Consider a version of our model with costly money holdings. We will assume that these costs come in the form of labor costs associated with each unit of real balances the individual holds. This cost function is then given by

$$Z_t H_t = \phi M_{t+1}/P_t,$$

where H_t is the number of extra labor units the individual must provide if his real balances are M_t/P_t and his productivity level is Z_t. The parameter ϕ governs the sensitivity of productive labor ($Z_t H_t$) to real balances. The total number of units that the individual must work is therefore given by $L_t + H_t$. Hence, his disutility from labor effort is $v(L_t + H_t)$.

The household's problem can be written as

$$\max_{\{C_t, L_t, H_t, M_{t+1}, B_{t+1}\}_{t=1,2}} \log(C_1) - v(L_1 + H_1) + \beta\left[\log(C_2) - v(L_2 + H_2)\right] + \beta^2 V(M_3, B_3) \text{ subject to}$$

$$\begin{aligned} M_t &\geq P_t C_t, \\ Z_t H_t &= \phi M_{t+1}/P_t, \\ P_t Z_t L_t + [M_t - P_t C_t] + B_t + T_t &\geq M_{t+1} + q_t B_{t+1} \text{ for } t = 1, 2. \end{aligned}$$

A) [10 pts.] Since the constraint on H_t is an equality constraint, I suggest using it to substitute out for H_t in your preferences. (After this we can drop the constraint since it is already being enforced and H_t only appears implicitly.) With this, the problem is fairly similar to what we had before. Write down the Lagrangian for the individual's problem.

B) [10 pts.] Derive the first-order conditions for your choice variables. This will consist of C_t, L_t, M_{t+1},and B_{t+1} if you follow my suggestion in (A). Your first-order condition for M_{t+1} should now include a new labor cost term, and your marginal disutility should reflect the fact that (implicitly) with $H_t > 0$, the marginal disutility of labor is now higher fixing L_t.

C) [10 pts.] Can we have negative interest rates ($q_t > 1$) in this version of our model? You can answer this using the FOCs for money and bonds.

D) [10 pts.] Assume that the productivity and money grow at constant rates

$$Z_t = (1+g)^t Z_0, \text{ and } \bar{M}_{t+1} = (1+\tau)^t M_0.$$

What are the level of the transfers T_t an individual must receive in the asset market? What is the goods market clearing condition. Assume the cash-in-advance constraint binds and use this to derive the price level P_t.

E) [10 pts.] Just as we did in the original example, you can use the FOC for money at time t, the FOC for labor at time t and the FOC for consumption at $t+1$ to get a similar expression to what we had before plus one extra term involving the labor cost of holding money at time t. [Getting rid of the multipliers.]

F) [10 pts.] Assume that our model admits a steady state in which $L_t = L$ (i.e. is constant) and the cash-in-advance constraint binds. Use this to derive an expression for P_t where we substitute $\bar{M}_t$ for M_t using money market clearing and for C_t using the goods market clearing condition.

G) [10 pts.] Derive a stationary version of the condition in (E) which only needs to be solved for L. To do this substitute for C_t using the goods market clearing condition, and for P_t using the condition from (F). In doing so, assume that

$$v(L_t) = \frac{L_t^{1+\gamma}}{1+\gamma}$$

H) [10 pts.] How would you expect changes in ϕ to impact on the steady state value of L?