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Advanced Econometrics
PhD in Economics
Exercise sheet 5 - Univariate time series modelling
(version 16/11/2020)

1. Which of the following autoregressive processes are stationary? [where $\varepsilon_t \sim WN(0, \sigma_\varepsilon^2)$]
   (a) $Y_t = 5 - 0.2Y_{t-1} + 0.08Y_{t-2} + \varepsilon_t$;
   (b) $\Phi(L)Y_t = \varepsilon_t$, where $\Phi(0.1 \pm 3i) = 0, \Phi(-1 \pm 0.1i) = 0$, $\Phi(0.9 \pm 0.4i) = 0$ and $i = \sqrt{-1}$. In this $AR(p)$ process what is the value of p?

2. Which of the following moving average processes are invertible? [where $\varepsilon_t \sim WN(0, \sigma_\varepsilon^2)$]
   (a) $Y_t = 2 + \varepsilon_t + 0.1\varepsilon_{t-1} - 0.3\varepsilon_{t-2}$.
   (b) $Y_t = -1 + \varepsilon_t - 0.99\varepsilon_{t-1} - 0.01\varepsilon_{t-2}$.

3. Consider the stationary $AR(2)$ process $Y_t = c + \phi_1 Y_{t-1} + \phi_2 Y_{t-2} + \varepsilon_t$, where $\varepsilon_t \sim WN(0, \sigma_\varepsilon^2)$.
   (a) Derive an expression for $\mu = E(Y_t)$.
   (b) Derive and expression for $\gamma_0 = var(Y_t)$ and for $\rho_j = \gamma_j/\gamma_0$, where $\gamma_j = E\left[(Y_t - \mu)(Y_{t-j} - \mu)\right]$, $(j = 1, 2, ...)$.
   (c) Under what conditions is Y_t stationary?
   (d) Write down the $MA(\infty)$ representation of the $AR(2)$ process.

4. Consider the stationary $ARMA(1,1)$ process $Y_t = c + \phi_1 Y_{t-1} + \varepsilon_t + \theta_1 \varepsilon_{t-1}$, where $\varepsilon_t \sim WN(0, \sigma_\varepsilon^2)$.
   (a) Derive an expression for $\mu = E(Y_t)$.
   (b) Derive and expression for $\gamma_0 = var(Y_t)$ and for $\rho_j = \gamma_j/\gamma_0$ where $\gamma_j = E\left[(Y_t - \mu)(Y_{t-j} - \mu)\right]$, $(j = 1, 2, ...)$.
   (c) Under what conditions is Y_t stationary? Under what conditions is Y_t invertible?
   (d) Write down the $MA(\infty)$ representation of $ARMA(1,1)$ process assuming that it is stationary.
   (e) Write down the $AR(\infty)$ representation of $ARMA(1,1)$ process assuming that it is invertible.