UNIVERSITY OF LISBON

ISEG- Lisbon School of Economics and Management

Exam - January 2021

Advanced Econometrics

Module Convenor:
Paulo M.D.C. Parente

Use answer booklets of ISEG (folhas de teste do ISEG)

APPENDIX: TABLE OF THE STANDARD NORMAL DENSITY FUNCTION

Instructions (please read before starting): Write in a clear legible manner in ink/ballpoint. Do not use pencils or erasable pens. Approved calculators are permitted. Only one sheet (2 sides A4) of notes made exclusively by the student may be consulted (no material distributed by the teacher in any form is allowed). Whenever conducting a test use a 5% significance level unless stated otherwise. Also be sure to state null and alternative hypotheses, null distribution (with degrees of freedom), rejection criterion (critical values and rejection region) and outcome. If you are asked to derive something, give all intermediate steps also. Do not answer questions with a "yes" or "no" only, but carefully justify your answer.

1. Let $X \sim Gamma(\alpha, \theta)$ and $(X_1, X_2, ..., X_n)$ be a random sample drawn from the distribution of X. The probability density function of the $Gamma(\alpha, \theta)$ random variable given by

$$f(x|\alpha, \theta) = \begin{cases} \left(\frac{1}{\Gamma(\alpha)\theta^\alpha}\right) x^{\alpha-1} \exp\{-x/\theta\} & x > 0 \\ 0 & x \leq 0 \end{cases},$$

where

$$\Gamma(\alpha) = \int_0^{+\infty} e^{-y} y^{\alpha-1} dy.$$

We know that $\mathrm{E}(X) = \alpha\theta$ and $\mathrm{Var}(X) = \alpha\theta^2$. Assume that the value of $\alpha > 0$ is known.

   (a) *(Marks 2)* Find the maximum likelihood estimator $\hat{\theta}$ of θ based on a random sample $(X_1, ..., X_n)$ using the first order conditions.

   (b) *(Marks 2)* Calculate $\mathrm{E}(\hat{\theta})$ and $\mathrm{Var}(\hat{\theta})$.

   (c) *(Mark 1)* Show that the expected value of $s(X, \theta) = \partial \log[f(X|\alpha, \theta)]/\partial\theta$ is zero.

   (d) *(Marks 2)* Show that the Information identity holds when the true probability density function of X is given by $f(x|\alpha, \theta)$.

2. A researcher would like to explain the probability of mobile phone users buying a new mobile phone in 2007. He considers the model

$$\mathcal{P}(mobile = 1|inc, agem.young) = \Phi(\beta_1 + \beta_2 inc + \beta_3 agem + \beta_4 young),$$

where $\Phi(.)$ is the cumulative distribution function of the standard normal random variable and

mobile - is a binary variable that take value 1
if the individual bought a new mobile phone in 2007 and zero otherwise,
inc - income of the individual in thousands of euros,
agem - age of the previous mobile phone
young - dummy variable that take value 1 if the individual is less than 30 years old and zero otherwise.

He obtains the following results using a random sample of size 325.

Equation 1

Probit Estimation

Dependent variable: mobile

Number of observations=325	Scaled R-Squared=0.049440
Number of positive obs.=153	LR (zero slopes)= 16.1803 [0.001]
Mean of dep. Var.=0.470769	Scwarz B.I.C. =228.195
Sum of squared residuals=77.1431	Log likelihood =-216.627
R-Squared=0.047302	
Fraction of Correct Predictions=0.612308	

Parameters	Estimate	Standard Error	t-statistic	p-value
Intercept	-0.267766	0.324233	-0.825843	0.409
inc	-0.037365	0.024116	-1.54943	0.121
agem	0.095022	0.029242	3.24949	0.001
young	0.450669	0.200742	2.24502	0.025

Equation 2

Probit Estimation

Dependent variable: mobile

Number of observations=325	Scaled R-Squared=0.014329
Number of positive obs.=153	LR (zero slopes)= 4.66612 [0.031]
Mean of dep. Var.=0.470769	Scwarz B.I.C. =228.168
Sum of squared residuals=79.8279	Log likelihood =-222.384
R-Squared=0.014134	
Fraction of Correct Predictions=0.553846	

Parameters	Estimate	Standard Error	t-statistic	p-value
Intercept	0.431738	0.244898	1.76293	0.078
inc	-0.050723	0.023598	-2.14946	0.032

(a) Using the results in equation 1, compute the approximate estimates of the following quantities and interpreted the results obtained:

i. *(Mark 1)* $\partial E(mobile|inc, agem, young)/\partial inc$ when $inc = 12$, $agem = 2$ and $young = 1$.

ii. *(Mark 1)* $\partial E(mobile|inc, agem, young)/\partial agem$ when $inc = 12$, $agem = 2$ and $young = 0$.

iii. *(Mark 1)*

$$E(mobile|inc, agem, young = 1) - E(mobile|inc, agem, young = 0)$$

when $inc = 12$, $agem = 2$.

(b) *(Mark 1)* Test the joint hypothesis that the variables *agem* and *young* are not relevant.

3. *(Marks 3)* Consider the random variable Y, defined as the number of times that an individual went to a restaurant in the previous week. Assume that for part of the population the distribution of Y conditional on X is Poisson with parameter $\lambda = \exp(X'\beta_0)$, where X is a vector of exogenous regressors. For the remaining individuals of the population, for professional reasons, the variable Y is always greater than 4. The probability that an individual belongs to this latter part of the population conditional on X is given by π_0, where π_0 does not vary with X. Unfortunately, the variable Y is observed with errors so large that cannot be used for estimation purposes. However, we can observe without errors the binary variable $D = I\left(Y > 0\right)$, where $I(\cdot)$ is the indicator function. Assume that a random sample $\left\{(D_i, X_i')'\right\}_{i=1}^n$ drawn from the distribution of $(D, X')'$ is available. We would like to estimate π_0 and β_0. Obtain $P\left(D = 1|X\right)$ and write down the log-likelihood function of this problem.

4. Consider the stationary $ARMA(1,1)$ process

$$Y_t = c + \phi_1 Y_{t-1} + \varepsilon_t + \theta_1 \varepsilon_{t-1},$$

where $|\phi_1| < 1$, $|\theta_1| < 1$ and ε_t is a white noise process with mean zero and variance σ_ε^2.

(a) *(Mark 1)* Derive an expression for $\mu = E(Y_t)$.

(b) *(Marks 3)* Derive and expression for $\gamma_0 = var(Y_t)$ and for $\rho_j = \gamma_j/\gamma_0$ where $\gamma_j = E\left[(Y_t - \mu)(Y_{t-j} - \mu)\right]$, $(j = 1, 2, ...)$.

(c) *(Mark 1)* Write down the $MA(\infty)$ representation of the $ARMA(1,1)$ process.

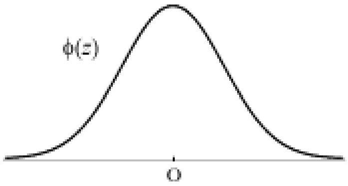
(d) *(Mark 1)* Write down the $AR(\infty)$ representation of the $ARMA(1,1)$ process.

[END OF PAPER]

APPENDIX

TABLE– STANDARD NORMAL DENSITY FUNCTION

$$\phi(z) = \frac{1}{\sqrt{2\pi}} e^{-z^2/2}$$



z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.0	.3989	.3989	.3989	.3988	.3986	.3984	.3982	.3980	.3977	.3973
.1	.3970	.3965	.3961	.3956	.3951	.3945	.3939	.3932	.3925	.3918
.2	.3910	.3902	.3894	.3885	.3876	.3867	.3857	.3847	.3836	.3825
.3	.3814	.3802	.3790	.3778	.3765	.3752	.3739	.3725	.3712	.3697
.4	.3683	.3668	.3653	.3637	.3621	.3605	.3589	.3572	.3555	.3538
.5	.3521	.3503	.3485	.3467	.3448	.3429	.3410	.3391	.3372	.3352
.6	.3332	.3312	.3292	.3271	.3251	.3230	.3209	.3187	.3166	.3144
.7	.3123	.3101	.3079	.3056	.3034	.3011	.2989	.2966	.2943	.2920
.8	.2897	.2874	.2850	.2827	.2803	.2780	.2756	.2732	.2709	.2685
.9	.2661	.2637	.2613	.2589	.2565	.2541	.2516	.2492	.2468	.2444
1.0	.2420	.2396	.2371	.2347	.2323	.2299	.2275	.2251	.2227	.2203
1.1	.2179	.2155	.2131	.2107	.2083	.2059	.2036	.2012	.1989	.1965
1.2	.1942	.1919	.1895	.1872	.1849	.1826	.1804	.1781	.1758	.1736
1.3	.1714	.1691	.1669	.1647	.1626	.1604	.1582	.1561	.1539	.1518
1.4	.1497	.1476	.1456	.1435	.1415	.1394	.1374	.1354	.1334	.1315
1.5	.1295	.1276	.1257	.1238	.1219	.1200	.1182	.1163	.1145	.1127
1.6	.1109	.1092	.1074	.1057	.1040	.1023	.1006	.0989	.0973	.0957
1.7	.0940	.0925	.0909	.0893	.0878	.0863	.0848	.0833	.0818	.0804
1.8	.0790	.0775	.0761	.0748	.0734	.0721	.0707	.0694	.0681	.0669
1.9	.0656	.0644	.0632	.0620	.0608	.0596	.0584	.0573	.0562	.0551
2.0	.0540	.0529	.0519	.0508	.0498	.0488	.0478	.0468	.0459	.0449
2.1	.0440	.0431	.0422	.0413	.0404	.0396	.0387	.0379	.0371	.0363
2.2	.0355	.0347	.0339	.0332	.0325	.0317	.0310	.0303	.0297	.0290
2.3	.0283	.0277	.0270	.0264	.0258	.0252	.0246	.0241	.0235	.0229
2.4	.0224	.0219	.0213	.0208	.0203	.0198	.0194	.0189	.0184	.0180
2.5	.0175	.0171	.0167	.0163	.0158	.0154	.0151	.0147	.0143	.0139
2.6	.0136	.0132	.0129	.0126	.0122	.0119	.0116	.0113	.0110	.0107
2.7	.0104	.0101	.0099	.0096	.0093	.0091	.0088	.0086	.0084	.0081
2.8	.0079	.0077	.0075	.0073	.0071	.0069	.0067	.0065	.0063	.0061
2.9	.0060	.0058	.0056	.0055	.0053	.0051	.0050	.0048	.0047	.0046
3.0	.0044	.0043	.0042	.0040	.0039	.0038	.0037	.0036	.0035	.0034
3.1	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026	.0025	.0025
3.2	.0024	.0023	.0022	.0022	.0021	.0020	.0020	.0019	.0018	.0018
3.3	.0017	.0017	.0016	.0016	.0015	.0015	.0014	.0014	.0013	.0013
3.4	.0012	.0012	.0012	.0011	.0011	.0010	.0010	.0010	.0009	.0009