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ISEG
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Advanced Econometrics
PhD in Economics
Exercise sheet 7 - Multivariate time series modelling
(version 11/12/2019)

1. Consider the following equations for a bivariate $VAR(1)$ model:

$$\begin{aligned} y_t &= 0.4y_{t-1} + 0.1x_{t-1} + \varepsilon_{1t}, \\ x_t &= 0.2y_{t-1} + 0.5x_{t-1} + \varepsilon_{2t}. \end{aligned}$$

and let $\varepsilon_t = (\varepsilon_{1t}, \varepsilon_{2t})'$ be a white noise process with variance-covariance matrix $\Omega = \operatorname{diag}\{16, 25\}$.

   (a) Is the above $VAR(1)$ process stationary?

   (b) Obtain the values of the elements of the matrices Ψ_ℓ, for $\ell = 0, 1, 2$ in the infinite moving average representation

$$z_t = \sum\nolimits_{\ell=0}^{\infty} \Psi_\ell \varepsilon_{t-\ell},$$

   where $z_t = (y_t, x_t)'$.

   (c) Obtain the impulse response function for y_t to a shock to the variable x_t of size σ_2 , for horizons$\ell = 0, 1, 2$, where σ_2 is the standard deviation of ε_{2t}.

2. Consider the $VAR(2)$ model $X_t = \Phi_1 X_{t-1} + \Phi_2 X_{t-2} + \varepsilon_t$, where $X_t = (x_{1t}, x_{2t})', \varepsilon_t$ is a (2×1) white noise process with

$$E[\varepsilon_t \varepsilon_t'] = \begin{bmatrix} \sigma_1^2 & \sigma_{12} \\ \sigma_{12} & \sigma_2^2 \end{bmatrix}.$$

   (a) Let

$$\Phi_1 = \begin{bmatrix} 0.6 & 0.8 \\ 0.8 & 0 \end{bmatrix}, \Phi_2 = \begin{bmatrix} 0 & -0.48 \\ 0 & 0 \end{bmatrix}.$$

   Obtain the roots of the characteristic equation and show that the process is stationary.

   (b) Show that if Φ_1 and Φ_2 satisfy the stationary condition, the weight matrices in $X_t = \sum_{j=0}^{\infty} \Psi_j \varepsilon_{t-j}$ satisfy

$$\Psi_0 = I_k, \Psi_1 = \Phi_1, \Psi_j = \Phi_1 \Psi_{j-1} + \Phi_2 \Psi_{j-2}, j \geq 2.$$

   (c) Obtain the impulse response function for x_{1t} to a shock to the variable x_{2t} of size σ_2 , for horizons $l = 0, 1, 2$.

3. Consider the following structural VAR model:

$$B_0 y_t = B_1 y_{t-1} + u_t$$

where $E(u_t) = 0$ and $var(u_t) = D$ and u_t is independent and identically distributed and B_0 is full rank.

(a) Derive the reduced form of the structural VAR model.

(b) Outline how to check for identification of the parameters in the structural VAR model using the order condition.

(c) Use the order condition to determine which of the following bivariate structural VAR models are identified and for the models that are identified show how to obtain the each element of B_0 and D from the parameters of the reduced form model

$$\text{Model 1:} \quad B_0 = \begin{bmatrix} \alpha_{11} & \alpha_{12} \\ \alpha_{21} & \alpha_{22} \end{bmatrix}, D = \begin{bmatrix} d_{11} & d_{12} \\ d_{21} & d_{22} \end{bmatrix}$$

$$\text{Model 2:} \quad B_0 = \begin{bmatrix} \alpha_{11} & \alpha_{12} \\ \alpha_{21} & \alpha_{22} \end{bmatrix}, D = \begin{bmatrix} d_{11} & 0 \\ 0 & d_{22} \end{bmatrix}$$

$$\text{Model 3:} \quad B_0 = \begin{bmatrix} 1 & \alpha_{12} \\ \alpha_{21} & 1 \end{bmatrix}, D = \begin{bmatrix} d_{11} & 0 \\ 0 & d_{22} \end{bmatrix}$$

$$\text{Model 4:} \quad B_0 = \begin{bmatrix} 1 & 0 \\ \alpha_{21} & 1 \end{bmatrix}, D = \begin{bmatrix} d_{11} & 0 \\ 0 & d_{22} \end{bmatrix}.$$