LISBON SCHOOL OF ECONOMICS & MANAGEMENT
UNIVERSIDADE DE LISBOA

Financial Markets and Investments

Raquel M. Gaspar

COMPUTER ASSIGNEMENT 2

Suppose you can invest in three risky assets A, B, C and deposit without risk at an interest rate of 3% or borrow at a rate of 7%. In addition we have the following estimates about the risky assets: $\bar{R}_A = 15\%, \bar{R}_B = 10\%$, $\bar{R}_C = 20\%$, $\sigma_A = 10\%$, $\sigma_B = 6\%$, $\sigma_C = 15\%$, $\rho_{AB} = 0.4, \rho_{BC} = 0.3$, and $\rho_{AC} = 0.5$.

1. Consider the above described market conditions.
   (a) Represent in the mean-variance space $(\sigma, \bar{R})$ the basic assets and determine the mean-variance inputs.
   (b) Explain its shape and derive the mathematical expression(s) for the efficient frontier (EF):
      (i) when we consider just combinations of the three risky assets.
      (ii) when we consider combinations of the three risky assets with the deposit and borrowing rates.
   (c) Explain the shape of the investment opportunity set (IOS) for scenarios b(i) and b(ii).
   (d) Represent the EF in mean-variance space and conclude about the efficiency of assets A, B and C?
   (e) Show it is possible to recover the exact same envelop hyperbola formula as combinations of the two tangent portfolios.

2. Suppose the returns of A, B, C are normally distributed. Determine and represent in the mean-variance space $(\sigma, \bar{R})$:
   (i) The Roy combination, *Roy*, of the three risk assets that has the lowest probability of returns lower than 0%.
   (ii) Taking in to account that it is possible to lend and borrow (at different rates), identify all portfolios that are as safe as the *Roy* portfolio in (i).
   (iii) The Kataoka combination of the risky assets, when one worries about the 10% worst scenarios.
   (iv) Taking in to account that it is possible to lend and borrow (at different rates), identify all portfolios that are as safe as the *Kataoka* portfolio in (iii).
   (v) Determine all portfolios for which $\Pr[R_p \leq 0\%) \leq 10\%]$ and identify the Telser portfolios, considering just risky assets and taking into account also lending and borrowing.

3. Consider an investor, Ms. Safe, that which to invest 10 000 euros and have asked for your advice. Her requirements are:
   (i) A volatility not bigger that 20%.
   (ii) A portfolio that verifies $\Pr[R_p \leq 0\%) \leq 10\%]$.
   (iii) A portfolio that maximizes the expected return, provided (i) and (ii) are verified.

4. Consider now shortselling is not allowed.

   (a) Determine the lower and upper limiting points of the envelop hyperbola that are consistent with the no shortselling restriction.

   (b) Determine the new efficient frontier and explain the similarities and differences when comparing to that of Question 1.

5. Suppose you would like to apply a constant correlation model (CCM) for in the described setup.

   (a) Determine the constant correlation parameter ρ and the mean-variance inputs under the CCM assumption.

   (b) Find out the new tangent portfolios – $T1$ (CCM) and $T2$ (CCM) – under the model assumption.

   (c) Represent the true envelop hyperbola of Question 1 and the CCM model implied envelop hyperbola and explain to which kind of investor would model risk be bigger?

6. Consider now the return correlations of assets A, B, C are $\rho_{AB} = 0.5, \rho_{BC} = 0$, and $\rho_{AC} = 0.25$.

   Redo the Exercise. Comment the results.