UNIVERSITY OF LISBON

ISEG- Lisbon School of Economics and Management

Exam - January 2020

Advanced Econometrics

Module Convenor:
Paulo M.D.C. Parente

Use answer booklets of ISEG (folhas de teste do ISEG)

Instructions (please read before starting): Write in a clear legible manner in ink/ballpoint. Do not use pencils or erasable pens. Approved calculators are permitted. Only one sheet (2 sides A4) of notes made exclusively by the student may be consulted (no material distributed by the teacher in any form is allowed). Whenever conducting a test use a 5% significance level unless stated otherwise. Also be sure to state null and alternative hypotheses, null distribution (with degrees of freedom), rejection criterion (critical values and rejection region) and outcome. If you are asked to derive something, give all intermediate steps also. Do not answer questions with a "yes" or "no" only, but carefully justify your answer.

Section A- Topics in Microeconometrics

1. Consider the random variable Y, defined as the number of occurrences of a particular phenomenon in a given period of time, and the vector of exogenous regressors X. Let D be an unobservable binary random variable. It is further known that:

$$
\begin{aligned}
P(Y=j|X,D=0) &= g(j,X,\beta), \quad j=0,1,2,\ldots \\
P(Y=0|X,D=1) &= 1, \\
E(Y|X,D) &= (1-D)\theta(X,\beta), \\
P(D=1|X) &= f(X,\gamma),
\end{aligned}
$$

where $g(j,X,\beta)$, $\theta(X,\beta)$ and $f(X,\gamma)$ are known functions. Assume that a random sample $\left\{(Y_i,X_i')'\right\}_{i=1}^{n}$ is available.

(a) *(2 Marks)* Write down the log-likelihood function for this problem.

(b) *(2 Marks)* Assume that $g(j,\, X,\, \beta)$ is unknown and that $\theta(X,\beta)$ and $f(X,\gamma)$ are known and correctly specified. Explain how γ and β can be estimated.

(c) *(3 Marks)* Assume that the function $f(X,\gamma)$ is unknown. Explain how β can be estimated.

(d) *(2 Marks)* Assume again that the function $f(X,\gamma)$ is known and that that D is observable and that a random sample $\left\{(D_i,X_i')'\right\}_{i=1}^{n}$ is available. Explain how γ can be estimated.

(e) *(1 Marks)* Assume that $f(X,\gamma)=\gamma$ where $\gamma\in[0,1]$ and that D is unobservable again. Explain briefly the difficulty that arises when the standard Wald and likelihood ratio statistics are used to test the hypothesis $H_0:\gamma=0$.

Section B - Topics in Time Series

2. Consider the GARCH(1,2) model,

$$
\begin{aligned}
x_t|F_{t-1} &\sim N(0,\sigma_t^2), \\
\sigma_t^2 &= \alpha_0+\alpha_1 x_{t-1}^2+\beta_1\sigma_{t-1}^2+\beta_2\sigma_{t-2}^2,
\end{aligned}
$$

where F_{t-1} is the the information set of all information until time $t-1$.

(a) *(2 marks)* State the conditions for stationarity of x_t and derive the marginal mean and variance of x_t.

(b) *(2 marks)* Show that the GARCH(1,2) process implies the following ARMA(2,2) representation for x_t^2 under the conditions stated in (a):

$$
\begin{aligned}
x_t^2 &= \alpha_0+(\alpha_1+\beta_1)x_{t-1}^2+\beta_2 x_{t-2}^2+\omega_t \\
&\quad -\beta_1\omega_{t-1}-\beta_2\omega_{t-2}
\end{aligned}
$$

where $E\left[\omega_t\right]=0$ and $cov\left(\omega_t,\omega_{t-j}\right)=0$ for $j=1,2,\ldots$

(c) *(2 marks)* Consider the multi-step forecast for the volatility of x_t. Define $\sigma_h^2(\ell) = var(x_{h+\ell}|F_h), \ell > 2$. Show that for $\ell > 2$ we have

$$\sigma_h^2(\ell) = \alpha_0 + (\alpha_1 + \beta_1)\sigma_h^2(\ell - 1) + \beta_2\sigma_h^2(\ell - 2)$$

3. In a number of different papers, Campbell and Shiller investigated the time-series relationship between stock prices and dividends. In this question we address this issue based on monthly data, 1927–1990, on log-dividends d_t and log-prices s_t of the S&P500 index. According to the theory d_t and s_t are cointegrated.

(a) *(1 Marks)* We first investigate whether d_t is an $I(1)$ process, by carrying out an augmented Dickey-Fuller test (the variable is called $LOGREALDIV$ in the table below). The results of the ADF regression are the following

Variable	Coefficient	Std. Error
LOGREALDIV(-1)	-0.005234	0.001708
D(LOGREALDIV(-1))	0.697754	0.035551
D(LOGREALDIV(-2))	0.064585	0.043297
D(LOGREALDIV(-3))	-0.142999	0.043307
D(LOGREALDIV(-4))	0.176805	0.035678
C	0.010636	0.003476
@TREND(1927M01)	5.67E-06	2.26E-06

where the dependent variable is $D(LOGREALDIV) = LOGREALDIV_t - LOGREALDIV_{t-1}$, C denotes an intercept and @TREND(1927M01) denotes a time trend. What do you conclude about the presence of a unit root ?

(b) *(2 Marks)* Assume that both s_t and d_t are $I(1)$. We test for cointegration between s_t (called $LOGREALPRICE$ in the table below) and d_t using Johansen's procedure. This leads to the following output:

Series: LOGREALPRICE LOGREALDIV
Unrestricted Cointegration Rank Test (Trace)

Hypothesized No. of CE(s)	Eigenvalue	Trace Statistic	0.05 Critical Value	Prob.
None	0.039330	33.29837	15.49471	0.0000
At most 1	0.003228	2.483051	3.841466	0.1151

1 Cointegrating Equation(s): Log likelihood 3867.032

Normalized cointegrating coefficients (standard error in parentheses)

LOGREALPRIC	LOGREALDIV
1.000000	-1.721045
	(0.14532)

What do you conclude about the number of cointegrating relations?

(c) *(1 Marks)* Do you find evidence for stationarity of $(d_t - s_t)$?

[END OF PAPER]