Master in Actuarial Science

Models in Finance - Computational Exam

04-02-2020

Time allowed: One and a half hours (90 minutes)

Instructions:

1. This paper contains 3 questions and comprises 3 pages including the title page.

2. You have 5 minutes of reading time. You must not start working in your answers until instructed to do so.

3. Marks are shown in brackets. Total marks: 50.

4. The Formulae and Tables for Actuarial Examinations (the 2002 edition) may be used and also the document about the R Package fOptions - fOptions: Rmetrics - Pricing and Evaluating Basic Options.

5. You should save and present all your R Scripts, Console outputs and plots. All the files should be saved with your name in the file name.

1. Consider the recombining Binomial model with one risky asset which pays no dividends, such that the factor u (up step factor of the price) is $u = 1.04$ per time period and consider 20 time periods. the initial value of the risky asset is $S_0 = 30$ and the interest rate is 6% per year and the time until maturity 9 months. In order to answer the following questions, use the R package fOptions and the appropriate R functions implemented in this package. In order to answer the following questions, use the R package fOptions and the appropriate R functions implemented in this package.

   (a) Calculate the annual volatility for this Binomial model and calculate the price of an American put option with strike $K = 32$ and time until maturity 9 months. (8)

   (b) Plot the binomial tree for the option prices considered in part (a) but now for 10 periods. Compare the price of the American put option with the price of the European put option with the same maturity and strike. If the same procedure is applied to an European call option and an American call option, what would be the result? (7)

2. Consider the Black-Scholes model with one risky asset which pays a continuously compounded dividend rate q.

   (a) Implement in R a function which prices put options at time $t = 0$, where the function depends on the parameters $x = S_0$ (initial price of the risky asset), maturity time T, interest rate r, volatility σ, dividend rate q and strike K. Implement this function without using any R package for option pricing. (9)

   (b) Use the function you implemented in part (a) to price a put option for $S_0 = 70$, $K = 74$, $r = 0.05$, time until maturity 15 months, volatility $\sigma = 0.25$ and dividend rate $q = 0.02$. Calculate also the delta for the put option at time 0 for initial price S_0 and for an initial price very close to S_0 (you can use here the fOptions package functions) and use these values of the delta to estimate the greek "gamma". Compare this gamma with the one obtained by using the appropriate function of the fOptions package. (10)

3. Consider the real prices of European call options on the S&P 500 index (SPX) with $S_0 = 1921.42$,(price at October 1, 2015), annual interest rate $r = 0.002$, dividend yield $q = 0.02$, and maturity on October 16, 2015. These option prices are listed in the file real_data.csv for different values of the strike. In this question you can use appropriate functions defined in the package fOptions.

(a) For the call option with strike $K = 1900$ and real price 47.9, calculate the implied volatility. Calculate also the implied volatility for the call option with strike $K = 1500$ and real price 425.6. (6)

(b) Calculate the implied volatilities for all the call options in the file real_prices_calls.csv and plot the implied volatilities against the strike. Comment the results. (10)
Hint - you can consider a function with the following structure in order to generate a vector with the implied volatilities:
volatilites <- sapply(seq_along(real_data$strike), function(i) GBSVolatility(price = real_data$ask[i], ...))