Soluções dos exercícios de Cálculo Diferencial - I

1. $(f^{-1})'(3) = \frac{1}{9}$.

2. a) $y = \frac{1}{3}x - \frac{1}{3}$; b) $y = x - 1$; c) $y = \frac{1}{60}x + \frac{5}{3}$.

3. a) $\text{argsinh}(x) = \ln(x + \sqrt{1+x^2})$; b) $\text{argsinh}'(x) = \frac{1}{\sqrt{x^2+1}}$; c) $\text{argsinh}'(0) = 1$.

4. –

5. (a)

   (i) $P_1(x) = 1 + x$, $P_2(x) = 1 + x + \frac{1}{2}x^2$, $P_3(x) = 1 + x + \frac{1}{2}x^2 + \frac{1}{6}x^3$.

   (ii) $P_1(x) = 1 - \frac{1}{2}(x-1) = \frac{3}{2} - \frac{1}{2}x$, $P_2(x) = 1 - \frac{1}{2}(x-1) + \frac{3}{8}(x-1)^2 = \frac{15}{8} - \frac{5}{4}x + \frac{3}{8}x^2$,

   $P_3(x) = 1 - \frac{1}{2}(x-1) + \frac{3}{8}(x-1)^2 - \frac{15}{48}(x-1)^3$.

   (iii) $P_1(x) = x - 1$, $P_2(x) = x - 1 - \frac{1}{2}(x-1)^2 = -\frac{3}{2} + 2x - \frac{1}{2}x^2$,

   $P_3(x) = x - 1 - \frac{1}{2}(x-1)^2 + \frac{1}{3}(x-1)^3$.

   (iv) $P_1(x) = 4 + \frac{1}{8}(x-16) = 2 + \frac{1}{8}x$, $P_2(x) = 4 + \frac{1}{8}(x-16) - \frac{1}{512}(x-16)^2 = \frac{3}{2} + \frac{3}{16}x - \frac{1}{512}x^2$.

   $P_3(x) = 4 + \frac{1}{8}(x-16) - \frac{1}{512}(x-16)^2 + \frac{1}{98304}(x-16)^3$.

   (c) $\ln(1.1) \approx 0.095$; $\sqrt[10]{e} \approx 1.105$; $\frac{1}{\sqrt{0.8}} \approx 1.115$; $\sqrt{17} \approx \frac{2111}{512}$.

6.

   (a) $\frac{33}{8}$

   (b) $\sqrt{x} = 4 + \frac{1}{8}(x-16) - \frac{1}{8c^{\frac{3}{2}}}(x-16)^2$, $c \in]16; x[$.

7.

   (a) $e^x = 1 + x + \frac{1}{2}x^2 + \frac{1}{6}e^c x^3$.

   (b) $\ln(x) = x - 1 - \frac{1}{2}(x-1)^2 + \frac{1}{3c^3}(x-1)^3$, $c \in]1; x[$.

   (c) $e^{\frac{1}{10}} = 1 + \frac{1}{10} + \frac{1}{200} + \frac{1}{6000}e^c$, $c \in]0; \frac{1}{10}[$. Assim $\left|e^{\frac{1}{10}} - \left(1 + \frac{1}{10} + \frac{1}{200}\right)\right| \leq \frac{e^c}{6000} \leq \frac{e^{\frac{1}{10}}}{6000} \leq \frac{1}{4000}$,

   onde se tomou, por exemplo, $e^{\frac{1}{10}} \leq 3^{\frac{1}{6}} \leq \frac{3}{2}$.

   $\ln(1.1) = 0.1 - \frac{1}{2}0.1^2 + \frac{1}{3c^3}0.1^3$, $c \in]1; 1.1[$. Assim, $|\ln(1.1) - (0.1 - \frac{1}{2}0.1^2)| = \frac{0.1^3}{3c^3} \leq \frac{1}{3000}$.

8.

   (a) $1 + 3x + \frac{3^2}{2!}x^2 + \frac{3^3}{3!}x^3 + \cdots + \frac{3^n}{3!}x^n$.

   (b) $\ln(1+2x) = 2x - 2x^2 + \frac{8}{3}x^3 - 4x^4 + \cdots + (-1)^{n+1}\frac{2^n}{n}x^n + x^n\epsilon(x^n)$.

   (c) Para n impar, $2x + \frac{2^3}{3!}x^3 + \frac{2^5}{5!}x^5 + \cdots + \frac{2^n}{n!}x^n$.
   Para n par, o polinómio é idêntico ao de ordem $n-1$.

   (d) $1 + \frac{1}{2}x - \frac{1}{8}x^2 + \frac{1}{16}x^3 + \cdots + \frac{(-1)^{n-1}(2n-3)}{2^n n!}x^n$.

9.

   (a) $-\frac{\sqrt{2}}{\sqrt{3}}$, maximizante local; $\frac{\sqrt{2}}{\sqrt{3}}$, minimizante local;

   (b) -1, maximizante local; 2, minimizante local;

(c) 1, maximizante local; 3, minimizante local;

(d) -1, maximizante local; 1, minimizante local;

(e) Sem pontos críticos;

(f) 1, maximizante local;

(g) $\frac{1}{\ln 3}$, maximizante local;

(h) Sem pontos críticos;

(i) -1, minimizante local;

(j) e^{-1}, minimizante local;

(k) $-3-\sqrt{10}$, maximizante local; $-3+\sqrt{10}$, minimizante local; 1 e -1, pontos de sela;

(l) 1 e -3, maximizantes locais; $\sqrt{3}$ e $-\sqrt{3}$, minimizantes locais.

10.

(a) Sem pontos de inflexão;

(b) Sem pontos de inflexão;

(c) 0;

(d) 3, -3, $-\frac{3}{\sqrt{5}}$, $\frac{3}{\sqrt{5}}$;

(e) $-2\sqrt{3}$, 0, $2\sqrt{3}$;

(g) -2;

(h) $-\frac{1}{\sqrt{6}}$, $\frac{1}{\sqrt{6}}$;

(i) Sem pontos de inflexão;

(k) Sem pontos de inflexão;

(l) -5 e $\frac{1}{2}$.

11. a) $e^{\frac{1}{\sqrt{2}}}$; b) $\frac{1}{3}$; c) $\frac{5}{2}$; d)$\frac{1}{3}$; e) $\frac{a}{b}$; f) 1; g) $\frac{\alpha^2}{\beta^2}$; h) $\frac{3}{5}$; i) $\frac{1}{5}$; j) 0;
l) Não existe (os limites laterais são distintos); m) $-\frac{2}{3}$; n) 1; o) $\sqrt{e}$; p) $e^{-\frac{1}{6}}$.