LISBON SCHOOL OF ECONOMICS & MANAGEMENT
UNIVERSIDADE DE LISBOA

Microeconomics

January 17, 2020

Answer to 3 groups only

1,5 hours

Group 1

1. **(2 marks) Show that if a production function $y = f(x)$, where $x \in R^n$, exhibits increasing returns to scale, the long run profit maximization problem does not have a solution.**

A: The function f(.) exhibits increasing returns to scale if and only if $f(tx) > tf(x)$ for all $t > 1$ and for all x $x \in R^n$. Now assume, by contradiction, that x* maximizes the firm's long run profit. Then maximum profit is $\pi(p,w) = pf(x^*)-wx^*$. If the firm doubles scale and uses 2x* instead of x*, profits become $pf(2x^*)-w(2x^*)$. Since, by definition of increasing returns to scale, $f(2x^*)>2f(x^*)$, we have $pf(2x^*)-w(2x^*) > p2f(x^*)-w2x^* = 2\,\pi(p,w)$. Then, by increasing the scale, profits increase and we reach a contradiction: x* does not maximize profits.

2. **Consider a firm with a technology represented by $y = x^\alpha z^\beta$, with $\alpha, \beta > 0$ and $\alpha + \beta < 1$.**
   a) **(10/3 marks) Determine the conditional input demand functions and the long run cost function.**

A: Solve the cost minimization problem for the Cobb Douglas case, i.e, find x and z so as to Min $w_x x + w_z z$ s.t. $x,z \geq 0$ and $x^\alpha z^\beta \geq y$. At the solution, we have $x^* = y^{1/\alpha+\beta}(\alpha w_z/\beta w_x)^{\beta/\alpha+\beta}$ and $z^* = y^{1/\alpha+\beta}(\beta w_x/\alpha w_z)^{\alpha/\alpha+\beta}$. The cost function is $c(y,w_x,w_z) = y^{1/\alpha+\beta} w_x^{\alpha/\alpha+\beta} w_z^{\beta/\alpha+\beta}[(\alpha/\beta)^{\beta/\alpha+\beta}+(\beta/\alpha)]^{\alpha/\alpha+\beta}$

   b) **(4/3 marks) Using the information obtained in a), how would you determine the individual supply function? (Write down the problem, but do not solve it.)**

   A: Use the long run cost function to solve the profit maximization problem, i.e, find y so as to Max $py - y^{1/\alpha+\beta} w_x^{\alpha/\alpha+\beta} w_z^{\beta/\alpha+\beta}[(\alpha/\beta)^{\beta/\alpha+\beta}+(\beta/\alpha)]^{\alpha/\alpha+\beta}$ s.t. $y \geq 0$. The solution to this problem is the individual supply function as long as $p \geq \min AC(y)$.

Group 2

Consider a duopoly with demand given by $Q = 100 - 5p$. Firm 1 has marginal cost equal to 14 and firm 2 has marginal cost equal to 12. Both firms have 0 fixed costs. Suppose that firms compete in quantities.

a) **(10/3 marks) Assume firms choose quantities simultaneously. Determine the Cournot (Nash) equilibrium of this game.**
   A: Solve both firms profit maximization problems. For firm 1 the problem is finding q_1 to Max $q_1(20 - q_1/5 - q_2/5) - 14\,q_1$; for firm 2 it is finding q_2 to Max $q_2(20 - q_1/5 - q_2/5) - 12\,q_2$. Firms' best-replies are given by $q_1(q_2) = 15 - q_2/2$; $q_2(q_1) = 20 - q_1/2$. Obtain the Nash equilibrium (20/3, 50/3) by intersecting the two best-replies.

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b) **(10/3 marks) Now suppose that the two firms play an extensive-form game, where firm 1 is the leader, choosing how much to produce first. Firm 2 then observes the choice of firm1 and chooses how much to produce. Determine the subgame perfect equilibrium of the game.**
   A: Firm 1 incorporates in its own profit maximization problem firms 2's best-reply. So, firm 1's problem becomes Max $q_1[20 - q_1/5 - 1/5*(20 - q_1/2)] - 14\,q_1$. Solving the problem one gets $q_1 = 10$ and, substituting in firm 2's best reply, we have $q_2 = 15$. It follows that SPE* = $\{(10, 20 - q_1/2)_{q_1 \in [0,40]}\}$.

Group 3

1. **(10/3 marks) Assume that good 1 is inferior and that its price decreases. Graphically represent Hicksian and Marshallian demands for good 1. Shade the area that represents the compensating variation.**

A: The Marshallian and Hicksian demand curves intersect at the initial price, but the latter is steeper than the former. The compensating variation is represented by the area below the Hicksian demand curve between the two price levels.

2. **(10/3 marks) A monopolist faces the demand curve $D(p) = (A/p)^2$, $A > 0$, constant marginal costs c, $c > 0$, and no fixed costs. Determine the profit maximising price and output.**

A: Finding the q to Max $yA/\sqrt{y}$-cy = Max $A\sqrt{y}$-cy, one obtains $y^* = (A/ 2c)^2$; it follows that p = 2c and profits are $A^2/ 4c$.

Group 4

1. **(2 marks) Comment on the following sentence: "In a strategic form game G, iterated elimination of (weakly) dominated strategies never eliminates a Nash equilibrium."**

A: False. The following game provides a counter example, since the set of Nash equilibria is {(U,L), (D,R)}, but D and R are (weakly) dominated strategies.

	L	R
U	1,1	0,0
D	0,0	0,0

2. **(14/3 marks) Players 1 and 2 face an incomplete information game. Player 1 does not know the type of player 2, believing that he is type I with probability 1/2 and type II with probability 1/2. Check if there are Bayes-Nash equilibria in pure strategies.**

Type I

	L	R
U	3,1	3,5
D	-2,1	4,4

Type II

	L	R
U	3,2	3,4
D	-2,2	4,1

A: Let p be the probability that player 1 chooses U, q be the probability that player 2, type I chooses L, and r be the probability that player 2, type II chooses L. To find player 1's best reply, compare u(U,q,r) = 3 with u(D,q,r) = 1/2*[-2*q+4*(1-q)] + 1/2*[-2*r+4*(1-r)] = 4-3q-3r. So, the best reply is p = 1 if 3q + 3r > 1; $0 \leq p \leq 1$ if 3q + 3r = 1;p = 0 if 3q + 3r < 1. As for player 2, type I, R is dominant; so, q(p) = 0. Finally, player 2, type II has: u(p, L) = 2 and u(p, R) = 4p + (1-p) = 1 + 3p; the best-reply is r(p) = 1 if p < 1/3, $0 \leq r(p) \leq 1$ if p = 1/3, and r(p) = 0 if p > 1/3. Intersecting the best-replies, one can see that there are no BN equilibria in pure strategies.