# UNIVERSITY OF LISBON 

# ISEG- Lisbon School of Economics and Management 

Exam - January 2019

Advanced Econometrics

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Use an answer booklet of ISEG (folha de teste do ISEG)

Instructions (please read before starting): Write in a clear legible manner in ink/ballpoint. Do not use pencils or erasable pens. Calculators are permitted. If you are asked to derive something, give all intermediate steps also. Do not answer questions with a "yes" or "no" only, but carefully justify your answer.

## Section A - Topics in Microeconometrics

## Question 1

Consider a random sample $\left\{\left(Y_{i}, X_{i}^{\prime}\right)^{\prime}\right\}_{i=1}^{n}$ and the following binary choice model:

$$
p_{i} \equiv P\left[Y_{i}=1 \mid X_{i}\right]=G\left(X_{i}^{\prime} \beta\right) \quad i=1, \ldots, n,
$$

where $Y_{i}$ is a binary random variable that can take values 0 or $1, X_{i}$ is a $k$-vector of explanatory variables, and $\beta$ is a $k$-vector of parameters.
(a) (2 marks) Suppose that the function $G(\cdot)$ is such that $G\left(X_{i}^{\prime} \beta\right)=X_{i}^{\prime} \beta$. Address the problems that would arise by estimating the above model by Ordinary Least Squares. Discuss how you could overcome these problems by imposing suitable restrictions on the function $G(\cdot)$.
(b) (2 marks) Discuss the Latent Variable Threshold Model, including any identification issue that might arise. Show how this framework allows to model $P\left[Y_{i}=1 \mid X_{i}\right]$.
(c) Consider now the Logit model and $k=1$ :
(i) (2 marks) Show that the expected value of the score vector evaluated at the true value of the parameter is zero.
(ii) (2 marks) Obtain the marginal effect for the Logit model $\partial E\left(Y_{i} \mid X_{i}\right) / \partial X_{i}$, and discuss how it is related to the parameter $\beta$.

## Section B - Topics in Time Series

## Question 2

Consider the stationary $M A(2)$ process $Y_{t}=c+\varepsilon_{t}+\theta_{1} \varepsilon_{t-1}+\theta_{2} \varepsilon_{t-2}$, where $\varepsilon_{t}$ is a white noise process with variance $\sigma_{\varepsilon}^{2}$ and $c$ is a constant.
(a) (2 marks) Derive an expression for $\mu=E\left(Y_{t}\right)$.
(b) (2 marks) Derive an expression for $\gamma_{0}=\operatorname{var}\left(Y_{t}\right)$ and for $\rho_{j}=\gamma_{j} / \gamma_{0}$, where $\gamma_{j}=$ $E\left[\left(Y_{t}-\mu\right)\left(Y_{t-j}-\mu\right)\right],(j=1,2, \ldots)$.
(c) (2 marks) Write down the $A R(\infty)$ representation of $Y_{t}^{*}=Y_{t}-E\left(Y_{t}\right)$ assuming that it is invertible.

## Question 3

Consider the $\operatorname{VAR}(1)$ model $z_{t}=\Phi_{1} z_{t-1}+\varepsilon_{t}$ where $z_{t}=\left(z_{1 t}, z_{2 t}\right)^{\prime}$ and $\varepsilon_{t}$ is a $2 \times 1$ vector of white noise processes with

$$
\operatorname{var}\left(\varepsilon_{t}\right)=\left[\begin{array}{cc}
\sigma_{1}^{2} & \sigma_{12} \\
\sigma_{12} & \sigma_{2}^{2}
\end{array}\right] .
$$

(a) (2 marks) Let

$$
\Phi_{1}=\left[\begin{array}{cc}
0.4 & 0.2 \\
-0.2 & 0.8
\end{array}\right]
$$

Obtain the roots of the characteristic equation and show that the process is stationary.
(b) (2 marks) Obtain the values of the elements of the matrices $\Psi_{\ell}$, for $\ell=0,1,2$ in the infinite moving average representation

$$
z_{t}=\sum_{\ell=0}^{\infty} \Psi_{\ell} \varepsilon_{t-\ell} .
$$

(c) (2 marks) Obtain the impulse response function for $z_{1 t}$ to a shock to the variable $z_{2 t}$ of size $\sigma_{2}$, for horizons $\ell=0,1,2$.

## [END OF PAPER]

