Models in Finance - Part 20

Master in Actuarial Science

João Guerra

ISEG

Credit risk

- Before, we have assumed that bonds are default free.
- This is not a reasonable assumption for corporate bonds and some government bonds (for example, if the government does not control the currency).
- It can be reasonable for some government bonds.
- Credit risk changes with the market and good practice is to assess both current and potential exposure. The current exposure is the current market value of the asset, the future exposure should be based on a wide range of future scenarios, with different default probabilities.

Credit risk

- The default of a bond can be triggered by a credit event of the type:
- (i) failure to pay capital or a coupon.
- (ii) actions associated with bankruptcy or insolvency laws.
- (iii) rating downgrade of the bond by a rating agency such as Standard and Poor's or Moody's or Fitch.
- (iv) repudiation/moratorium.
- (v) restructuring– the terms of the obligation are changed and the new terms are less attractive to the debt holder (e.g. reduction in the interest rate, re-scheduling, change in principal).
- Recovery rate: fraction of the defaulted amount that can be recovered through bankruptcy proceedings or other forms of settlement.
- Credit risk is calculated as an expected loss:
  Expected Loss = Exposure x Probability of Default x Loss Given Default,
  where Loss Given Default (LGD) = 100% - Recovery Rate

Structural models

- Structural models: explicit models of a corporate entity issuing both equity and debt.
- These models link default events explicitly to the fortunes of the issuing corporate entity.
- These models can give an insight into the nature of default and the interaction between bond holders and equity holders.
- Examples of a structural model: the Merton model or First Passage models.

Reduced form models

- Reduced form models: statistical models which use observed market statistics rather than specific data relating to the issuing corporate entity.
- The market statistics most commonly used are the credit ratings issued by credit rating agencies such as Standard and Poor's, Moody's or Fitch.
- These models use market statistics along with data on the default-free market to model the movement of the credit rating of the bonds.
- The output of these models is a distribution of the time to default.

Intensity-based models

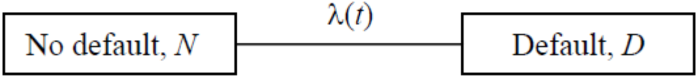
- An intensity-based model is a particular type of reduced form model.
- These models are defined in continuous-time and they model the "jumps" between different states (usually credit ratings) using transition intensities.
- Examples: two-state model for credit ratings with a deterministic transition intensity and the Jarrow-Lando-Turnbull model.

The Merton model

- Consider that a corporate entity has issued both equity E_t and debt D_t such that its total value at time t is $V_t = D_t + E_t$.
- Assume a firm has issued a single zero coupon bond with face of D which matures at time T. At time T the remainder of the value of the corporate entity will be distributed amongst the equity holders. Therefore
$$E_T = max\{V_T - D, 0\}. \tag{1}$$
- Default situation: if $V_T < D$.
- The value of a firm's equity E_t is equal to the value of an European call option on the value of the firm with maturity T and a strike price D, and the present value of the debt is $D_t = V_t - E_t$.
- In the Black-Scholes formula for a call option, the term $\Phi(d_2)$ is the risk-neutral probability that the option is exercised. In this context, this implies that $1 - \Phi(d_2)$ is the risk neutral probability of default.

Two-state model with constant intensity

- In continuous time, consider a model with two states: N (not previously defaulted) and D (previously defaulted).
- Assume that the interest rate term structure is deterministic: $r(t) = r$ for all t.
- The transition intensity, under the real world measure P, from N to D is denoted by $\lambda(t)$.



Two state model with constant intensity

- The state D is an absorbing state.
- Let $X(t)$ be the state at time t. The transition intensity $\lambda(t)$ is such that (under P)

$$P\left[X(t+dt)=N|X(t)=N\right]=1-\lambda(t)dt+o(dt) \quad \text{as } dt\longrightarrow 0,$$
$$P\left[X(t+dt)=D|X(t)=N\right]=\lambda(t)dt+o(dt) \qquad \text{as } dt\longrightarrow 0.$$

- Define the stopping time τ (time of default):

$$\tau=\inf\left\{t:X\left(t\right)=D\right\}.$$

- Define the number of defaults as the counting process $N(t)$:

$$N(t)=\begin{cases}0 & \text{if } \tau>t,\\ 1 & \text{if } \tau\leq t.\end{cases}$$

Two state model with deterministic intensity

- Assume that if the corporate entity defaults all bond payments will be reduced by a deterministic factor $(1-\delta)$ where δ is the recovery rate.
- If a bond is due to pay 1 at time T, the actual payment at time T will be 1 if $\tau > T$ and δ if $\tau \leq T$.
- Let $B(t,T)$ be the price at time t of a zero-coupon bond. Then there exists a risk-neutral measure Q equivalent to P under which:

$$\begin{aligned} B(t,T) &= e^{-r(T-t)} E_Q \left[Payoff \text{ at } T | \mathcal{F}_t \right] \\ &= e^{-r(T-t)} E_Q \left[1 - (1-\delta) N(T) | \mathcal{F}_t \right]. \end{aligned}$$

- If q is the risk neutral probability that default occurs before T, we can write

$$B(t,T) = e^{-r(T-t)} (1 - q + \delta q)$$

Two state model with constant intensity

- It can be proved that:

$$E_Q\left[N\left(T\right)|N(t)=0\right]=E_Q\left[1-\exp\left(-\int_t^T\widetilde{\lambda}\left(s\right)ds\right)\right].$$

- Assuming that $\widetilde{\lambda}\left(s\right)$ is deterministic, this implies that:

$$B(t,T)=e^{-r(T-t)}\left[1-(1-\delta)\left(1-\exp\left(-\int_t^T\widetilde{\lambda}\left(s\right)ds\right)\right)\right]$$

which is equivalent to:

$$\widetilde{\lambda}\left(s\right)=-\frac{\partial}{\partial s}\log\left[e^{r(s-t)}B(t,s)-\delta\right]$$

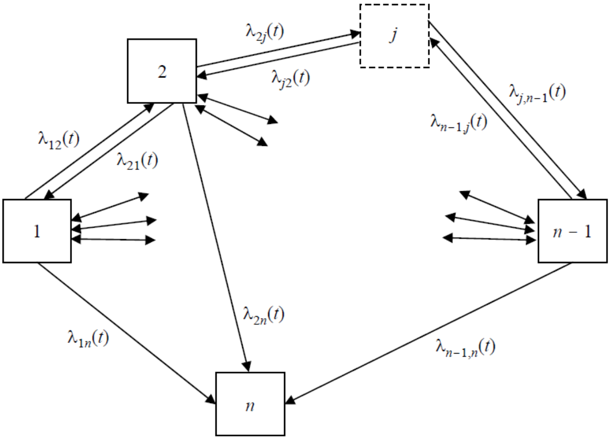
- Note: $\widetilde{\lambda}\left(s\right)$ is the transition intensity under Q.
- From the bond term structures and making an assumption about the recovery rate allows the implied risk-neutral transition intensities to be determined.

The Jarrow-Lando-Turnbull model

- In this model there are $n-1$ credit ratings plus default $(n$ states$)$.
- $\lambda_{ij}(t)$: transition intensities, under the real-world measure P, from state i to state j at time t.
- If $X(t)$ is the state or credit rating at time t, then, for $i, j = 1, \ldots, n-1$,

$$P\left[X(t+dt) = j | X(t) = i\right] =$$
$$= \begin{cases} \lambda_{ij}(t)dt + o(dt) & \text{for } j \neq i \\ 1 - \sum_{i \neq j} \lambda_{ij}(t)dt + o(dt) = \lambda_{ii}(t)dt + o(dt) & \text{for } j = i \end{cases}.$$

The Jarrow-Lando-Turnbull model



The Jarrow-Lando-Turnbull model

- The state n (default) is absorbing: $\lambda_{nj}(t) = 0$ for all j and for all t.
- $n \times n$ intensity matrix:

$$\Lambda(t) = [\lambda_{ij}(t)]_{i,j=1}^{n}.$$

- Define, for $s > t$, the transition probabilities:

$$p_{i,j}(t,s) = P[X(s) = j | X(t) = i].$$

- Matrix of transition probabilities:

$$\Pi(t,s) = [p_{ij}(t,s)]_{i,j=1}^{n}.$$

The Jarrow-Lando-Turnbull model

- It can be shown that:

$$\Pi(t,s) = \exp\left[\int_t^s \Lambda(u)\, du\right].$$

- It can be shown that there exists a risk-neutral measure Q equivalent to P such that the price of a zero-coupon bond maturing at time T, which pays 1 if default has not yet occurred and δ if default has occurred and for which the credit rating of the underlying corporate entity is i is given by:

$$V(t,T,X(t)) = B(t,T)\left[1-(1-\delta)P_Q\left[X(T)=n|\mathcal{F}_t\right]\right].$$