UNIVERSITY OF LISBON

ISEG- Lisbon School of Economics and Management

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Advanced Econometrics

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Duration: TWO HOURS

Use a separate answer booklet of ISEG (folha de teste do ISEG) for each section

This is a closed note paper.

Material in the annex: Statistical Tables

Instructions (please read before starting): Write in a clear legible manner in ink/ballpoint. Do not use pencils or erasable pens. Calculators are permitted. Whenever conducting a test use a 5% significance level unless stated otherwise. Also be sure to state null and alternative hypotheses, null distribution (with degrees of freedom), rejection criterion (critical values and rejection region) and outcome. If you are asked to derive something, give all intermediate steps also. Do not answer questions with a "yes" or "no" only, but carefully justify your answer.

Section A

1. Consider the latent variable $y^* = \beta_0 + \beta_1\, log(x_1) + \beta_2\, x_2 + u$ where β_j with $j = 0, 1, 2$ are unknown coefficients, x_j with $j = 1, 2$ are explanatory variables and u is a continuously normally distributed variable, independent of $x_j, : j = 1, 2$ with $E(u|x_1, x_2) = 0$ and $Var(u|x_1, x_2) = \sigma^2$. Suppose a binary variable y verifying, $y = 1[y^* > 0]$.

   (a) *(1 Mark)* Deduce $P(y = 1|x_1, x_2)$.

   (b) *(1 Mark)* Deduce the partial effect of x_1 on the $P(y = 1|x_1, x_2)$.

   (c) *(1 Mark)* Comment on the use of the estimates obtained with the STATA command "probit y x1 x2" to estimate the partial effect deduced in (b).

2. Consider the variable y which is equal to: 1 if an individual has not subscribed an internet service at home, 2 if she or he has subscribed the service in firm A and 3 if she or he has subscribed the service in firm B.

   (a) *(1 Mark)* Identify and write the model estimated with the command in line 2 in the Annex, characterizing the nature of variable y.

   (b) *(1 Mark)* Knowing that x_3 is a dummy variable equal to 1 if the individual is a female, interpret the results obtained with the command in line 3 and write the expression used to obtain it.

   (c) *(1.5 Mark)* Deduce $P(Y = 2|x_1, x_2, x_3)/P(Y = 1|x_1, x_2, x_3)$ and interpret.

Section B - Topics in time series

1. Consider the linear process $X_t = \sum_{i=0}^{\infty} \psi_i \varepsilon_{t-i}$, where ε_t is a white noise process with mean zero and variance σ_ε^2 and assume that $\sum_{j=0}^{\infty} \left|\psi_j\right| < \infty$.

   (a) *(1 Mark)* Show that $E[X_t] = 0$ and $Var(X_t) = \sigma_\varepsilon^2 \sum_{j=0}^{\infty} \psi_j^2$.

   (b) *(2 Marks)* Show that the Autocorrelation function is $\rho_j = \sum_{i=0}^{\infty} \psi_i \psi_{i+j} / \sum_{j=0}^{\infty} \psi_j^2$, $j > 0$.

   (c) *(1.5 Mark)* Suppose that $\psi_0 = 1$ and $\psi_i = \phi_1^{i-1}(\phi_1 + \theta_1)$, $i \geq 1$. Use the result obtained in (b) to show that

$$\rho_j = \phi_1^{j-1}(\phi_1 + \theta_1)\frac{1 + \phi_1\theta_1}{1 + \theta_1^2 + 2\phi_1\theta_1}.$$

2. Consider the $VAR(1)$ model $z_t = \Phi_1 z_{t-1} + \varepsilon_t$ where $z_t = (z_{1t}, z_{2t})'$ and ε_t is a 2×1 multivariate white noise process with positive definite variance-covariance matrix

$$var(\varepsilon_t) = \begin{bmatrix} 1 & \sigma_{12} \\ \sigma_{12} & 1 \end{bmatrix}.$$

Let

$$\Phi_1 = \begin{bmatrix} 0.4 & 0.2 \\ -0.2 & 0.8 \end{bmatrix}.$$

Show that $z_{1,t}$ follows a ARMA(2,1) process by proving that:

(a) *(2.5 Marks)* $z_{1,t} = 1.2z_{1,t-1} - 0.36z_{1,t-2} + 0.2\varepsilon_{2,t-1} + \varepsilon_{1t} - 0.8\varepsilon_{1t-1}$.

(b) *(2 Marks)* $u_t = 0.2\varepsilon_{2,t-1} + \varepsilon_{1t} - 0.8\varepsilon_{1t-1}$ follows a MA(1) process.

3. In this question we investigate whether there is a long run relationship between stock prices and real economic activity. Let $lsp500$ be the natural logarithm of S&P 500 index, lip be the natural logarithm of index of industrial production and consider the sample from January, 1947 to June, 1993 (558 observations.)

   (a) *(1.5 Marks)* Consider the information provided in the following table on the ADF and Phillips-Perron tests (intercept, time trend).

	ADF		Phillips-Perron
	TEST STATISTIC	LAG-LENGTH	TEST STATISTIC
$lsp500$	−1.974	2	−1.92867
lip	−2.27162	2	−2.08367

   Are the variables $lsp500$ and lip stationary?

   (b) *(1 Marks)* The results of the ADF regression of the residuals of the regression of $lsp500$ on lip is −1.57262 (lag-length 2 lags, intercept, no time trend). Are $lsp500$ and lip cointegrated?

   (c) *(1 Marks)* Consider the results of the Johansen cointegration test with $lsp500$ and lip (with lag length 3)

   Johansen tests of H_0 : rank= r vs. H_1 : rank> r
   (No deterministic trend)

r	Trace statistic	5% Critical Value	1% Critical Value
0	24.1486	19.96	24.60
1	3.09108	9.24	12.97

   Does it appear that $lsp500$ and lip have a long-run relationship according to the Johansen cointegration test at 5% level ? How about at 1% level? Justify your answer.

   (d) *(1 Marks)* Assume that $lsp500$ and lip are cointegrated and consider the following output:

Dependent Variable is $lsp500$
558 observations used for estimation.
Estimation Method: Saikkonen-Stock-Watson Efficient Least Squares

	Estimate	Std. Err.	t Ratio	p-Value
Intercept	-2.51695	0.09652	-26.077	0
lip	1.71547	0.02334	73.499	0

Lead/lag length of 8 selected by Schwarz criterion.

Is $lsp500 - 2 \times lip$ stationary?

[END OF PAPER]

ANNEX

1. tab y

```
          y |      Freq.     Percent        Cum.
------------+-----------------------------------
          1 |        826       43.94       43.94
          2 |        502       26.70       70.64
          3 |        552       29.36      100.00
------------+-----------------------------------
      Total |      1,880      100.00
```

2. mlogit y x1 x2 i.x3, baseoutcome(1)

```
                                                Number of obs     =      1,880
                                                LR chi2(6)        =     779.14
                                                Prob > chi2       =     0.0000
Log likelihood = -1629.0779                     Pseudo R2         =     0.1930

------------------------------------------------------------------------------
           y |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
1            |  (base outcome)
-------------+----------------------------------------------------------------
2            |
          x1 |  -.4700637   .0451184   -10.42   0.000    -.5584942   -.3816332
          x2 |   2.360152   .2197886    10.74   0.000     1.929375     2.79093
        1.x3 |   .8990353   .1263129     7.12   0.000     .6514665    1.146604
       _cons |   4.049434   .5108222     7.93   0.000     3.048241    5.050627
-------------+----------------------------------------------------------------
3            |
          x1 |  -.0972707   .0475933    -2.04   0.041    -.1905519   -.0039895
          x2 |   3.185773   .2156872    14.77   0.000     2.763034    3.608512
        1.x3 |   .0201598   .1383609     0.15   0.884    -.2510225    .2913421
       _cons |  -.0458046    .558607    -0.08   0.935    -1.140654    1.049045
------------------------------------------------------------------------------
```

3. margins, dydx(x3) predict(outcome(2))

```
Average marginal effects                        Number of obs     =      1,880
Model VCE    : OIM

Expression   : Pr(status==2), predict(outcome(2))
dy/dx w.r.t. : educ exper 1.black

------------------------------------------------------------------------------
             |            Delta-method
             |      dy/dx   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
        1.x3 |   .1633938    .021143     7.73   0.000     .1219543    .2048332
------------------------------------------------------------------------------
```

Appendix G

Statistical Tables

TABLE G.1

Cumulative Areas under the Standard Normal Distribution

z	0	1	2	3	4	5	6	7	8	9
−3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
−2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
−2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
−2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
−2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
−2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
−2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
−2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
−2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
−2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
−2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
−1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
−1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
−1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
−1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
−1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
−1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
−1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
−1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
−1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
−1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
−0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
−0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
−0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
−0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
−0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
−0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
−0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483

(continued)

TABLE G.1 (Continued)

z	0	1	2	3	4	5	6	7	8	9
−0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
−0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
−0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990

Examples: If Z ~ Normal(0,1), then $P(Z \leq -1.32) = .0934$ and $P(Z \leq 1.84) = .9671$.
Source: This table was generated using the Stata® function normprob.

TABLE G.2

Critical Values of the *t* Distribution

		Significance Level				
	1-Tailed:	**.10**	**.05**	**.025**	**.01**	**.005**
	2-Tailed:	**.20**	**.10**	**.05**	**.02**	**.01**
Degrees of Freedom	1	3.078	6.314	12.706	31.821	63.657
	2	1.886	2.920	4.303	6.965	9.925
	3	1.638	2.353	3.182	4.541	5.841
	4	1.533	2.132	2.776	3.747	4.604
	5	1.476	2.015	2.571	3.365	4.032
	6	1.440	1.943	2.447	3.143	3.707
	7	1.415	1.895	2.365	2.998	3.499
	8	1.397	1.860	2.306	2.896	3.355
	9	1.383	1.833	2.262	2.821	3.250
	10	1.372	1.812	2.228	2.764	3.169
	11	1.363	1.796	2.201	2.718	3.106
	12	1.356	1.782	2.179	2.681	3.055
	13	1.350	1.771	2.160	2.650	3.012
	14	1.345	1.761	2.145	2.624	2.977
	15	1.341	1.753	2.131	2.602	2.947
	16	1.337	1.746	2.120	2.583	2.921
	17	1.333	1.740	2.110	2.567	2.898
	18	1.330	1.734	2.101	2.552	2.878
	19	1.328	1.729	2.093	2.539	2.861
	20	1.325	1.725	2.086	2.528	2.845
	21	1.323	1.721	2.080	2.518	2.831
	22	1.321	1.717	2.074	2.508	2.819
	23	1.319	1.714	2.069	2.500	2.807
	24	1.318	1.711	2.064	2.492	2.797
	25	1.316	1.708	2.060	2.485	2.787
	26	1.315	1.706	2.056	2.479	2.779
	27	1.314	1.703	2.052	2.473	2.771
	28	1.313	1.701	2.048	2.467	2.763
	29	1.311	1.699	2.045	2.462	2.756
	30	1.310	1.697	2.042	2.457	2.750
	40	1.303	1.684	2.021	2.423	2.704
	60	1.296	1.671	2.000	2.390	2.660
	90	1.291	1.662	1.987	2.368	2.632
	120	1.289	1.658	1.980	2.358	2.617
	∞	1.282	1.645	1.960	2.326	2.576

Examples: The 1% critical value for a one-tailed test with 25 *df* is 2.485. The 5% critical value for a two-tailed test with large (> 120) *df* is 1.96.
Source: This table was generated using the Stata® function invttail.

TABLE G.3a

10% Critical Values of the *F* Distribution

Denominator Degrees of Freedom	Numerator Degrees of Freedom: 1	2	3	4	5	6	7	8	9	10
10	3.29	2.92	2.73	2.61	2.52	2.46	2.41	2.38	2.35	2.32
11	3.23	2.86	2.66	2.54	2.45	2.39	2.34	2.30	2.27	2.25
12	3.18	2.81	2.61	2.48	2.39	2.33	2.28	2.24	2.21	2.19
13	3.14	2.76	2.56	2.43	2.35	2.28	2.23	2.20	2.16	2.14
14	3.10	2.73	2.52	2.39	2.31	2.24	2.19	2.15	2.12	2.10
15	3.07	2.70	2.49	2.36	2.27	2.21	2.16	2.12	2.09	2.06
16	3.05	2.67	2.46	2.33	2.24	2.18	2.13	2.09	2.06	2.03
17	3.03	2.64	2.44	2.31	2.22	2.15	2.10	2.06	2.03	2.00
18	3.01	2.62	2.42	2.29	2.20	2.13	2.08	2.04	2.00	1.98
19	2.99	2.61	2.40	2.27	2.18	2.11	2.06	2.02	1.98	1.96
20	2.97	2.59	2.38	2.25	2.16	2.09	2.04	2.00	1.96	1.94
21	2.96	2.57	2.36	2.23	2.14	2.08	2.02	1.98	1.95	1.92
22	2.95	2.56	2.35	2.22	2.13	2.06	2.01	1.97	1.93	1.90
23	2.94	2.55	2.34	2.21	2.11	2.05	1.99	1.95	1.92	1.89
24	2.93	2.54	2.33	2.19	2.10	2.04	1.98	1.94	1.91	1.88
25	2.92	2.53	2.32	2.18	2.09	2.02	1.97	1.93	1.89	1.87
26	2.91	2.52	2.31	2.17	2.08	2.01	1.96	1.92	1.88	1.86
27	2.90	2.51	2.30	2.17	2.07	2.00	1.95	1.91	1.87	1.85
28	2.89	2.50	2.29	2.16	2.06	2.00	1.94	1.90	1.87	1.84
29	2.89	2.50	2.28	2.15	2.06	1.99	1.93	1.89	1.86	1.83
30	2.88	2.49	2.28	2.14	2.05	1.98	1.93	1.88	1.85	1.82
40	2.84	2.44	2.23	2.09	2.00	1.93	1.87	1.83	1.79	1.76
60	2.79	2.39	2.18	2.04	1.95	1.87	1.82	1.77	1.74	1.71
90	2.76	2.36	2.15	2.01	1.91	1.84	1.78	1.74	1.70	1.67
120	2.75	2.35	2.13	1.99	1.90	1.82	1.77	1.72	1.68	1.65
∞	2.71	2.30	2.08	1.94	1.85	1.77	1.72	1.67	1.63	1.60

Example: The 10% critical value for numerator $df = 2$ and denominator $df = 40$ is 2.44.
Source: This table was generated using the Stata® function invFtail.

TABLE G.3b

5% Critical Values of the *F* Distribution

Denominator Degrees of Freedom	Numerator Degrees of Freedom: 1	2	3	4	5	6	7	8	9	10
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90	2.85
12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80	2.75
13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71	2.67
14	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65	2.60
15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59	2.54
16	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54	2.49
17	4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.49	2.45
18	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46	2.41
19	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42	2.38
20	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39	2.35
21	4.32	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.37	2.32
22	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34	2.30
23	4.28	3.42	3.03	2.80	2.64	2.53	2.44	2.37	2.32	2.27
24	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30	2.25
25	4.24	3.39	2.99	2.76	2.60	2.49	2.40	2.34	2.28	2.24
26	4.23	3.37	2.98	2.74	2.59	2.47	2.39	2.32	2.27	2.22
27	4.21	3.35	2.96	2.73	2.57	2.46	2.37	2.31	2.25	2.20
28	4.20	3.34	2.95	2.71	2.56	2.45	2.36	2.29	2.24	2.19
29	4.18	3.33	2.93	2.70	2.55	2.43	2.35	2.28	2.22	2.18
30	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21	2.16
40	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12	2.08
60	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04	1.99
90	3.95	3.10	2.71	2.47	2.32	2.20	2.11	2.04	1.99	1.94
120	3.92	3.07	2.68	2.45	2.29	2.17	2.09	2.02	1.96	1.91
∞	3.84	3.00	2.60	2.37	2.21	2.10	2.01	1.94	1.88	1.83

Example: The 5% critical value for numerator $df = 4$ and large denominator df (∞) is 2.37.
Source: This table was generated using the Stata® function invFtail.

TABLE G-3c

1% Critical Values of the *F* Distribution

Denominator Degrees of Freedom	Numerator Degrees of Freedom: 1	2	3	4	5	6	7	8	9	10
10	10.04	7.56	6.55	5.99	5.64	5.39	5.20	5.06	4.94	4.85
11	9.65	7.21	6.22	5.67	5.32	5.07	4.89	4.74	4.63	4.54
12	9.33	6.93	5.95	5.41	5.06	4.82	4.64	4.50	4.39	4.30
13	9.07	6.70	5.74	5.21	4.86	4.62	4.44	4.30	4.19	4.10
14	8.86	6.51	5.56	5.04	4.69	4.46	4.28	4.14	4.03	3.94
15	8.68	6.36	5.42	4.89	4.56	4.32	4.14	4.00	3.89	3.80
16	8.53	6.23	5.29	4.77	4.44	4.20	4.03	3.89	3.78	3.69
17	8.40	6.11	5.18	4.67	4.34	4.10	3.93	3.79	3.68	3.59
18	8.29	6.01	5.09	4.58	4.25	4.01	3.84	3.71	3.60	3.51
19	8.18	5.93	5.01	4.50	4.17	3.94	3.77	3.63	3.52	3.43
20	8.10	5.85	4.94	4.43	4.10	3.87	3.70	3.56	3.46	3.37
21	8.02	5.78	4.87	4.37	4.04	3.81	3.64	3.51	3.40	3.31
22	7.95	5.72	4.82	4.31	3.99	3.76	3.59	3.45	3.35	3.26
23	7.88	5.66	4.76	4.26	3.94	3.71	3.54	3.41	3.30	3.21
24	7.82	5.61	4.72	4.22	3.90	3.67	3.50	3.36	3.26	3.17
25	7.77	5.57	4.68	4.18	3.85	3.63	3.46	3.32	3.22	3.13
26	7.72	5.53	4.64	4.14	3.82	3.59	3.42	3.29	3.18	3.09
27	7.68	5.49	4.60	4.11	3.78	3.56	3.39	3.26	3.15	3.06
28	7.64	5.45	4.57	4.07	3.75	3.53	3.36	3.23	3.12	3.03
29	7.60	5.42	4.54	4.04	3.73	3.50	3.33	3.20	3.09	3.00
30	7.56	5.39	4.51	4.02	3.70	3.47	3.30	3.17	3.07	2.98
40	7.31	5.18	4.31	3.83	3.51	3.29	3.12	2.99	2.89	2.80
60	7.08	4.98	4.13	3.65	3.34	3.12	2.95	2.82	2.72	2.63
90	6.93	4.85	4.01	3.54	3.23	3.01	2.84	2.72	2.61	2.52
120	6.85	4.79	3.95	3.48	3.17	2.96	2.79	2.66	2.56	2.47
∞	6.63	4.61	3.78	3.32	3.02	2.80	2.64	2.51	2.41	2.32

Example: The 1% critical value for numerator $df = 3$ and denominator $df = 60$ is 4.13.
Source: This table was generated using the Stata® function invFtail.

TABLE G-4

Critical Values of the Chi-Square Distribution

Degrees of Freedom	Significance Level: .10	.05	.01
1	2.71	3.84	6.63
2	4.61	5.99	9.21
3	6.25	7.81	11.34
4	7.78	9.49	13.28
5	9.24	11.07	15.09
6	10.64	12.59	16.81
7	12.02	14.07	18.48
8	13.36	15.51	20.09
9	14.68	16.92	21.67
10	15.99	18.31	23.21
11	17.28	19.68	24.72
12	18.55	21.03	26.22
13	19.81	22.36	27.69
14	21.06	23.68	29.14
15	22.31	25.00	30.58
16	23.54	26.30	32.00
17	24.77	27.59	33.41
18	25.99	28.87	34.81
19	27.20	30.14	36.19
20	28.41	31.41	37.57
21	29.62	32.67	38.93
22	30.81	33.92	40.29
23	32.01	35.17	41.64
24	33.20	36.42	42.98
25	34.38	37.65	44.31
26	35.56	38.89	45.64
27	36.74	40.11	46.96
28	37.92	41.34	48.28
29	39.09	42.56	49.59
30	40.26	43.77	50.89

Example: The 5% critical value with $df = 8$ is 15.51.
Source: This table was generated using the Stata® function invchi2tail.

Critical Values of the ADF test Statistic

Model	Significance Level (%)	Critical Values
No constant, no Trend	1	-2.5658
	5	-1.9393
	10	-1.6156
Constant, no trend	1	-3.4336
	5	-2.8621
	10	-2.5671
Constant + trend	1	-3.9638
	5	-3.4126
	10	-3.1279

Critical Values of the Engle Granger test Statistic

Number of regressors	Model	Significance Level (%)	Critical Values
1	Constant, no trend	1	-3.9001
		5	-3.3377
		10	-3.0462
1	Constant + trend	1	-4.3266
		5	-3.7809
		10	-3.4959
2	Constant, no trend	1	-4.2981
		5	-3.7429
		10	-3.4518
2	Constant + trend	1	-4.6676
		5	-4.1193
		10	-3.8344
3	Constant, no trend	1	-4.6493
		5	-4.1
		10	-3.811
3	Constant + trend	1	-4.9695
		5	-4.4294
		10	-4.1474
4	Constant, no trend	1	-4.9587
		5	-4.4185
		10	-4.1327
4	Constant + trend	1	-5.2497
		5	-4.7154
		10	-4.4345
5	Constant, no trend	1	-5.24
		5	-4.7048
		10	-4.4242
5	Constant +trend	1	-5.5127
		5	-4.9767
		10	-4.6999

Source: James MacKinnon (1991) "Critical Values for Cointegration Tests", in Engle, R. and Granger, C. Long-Run Economic Relationships, Oxford University Press.