Models in Finance - Class 24

Master in Actuarial Science

João Guerra

ISEG

December 10, 2019

Credit risk

- Before, we have assumed that bonds and obligations are default free.
- This is not a reasonable assumption for corporate bonds and some government bonds or obligations (in, particular, if the payments are in a currency that the government does not control).
- It can be reasonable for some government bonds and payment obligations (if the payments are in a currency that the government controls).
- A credit loss only exists if the counter-party defaults and the contract has value.
- Credit risk is calculated as an expected loss:

$$\text{Expected Loss} = \text{Exposure} \times \text{Probability of Default} \times \text{Loss Given Default}$$

- All the parameters have an implicit time dependence.
- The Loss Given Default (LGD) is the percentage of the exposure that will be lost on a default.

Credit risk

- The recovery rate is the reciprocal of the LGD (Recovery Rate $= 100\% - LGD$). Usually some value can be recovered when a counter-party defaults.
- Credit risk changes with the market and good practice is to assess both current and potential exposure. The current exposure is the current market value of the asset, the future exposure should be based on a wide range of future scenarios, with different default probabilities.
- The outcome of a default may be that the contracted payment stream is:
- (i) rescheduled.
- (ii) cancelled by the payment of an amount which is less than the default-free value of the original contract.
- (iii) continues at a reduced rate.
- (iv) totally wiped out.

Credit risk

- The default of a bond can be triggered by a credit event of the type:
- failure to pay capital or a coupon.
- bankruptcy.
- rating downgrade by a rating agency such as Standard and Poor's or Moody's or Fitch.
- repudiation (disputing the validity of a contract and refusing to honor its terms)/moratorium (delay in the payment of debts or obligations).
- restructuring– when the terms of the obligation are altered so as to make the new terms less attractive to the debt holder, such as a reduction in the interest rate, re-scheduling or change in principal.

Recovery rate: fraction of the defaulted amount that can be recovered through bankruptcy proceedings or other forms of settlement.

Structural models

- Structural models: explicit models of a corporate entity issuing both equity and debt.
- In these models, typically, default occurs when a stochastic variable (or process) hits a barrier representing default.
- Examples of a structural model: the Merton model or First Passage models.
- These models deliver an explicit link between a firm's default and the economic conditions and provide a sound basis for estimating default correlations amongst different firms.
- Disadvantage: identifying the correct model and estimating its parameters.

Reduced form models

- Reduced form models: statistical models which use observed market statistics rather than specific data relating to the issuing corporate entity.
- The market statistics most commonly used are the credit ratings issued by credit rating agencies such as Standard and Poor's, Moody's or Fitch.
- These models use market statistics along with data on the default-free market to model the movement of the credit rating of the bonds.
- The output of these models is a distribution of the time to default.
- Default is no longer tied to the firm value falling below a threshold-level, as in structural models.
- Default occurs according to some exogenous process.
- Disadvantage of these models: they sometimes lack the clarity of structural models.

Intensity-based models

- An intensity-based model is a particular type of reduced form model.
- These models are defined in continuous-time and they model the "jumps" between different states (usually credit ratings) using transition intensities.
- Examples: two-state model for credit ratings with a deterministic transition intensity and the Jarrow-Lando-Turnbull model.

The Merton model

- Consider that a corporate entity has issued both equity E_t and debt D_t such that its total value at time t is V_t:

$$V_t = D_t + E_t.$$

- Assume a firm has issued a single zero coupon bond with face value D and which matures at time T.
- At time T the remainder of the value of the corporate entity will be distributed amongst the equity holders.
- Default situation: if $V_T < D$.
- If default occurs, the bond holder receives V_T instead of D and the equity holders receive nothing at all.

The Merton model

- For the equity holders, this is equivalent to have a European call option on the assets of the company with maturity T and a strike price D. The value of Equity at maturity is

$$E_T = \max\{V_T - D, 0\}.$$

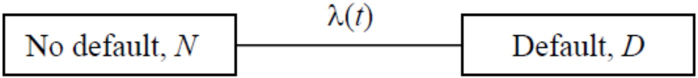
- The Merton model can be used to estimate the risk-neutral probability that the company will default.
- In the Black-Scholes world, it is easy to show that $\Phi(d_2)$ is the risk-neutral probability that a call will be exercised (it expires in the money). In this context, this means that

$$1 - \Phi(d_2) = \Phi(-d_2)$$

is the risk neutral probability that the firm is in default at time T.

Two-state model with constant intensity

- In continuous time, consider a model with two states: N (not previously defaulted) and D (previously defaulted).
- Assume that the interest rate term structure is deterministic: $r(t) = r$ for all t.
- The transition intensity, under the real world measure P, from N to D is denoted by $\lambda(t)$.



Two state model with constant intensity

- The state D is an absorbing state.
- Let $X(t)$ be the state at time t. The transition intensity $\lambda(t)$ is such that (under P)

$$P\left[X(t+dt)=N|X(t)=N\right]=1-\lambda(t)dt+o(dt) \quad \text{as } dt\longrightarrow 0,$$
$$P\left[X(t+dt)=D|X(t)=N\right]=\lambda(t)dt+o(dt) \quad \text{as } dt\longrightarrow 0.$$

- Define the stopping time τ (time of default):

$$\tau=\inf\left\{t:X\left(t\right)=D\right\}.$$

- Define the number of defaults as the counting process $N(t)$:

$$N(t)=\begin{cases}0 & \text{if } \tau>t,\\ 1 & \text{if } \tau\leq t.\end{cases}$$

Two state model with deterministic intensity

- Assume that if the corporate entity defaults all bond payments will be reduced by a deterministic factor $(1-\delta)$ where δ is the recovery rate.
- If a bond is due to pay 1 at time T, the actual payment at time T will be 1 if $\tau > T$ and δ if $\tau \leq T$.
- Let $B(t,T)$ be the price at time t of a zero-coupon bond. Then there exists a risk-neutral measure Q equivalent to P under which:

$$\begin{aligned} B(t,T) &= e^{-r(T-t)} E_Q\left[Payoff \text{ at } T | \mathcal{F}_t \right] \\ &= e^{-r(T-t)} E_Q\left[1 - (1-\delta) N(T) | \mathcal{F}_t \right]. \end{aligned}$$

Two state model with constant intensity

- It can be proved that:

$$E_Q\left[N\left(T\right)|N(t)=0\right]=E_Q\left[1-\exp\left(-\int_t^T\widetilde{\lambda}\left(s\right)ds\right)\right].$$

- Assuming that $\widetilde{\lambda}\left(s\right)$ is deterministic, this implies that:

$$B(t,T)=e^{-r(T-t)}\left[1-(1-\delta)\left(1-\exp\left(-\int_t^T\widetilde{\lambda}\left(s\right)ds\right)\right)\right]$$

which is equivalent to:

$$\widetilde{\lambda}\left(s\right)=-\frac{\partial}{\partial s}\log\left[e^{r(s-t)}B(t,s)-\delta\right]$$

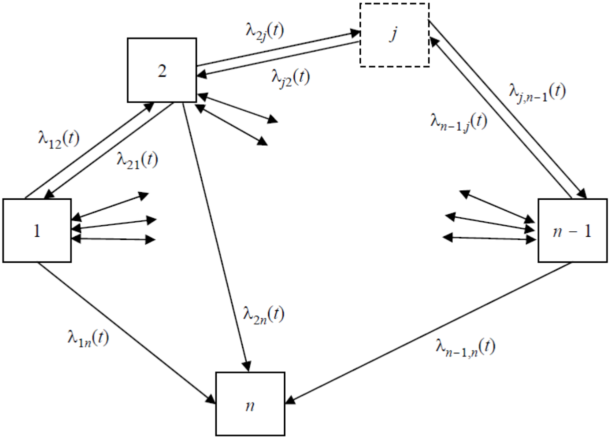
- Note: $\widetilde{\lambda}\left(s\right)$ is the transition intensity under Q.
- From the bond term structures and making an assumption about the recovery rate allows the implied risk-neutral transition intensities to be determined.

The Jarrow-Lando-Turnbull model

- In this model there are $n-1$ credit ratings plus default (n states).
- $\lambda_{ij}(t)$: transition intensities, under the real-world measure P, from state i to state j at time t.
- If $X(t)$ is the state or credit rating at time t, then, for $i,j=1,\ldots,n-1$,

$$P\left[X(t+dt)=j|X(t)=i\right]=$$
$$=\begin{cases} \lambda_{ij}(t)dt+o(dt) & \text{for } j\neq i \\ 1-\sum_{i\neq j}\lambda_{ij}(t)dt+o(dt)=\lambda_{ii}(t)dt+o(dt) & \text{for } j=i \end{cases}.$$

The Jarrow-Lando-Turnbull model



The Jarrow-Lando-Turnbull model

- We assume that the state n (default) is absorbing: $\lambda_{nj}(t) = 0$ for all j and for all t. Moreover, we assume that state $i+1$ is always more risky than state i.
- $n \times n$ intensity matrix:

$$\Lambda(t) = [\lambda_{ij}(t)]_{i,j=1}^{n}.$$

- Define, for $s > t$, the transition probabilities:

$$p_{i,j}(t,s) = P[X(s) = j | X(t) = i].$$

- Matrix of transition probabilities:

$$\Pi(t,s) = [p_{ij}(t,s)]_{i,j=1}^{n}.$$

The Jarrow-Lando-Turnbull model

- It can be shown that:

$$\Pi\left(t,s\right)=\exp\left[\int_{t}^{s}\Lambda\left(u\right)du\right].$$

- It can be shown that there exists a risk-neutral measure Q equivalent to P such that the price of a zero-coupon bond maturing at time T, which pays 1 if default has not yet occurred and δ if default has occurred and for which the credit rating of the underlying corporate entity is i is given by:

$$V(t,T,X\left(t\right))=B(t,T)\left[1-(1-\delta)P_{Q}\left[X(T)=n|\mathcal{F}_{t}\right]\right].$$

The JLT model with stochastic transition probabilities

- Let us assume that

$$\Pi(t, s) = \exp[\Lambda(s - t)]$$

  and $\mathit{\Lambda}$ is a diagonalisable matrix:

$$\mathit{\Lambda} = \Sigma D \Sigma^{-1},$$

  where D is a diagonal matrix with the eigenvalues of Λ and Σ is a matrix whose columns are the eigenvectors of Λ.
- Then

$$\Pi(t, T) = \Sigma \exp[D(T - t)] \Sigma^{-1}.$$

- We can consider stochastic transition probabilities by introducing a stochastic process $U(t)$ such that

$$\Lambda(t) = \Sigma D U(t) \Sigma^{-1}.$$