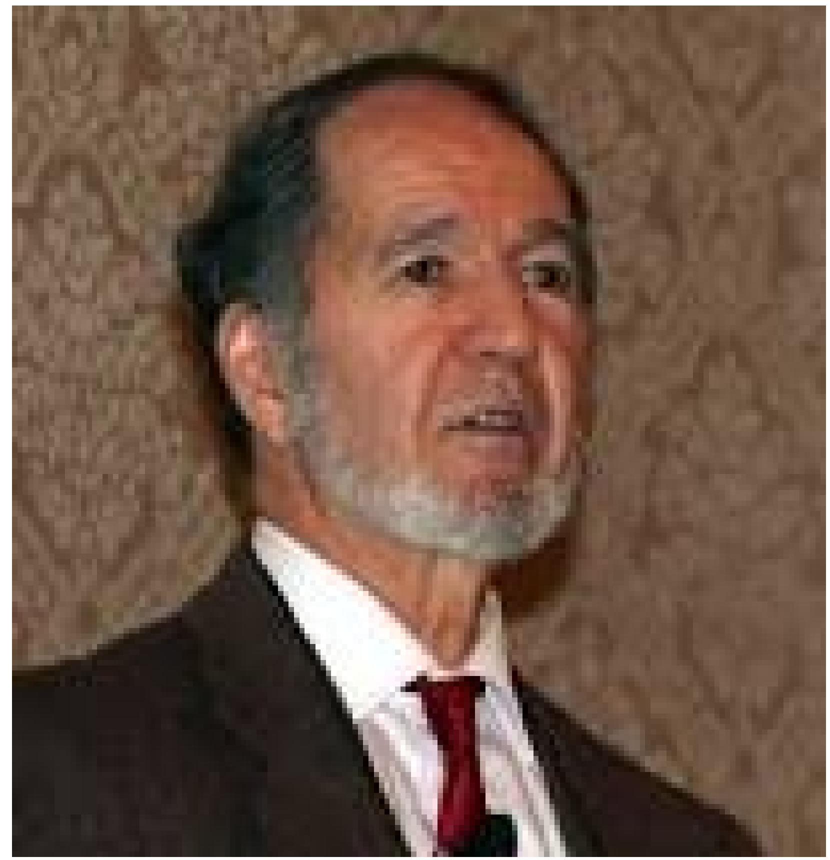
Can there be *too much* growth?

- Resource depletion / pollution / climate change / other environmental consequences
- Increasing income and wealth inequality
- Sacrificing present consumption

- The ‘degrowth’ debate: ecology, downscaling, anti-consumerism

Collapse, by Jared Diamond





The Easter island

- Discovered by the 8th or 9th century by fishermen from Polinesia
- The most remote place on the planet
- Only in 1722 did a Dutch ship notice the island

Statues looking at the seaside

A demographic tragedy

From hundreds of people (18th century), to 111 inhabitants by the end of the 19th century

And 30 thousand until the 17th century collapse

What happened?

The making of the statues

397 statues, some unfinished

A difficult task

Each hat weights 1§2 tons but still they were placed above the statues

Some historical details

- 30 thousand inhabitants, divided into 12 clans and territories
- 20 sculpters a month are required, plus 50 to 500 for transporting and putting in place the statue
- They were built between 1000 and 1600
- Since the 17th century, massive deforestation

- The answer: ecological collapse.

The collapse of Easter Island

- Some societies may be unable to avoid collapse.
- The collapse, in this case, is a result of lack of energy or other inputs, the reduction of surplus and food, and the resulting conflicts
- The surviving populations tore down the statues
- Diseases brought in by European sailors further decimated the population

Is there an ecological curse even in the case of more complex societies?

Is growth dangerous?