

FORMULÁRIO DE ESTATÍSTICA I

PROBABILIDADE

- $P(A \cap B \cap C) = P(A)P(B|A)P(C|A \cap B)$
- Sendo $\{A_1, A_2, \dots\}$ uma partição do espaço dos resultados com $P(A_j) > 0, j = 1, 2, \dots,$

$$P(B) = \sum_j P(A_j)P(B|A_j) \quad ; \quad P(A_j|B) = \frac{P(A_j)P(B|A_j)}{\sum_i P(A_i)P(B|A_i)}$$

VALOR ESPERADO, MOMENTOS E PARÂMETROS

	Discretas	Contínuas
$E[\psi(X)] =$	$\sum_x \psi(x)f_X(x)$	$\int_{-\infty}^{+\infty} \psi(x)f_X(x) dx$
$E[\psi(X, Y)] =$	$\sum_x \sum_y \psi(x, y)f_{X,Y}(x, y)$	$\int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \psi(x, y)f_{X,Y}(x, y) dx dy$
$E[\psi(X, Y) X = x] =$	$\sum_y \psi(x, y)f_{Y X=x}(y)$	$\int_{-\infty}^{+\infty} \psi(x, y)f_{Y X=x}(y) dy$

Momentos de ordem k	$\mu'_k = E(X^k)$	$\mu_k = E[(X - \mu)^k]$
Momentos de ordem r+s	$\mu'_{rs} = E(X^r Y^s)$	$\mu_{rs} = E\{(X - \mu_X)^r (Y - \mu_Y)^s\}$

$$\text{Var}(X) = E(X - \mu)^2 = E(X^2) - \mu^2;$$

$$\text{Cov}(X, Y) = E\{(X - \mu_X)(Y - \mu_Y)\} = E(XY) - E(X)E(Y) \quad ; \quad \rho_{X,Y} = \frac{\text{Cov}(X, Y)}{\sigma_X \sigma_Y}$$

$E(aX + bY) = aE(X) + bE(Y)$ e $\text{Var}(aX + bY) = a^2 \text{Var}(X) + b^2 \text{Var}(Y) + 2ab \text{Cov}(X, Y)$ com a, b constantes

$$E(Y) = E_X[E(Y|X)] \quad ; \quad \text{Var}(Y) = \text{Var}_X[E(Y|X)] + E_X[\text{Var}(Y|X)]$$

$$\text{Coeficiente de assimetria: } \gamma_1 = \frac{\mu_3}{\sigma^3}; \quad \text{Kurtosis: } \gamma_2 = \frac{\mu_4}{\sigma^4}$$

Quantil de ordem α (v.a. contínua, função distribuição $F_X(x)$ estritamente crescente): q_α - solução única da equação $F_X(q_\alpha) = \alpha$;

Amplitude Inter-Quartis: $\text{AIQ} = q_{0.75} - q_{0.25}$.

DISTRIBUIÇÕES TEÓRICAS

• UNIFORME (DISCRETA)

$$\text{Caso } f(x) = \frac{1}{n}, \quad x = 1, 2, \dots, n; \quad E(X) = \frac{n+1}{2}; \quad \text{Var}(X) = \frac{n^2-1}{12}$$

$$\text{Caso } f(x) = \frac{1}{m+1}, \quad x = 0, 1, 2, \dots, m; \quad E(X) = \frac{m}{2}; \quad \text{Var}(X) = \frac{m(m+2)}{12}$$

• BINOMIAL $X \sim B(n; \theta), (0 < \theta < 1)$

$$f(x|\theta) = \binom{n}{x} \theta^x (1-\theta)^{n-x}, \quad x = 0, 1, 2, \dots, n$$

$$E(X) = n\theta \quad ; \quad \text{Var}(X) = n\theta(1-\theta) \quad ; \quad M_X(s) = [(1-\theta) + \theta e^s]^n \quad ; \quad \gamma_1 = (1-2\theta)/\sigma$$

Propriedades:

- $X \sim B(n; \theta) \Leftrightarrow (n - X) \sim B(n; 1 - \theta)$
- $X_1 \sim B(n_1; \theta), X_2 \sim B(n_2; \theta), X_1 \text{ e } X_2 \text{ independentes} \Rightarrow X_1 + X_2 \sim B(n_1 + n_2, \theta)$
- **BERNOULLI** $X \sim B(1; \theta)$

- **POISSON** $X \sim \text{Po}(\lambda), (\lambda > 0)$

$$f(x | \lambda) = \frac{e^{-\lambda} \lambda^x}{x!}, x = 0, 1, 2, \dots; \quad E(X) = \lambda; \text{Var}(X) = \lambda; M_X(s) = \exp\{\lambda(e^s - 1)\}; \quad \gamma_1 = \lambda^{-1/2}$$

Propriedades:

- $X_1 \sim \text{Po}(\lambda_1), X_2 \sim \text{Po}(\lambda_2), X_1 \text{ e } X_2 \text{ independentes} \Rightarrow X_1 + X_2 \sim \text{Po}(\lambda_1 + \lambda_2)$
- Se $X \sim B(n; \theta)$, com n grande θ pequeno então $X \stackrel{a}{\sim} \text{Po}(n\theta)$

- **UNIFORME (CONTÍNUA)** $X \sim U(\alpha, \beta), (\alpha < \beta)$

$$f(x | \alpha, \beta) = \frac{1}{\beta - \alpha} \quad \alpha < x < \beta; \quad E(X) = \frac{\alpha + \beta}{2}; \text{Var}(X) = \frac{(\beta - \alpha)^2}{12}; M_X(s) = \frac{e^{s\beta} - e^{s\alpha}}{s(\beta - \alpha)}, \quad s \neq 0$$

- **NORMAL** $X \sim N(\mu, \sigma^2), (-\infty < \mu < +\infty, 0 < \sigma < +\infty)$

$$f(x | \mu, \sigma^2) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left\{-\frac{1}{2\sigma^2}(x - \mu)^2\right\}, \quad -\infty < x < +\infty$$

$$E(X) = \mu; \text{Var}(X) = \sigma^2; M_X(s) = \exp\left\{\mu s + \frac{\sigma^2 s^2}{2}\right\}; \quad \gamma_1 = 0; \gamma_2 = 3$$

Propriedades:

- Normal estandardizada $Z = \frac{X - \mu}{\sigma} \sim N(0, 1); \quad \phi(z) = \phi(-z) \text{ e } \Phi(z) = 1 - \Phi(-z)$
- $X_i \sim N(\mu, \sigma^2) (i = 1, 2, \dots, k) \text{ independentes} \Rightarrow Y = \sum_{i=1}^k X_i \sim N(k\mu, k\sigma^2) \text{ e } \bar{X} = \frac{1}{k} \sum_{i=1}^k X_i \sim N\left(\mu, \frac{\sigma^2}{k}\right)$
- $X_i \sim N(\mu_i, \sigma_i^2) (i = 1, 2, \dots, k) \text{ independentes} \Rightarrow \sum_{i=1}^k \alpha_i X_i \sim N(\mu_Y, \sigma_Y^2) \text{ com } \mu_Y = \sum_{i=1}^k \alpha_i \mu_i \text{ e } \sigma_Y^2 = \sum_{i=1}^k \alpha_i^2 \sigma_i^2$

- **EXPONENCIAL** $X \sim \text{Ex}(\lambda), (\lambda > 0); \quad X \sim \text{Ex}(\lambda) \Leftrightarrow X \sim G(1, \lambda)$

$$f(x | \lambda) = \lambda e^{-\lambda x}, x > 0 \quad E(X) = \frac{1}{\lambda}; \text{Var}(X) = \frac{1}{\lambda^2}; M_X(s) = \frac{\lambda}{\lambda - s}, s < \lambda; \quad \gamma_1 = 2; \gamma_2 = 9$$

Propriedades:

- $X_i \sim \text{Ex}(\lambda) (i = 1, 2, \dots, k) \text{ independentes} \Rightarrow \sum_{i=1}^k X_i \sim G(k, \lambda) \text{ e } \min_i X_i \sim \text{Ex}(k\lambda)$

- **GAMA** $X \sim G(\alpha, \lambda), (\lambda > 0, \alpha > 0)$

$$f(x | \alpha, \lambda) = \frac{\lambda^\alpha e^{-\lambda x} x^{\alpha-1}}{\Gamma(\alpha)}, \quad x > 0; \quad E(X) = \frac{\alpha}{\lambda}; \text{Var}(X) = \frac{\alpha}{\lambda^2}; M_X(s) = \left(\frac{\lambda}{\lambda - s}\right)^\alpha, s < \lambda; \quad \gamma_1 = \frac{2}{\sqrt{\alpha}}; \gamma_2 = 3 + \frac{6}{\alpha}$$

Propriedades:

- $X_i \sim G(\alpha_i; \lambda), (i = 1, 2, \dots, k) \text{ independentes} \Rightarrow \sum_{i=1}^k X_i \sim G\left(\sum_{i=1}^k \alpha_i; \lambda\right)$
- $X \sim G(\alpha, \lambda) \text{ então } Y = cX \sim G\left(\alpha, \frac{\lambda}{c}\right) \text{ onde } c \text{ constante positiva}$

- **QUI-QUADRADO** $X \sim \chi^2(n), (n \text{ inteiro positivo}).$

$$X \sim \chi^2(n) \Leftrightarrow X \sim G(n/2; 1/2); \quad E(X) = n; \text{Var}(X) = 2n; M_X(s) = (1 - 2s)^{-n/2}, s < \frac{1}{2}; \quad \gamma_1 = \sqrt{\frac{8}{n}}; \gamma_2 = 3 + \frac{12}{n}$$

Propriedades:

- $X_i \sim \chi^2_{(n_i)} (i = 1, 2, \dots, k) \text{ independentes} \Rightarrow \sum_{i=1}^k X_i \sim \chi^2_{(n)} \text{ com } n = \sum_{i=1}^k n_i$
- $X \sim G(n; \lambda) \Leftrightarrow 2\lambda X \sim \chi^2(2n)$

- $X_i \sim N(0,1)$, $(i=1,2,\dots,n)$ independentes $\Rightarrow \sum_{i=1}^n X_i^2 \sim \chi^2(n)$
- $X \sim \chi^2(n) \Rightarrow \sqrt{2X} - \sqrt{2n-1} \stackrel{a}{\sim} N(0,1)$

• **t-“STUDENT”**

$$T = \frac{U}{\sqrt{V/n}} \sim t(n) \text{ com } U \sim N(0,1) \text{ e } V \sim \chi^2(n) \text{ independentes}$$

$$E(T) = 0; \text{Var}(T) = \frac{n}{n-2} \text{ (} n > 2 \text{)}; \gamma_1 = 0; \gamma_2 = \frac{3(n-2)}{n-4} \text{ (} n > 4 \text{)}$$

Propriedade: • Sendo $T \sim t(n) \Rightarrow \lim_{n \rightarrow \infty} F_T(t|n) = \Phi(t)$

• **F-SNEDCOR**

$$F = \frac{U/m}{V/n} \sim F(m,n) \text{ com } U \sim \chi^2(m), V \sim \chi^2(n) \text{ (independentes)}$$

$$E(X) = \frac{n}{n-2} \text{ (} n > 2 \text{)}; \text{Var}(X) = \frac{2n^2(m+n-2)}{m(n-2)^2(n-4)} \text{ (} n > 4 \text{)}$$

Propriedades: • $X \sim F(m,n) \Rightarrow \frac{1}{X} \sim F(n,m)$ • $T \sim t(n) \Rightarrow T^2 \sim F(1,n)$

TEOREMA DO LIMITE CENTRAL E COROLÁRIOS

TLC: Sendo X_i iid com $E(X_i) = \mu$ e $\text{Var}(X_i) = \sigma^2 \Rightarrow \frac{\sum_{i=1}^n X_i - n\mu}{\sqrt{n}\sigma} = \frac{\bar{X} - \mu}{\sigma/\sqrt{n}} \stackrel{a}{\sim} N(0,1)$

Corolário: Sendo $X_i \sim B(1;\theta)$, iid $\Rightarrow \frac{\sum_{i=1}^n X_i - n\theta}{\sqrt{n\theta(1-\theta)}} \stackrel{a}{\sim} N(0,1)$

Correcção de continuidade: $P(a \leq X \leq b) \approx \Phi\left(\frac{b + \frac{1}{2} - n\theta}{\sqrt{n\theta(1-\theta)}}\right) - \Phi\left(\frac{a - \frac{1}{2} - n\theta}{\sqrt{n\theta(1-\theta)}}\right)$, com a e b inteiros

Corolário: Sendo $X \sim \text{Po}(\lambda)$, quando $\lambda \rightarrow +\infty \Rightarrow \frac{X - \lambda}{\sqrt{\lambda}} \stackrel{a}{\sim} N(0,1)$

Correcção de continuidade: $P(a \leq X \leq b) \approx \Phi\left(\frac{b + \frac{1}{2} - \lambda}{\sqrt{\lambda}}\right) - \Phi\left(\frac{a - \frac{1}{2} - \lambda}{\sqrt{\lambda}}\right)$, com a e b inteiros

AMOSTRAGEM. DISTRIBUIÇÕES POR AMOSTRAGEM

$$\bar{X} = \frac{\sum_{i=1}^n X_i}{n}; \quad S^2 = \frac{\sum_{i=1}^n (X_i - \bar{X})^2}{n} = \frac{\sum_{i=1}^n X_i^2}{n} - \bar{X}^2; \quad (n-1)S'^2 = n S^2$$

$$E(\bar{X}) = \mu; \quad \text{Var}(\bar{X}) = \frac{\sigma^2}{n}; \quad E(S^2) = \frac{n-1}{n} \sigma^2; \quad E(S'^2) = \sigma^2$$

• **DISTRIBUIÇÃO DO MÍNIMO E DO MÁXIMO**

$$G_1(x) = 1 - [1 - F(x)]^n; \quad G_n(x) = [F(x)]^n$$

- **GRANDES AMOSTRAS**

Caso geral

Média	$\frac{\bar{X} - \mu}{\sigma/\sqrt{n}} \stackrel{a}{\sim} N(0,1)$	$\frac{\bar{X} - \mu}{S'/\sqrt{n}} \stackrel{a}{\sim} N(0,1)$
Diferença de médias	$\frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1^2}{m} + \frac{\sigma_2^2}{n}}} \stackrel{a}{\sim} N(0,1)$	$\frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{S_1'^2}{m} + \frac{S_2'^2}{n}}} \stackrel{a}{\sim} N(0,1)$

População de Bernoulli

Proporção	$\frac{\bar{X} - \theta}{\sqrt{\frac{\theta(1-\theta)}{n}}} \stackrel{a}{\sim} N(0,1)$
Diferença de proporções	$\frac{\bar{X}_1 - \bar{X}_2 - (\theta_1 - \theta_2)}{\sqrt{\frac{\theta_1(1-\theta_1)}{m} + \frac{\theta_2(1-\theta_2)}{n}}} \stackrel{a}{\sim} N(0,1)$

- **POPULAÇÕES NORMAIS**

Média	$\frac{\bar{X} - \mu}{\sigma/\sqrt{n}} \sim N(0,1)$	$\frac{\bar{X} - \mu}{S'/\sqrt{n}} \sim t(n-1)$
Diferença de médias	$\frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1^2}{m} + \frac{\sigma_2^2}{n}}} \sim N(0,1)$	$Z = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{S_1'^2}{m} + \frac{S_2'^2}{n}}} \stackrel{a}{\sim} t(\nu)$
	$T = \frac{\frac{\bar{X}_1 - \bar{X}_2 - (\mu_1 - \mu_2)}{\sqrt{\frac{1}{m} + \frac{1}{n}}}}{\sqrt{\frac{(m-1)S_1'^2 + (n-1)S_2'^2}{m+n-2}}} \sim t(m+n-2)$	onde ν é o maior inteiro contido em r , $r = \frac{\left(\frac{s_1'^2}{m} + \frac{s_2'^2}{n}\right)^2}{\frac{1}{m-1} \left(\frac{s_1'^2}{m}\right)^2 + \frac{1}{n-1} \left(\frac{s_2'^2}{n}\right)^2}$
Variância	$\frac{nS^2}{\sigma^2} = \frac{(n-1)S'^2}{\sigma^2} \sim \chi^2(n-1)$	
Relação de variâncias	$\frac{S_1'^2}{S_2'^2} \frac{\sigma_2^2}{\sigma_1^2} \sim F(m-1, n-1)$	