University of Lisbon ISEG
2019/2020 Paulo M.D.C. Parente

Advanced Econometrics
PhD in Economics
2019/2020
Exercise Sheet 2 - Discrete Choice Models

1. Consider a latent variable modeled by $Y_i^* = X_i'\beta + \varepsilon_i$, with $\varepsilon_i \sim N(0,1)$. Suppose we observe only $Y_i = 1$ if $Y_i^* < U_i$ and $Y_i = 0$ if $Y_i^* \geq U_i$, where the upper limit U_i is a known constant for each individual (i.e., data) and may differ over individuals.

   (a) Find $\mathcal{P}[Y_i = 1|X_i, U_i]$. [Hint: Note that this differs from the standard case both due to presence of U_i and because the equalities are reversed with $Y_i = 1$ if $Y_i^* < U_i$.]

   (b) Provide details on an estimation method to consistently estimate β.

2. Consider the logit model with $\mathcal{P}[Y = 1|X_1] = \Lambda(\beta_0 + \beta_1 X_1)$, where $\Lambda(z) = e^z/(1+e^z)$.

   (a) Write down the first order conditions of the maximum likelihood estimator in an expanded form.

   (b) Write down the information matrix in an expanded form.

   (c) Derive the LM test of $H_0 : \beta_1 = 0$.

   (d) Show that the LM test is equal to $n \times r_{yx}^2$ where r_{yx}^2 is the squares of the empirical correlation coefficient between Y and X_1.

3. A data set consists of $n = n_1 + n_2 + n_3$ observations on Y and X. For the first n_1 observations, $Y = 1$ and $X = 1$. For the next n_2 observations, $Y = 0$ and $X = 1$. For the last n_3 observations, $Y = 0$ and $X = 0$. Prove that the Maximum Likelihood estimator does not exist.