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Valuing companies by cash flow discounting: ten methods and nine theories

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Abstract

Purpose – The aim of this paper is to answer the question: Do discounted cash flows valuation methods provide always the same value?

Design/methodology/approach – This paper is a summarized compendium of ten methods including: free cash flow; equity cash flow; capital cash flow; adjusted present value; business's risk-adjusted free cash flow and equity cash flow; risk-free rate-adjusted free cash flow and equity cash flow; economic profit; and economic value added.

Findings – All ten methods always give the same value.

Research limitations/implications – The disagreements among the various theories of firm valuation arise from the calculation of the value of the tax shields (VTS). The paper analyses nine different theories.

Originality/value – The paper is an analysis of ten methods of company valuation using discounted cash flows and nine different theories about the VTS.

Keywords Cash flow, Organizations, Discounted cash flow

Paper type Conceptual paper

1. Introduction

This paper is a summarized compendium of all the methods and theories on company valuation using discounted cash flows.

Section 2 shows the ten most commonly used methods for valuing companies by discounted cash flows:

(1) free cash flow discounted at the weighted average cost of capital (WACC);
(2) equity cash flows discounted at the required return to equity;
(3) capital cash flows discounted at the WACC before tax;
(4) adjusted present value (APV);
(5) the business's risk-adjusted free cash flows discounted at the required return to assets;
(6) the business's risk-adjusted equity cash flows discounted at the required return to assets;
(7) economic profit discounted at the required return to equity;
(8) economic value added (EVA) discounted at the WACC;

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(9) the risk-free rate-adjusted free cash flows discounted at the risk-free rate; and
(10) the risk-free rate-adjusted equity cash flows discounted at the required return to assets.

All ten methods always give the same value. This result is logical, since all the methods analyze the same reality under the same hypotheses; they differ only in the cash flows taken as the starting point for the valuation.

In section 3 the ten methods and nine theories are applied to an example. The nine theories are:

(1) No-cost-of-leverage. Assuming that there are no leverage costs. This theory appears in Fernández (2004a);
(2) Damodaran (1994). To introduce leverage costs, Damodaran assumes that the relationship between the levered and unlevered beta is [1]: $\beta_L = \beta u + D(1-T)\beta u/E$;
(3) Practitioners method. To introduce higher leverage costs, this method assumes that the relationship between the levered and unlevered beta is: $\beta_L = \beta u + D\,\beta u/E$;
(4) Harris and Pringle (1985) and Ruback (1995). All of their equations arise from the assumption that the leverage-driven value creation or value of tax shields (VTS) is the present VTS [2] discounted at the required return to the unlevered equity (Ku). According to them, $\mathrm{VTS} = \mathrm{PV}[D\mathrm{Kd}T; \mathrm{Ku}]$;
(5) Myers (1974), who assumes that the value of tax shields (VTS) is the present VTS discounted at the required return to debt (Kd). According to Myers:

$$\mathrm{VTS} = \mathrm{PV}[D\mathrm{Kd}T; \mathrm{Kd}]$$

(6) Miles and Ezzell (1980). They state that the correct rate for discounting the tax shield ($D\mathrm{Kd}T$) is Kd for the first year, and Ku for the following years;
(7) Miller (1977) concludes that the leverage-driven value creation or VTS is zero;
(8) With-cost-of leverage. This theory assumes that the cost of leverage is the present value of the interest differential that the company pays over the risk-free rate; and
(9) Modigliani and Miller (1963) calculate the VTS by discounting the present value of the tax savings due to interest payments of a risk-free debt (TDR_F) at the risk-free rate (R_F). Modigliani and Miller claim that:

$$\mathrm{VTS} = \mathrm{PV}[R_F; DTR_F].$$

Appendix 1 gives a brief overview of the most significant theories on discounted cash flow valuation.

Appendix 2 contains the valuation equations according to these theories.

Appendix 3 shows how the valuation equations change if the debt's market value is not equal to its nominal value.

Appendix 4 contains a list of the abbreviations used in the paper.

2. Ten discounted cash flow methods for valuing companies

There are four basic methods for valuing companies by discounted cash flows:

Method 1. Using the free cash flow and the WACC

Equation (1) indicates that the value of the debt (D) plus that of the shareholders' equity (E) is the present value of the expected free cash flows (FCF) that the company will generate, discounted at the weighted average cost of debt and shareholders' equity after tax (WACC):

$$E_0 + D_0 = \mathrm{PV}_0[\mathrm{WACC}_t; \mathrm{FCF}_t] \quad (1)$$

The definition of WACC or "weighted average cost of capital" is given by equation (2):

$$\mathrm{WACC}_t = \frac{[E_{t-1}K\mathrm{e}_t + D_{t-1}\mathrm{Kd}_t(1-T)]}{[E_{t-1} + D_{t-1}]} \quad (2)$$

Ke is the required return to equity, Kd is the cost of the debt, and T is the effective tax rate applied to earnings. $E_{t-1} + D_{t-1}$ are market values[3].

Method 2. Using the expected equity cash flow (ECF) and the required return to equity (Ke)

Equation (3) indicates that the value of the equity (E) is the present value of the expected equity cash flows (ECF) discounted at the required return to equity (Ke):

$$E_0 = \mathrm{PV}_0[\mathrm{Ke}_t; \mathrm{ECF}_t] \quad (3)$$

Equation (4) indicates that the value of the debt (D) is the present value of the expected debt cash flows (CFd) discounted at the required return to debt (Kd):

$$D_0 = \mathrm{PV}_0[\mathrm{Kd}_t; \mathrm{CFd}_t] \quad (4)$$

The expression that relates the FCF with the ECF is[4]:

$$\mathrm{ECF}_t = \mathrm{FCF}_t + \Delta D_t - I_t(1-T) \quad (5)$$

ΔD_t is the increase in debt and I_t is the interest paid by the company. It is obvious that $\mathrm{CFd} = I_t - \Delta D_t$

The sum of the values given by equations (3) and (4) is identical to the value provided by equation (1)[5]:

$$E_0 + D_0 = \mathrm{PV}_0[\mathrm{WACC}_t; \mathrm{FCF}_t] = \mathrm{PV}_0[\mathrm{Ke}_t; \mathrm{ECF}_t] + \mathrm{PV}_0[\mathrm{Kd}_t; \mathrm{CFd}_t].$$

Method 3. Using the capital cash flow (CCF) and the $WACC_{BT}$ (weighted average cost of capital, before tax)

The capital cash flows[6] are the cash flows available for all holders of the company's securities, whether these be debt or shares, and are equivalent to the ECF plus the cash flow corresponding to the debt holders (CFd).

Equation (6) indicates that the value of the debt today (D) plus that of the shareholders' equity (E) is equal to the capital cash flow (CCF) discounted at the weighted average cost of debt and shareholders' equity before tax ($\mathrm{WACC}_{\mathrm{BT}}$):

$$E_0 + D_0 = \mathrm{PV}[\mathrm{WACC}_{\mathrm{BT}t}; \mathrm{CCF}_t] \tag{6}$$

The definition of $\mathrm{WACC}_{\mathrm{BT}}$ is equation (7):

$$\mathrm{WACC}_{\mathrm{BT}t} = \frac{[E_{t-1}\mathrm{Ke}_t + D_{t-1}\mathrm{Kd}_t]}{[E_{t-1} + D_{t-1}]} \tag{7}$$

The expression (7) is obtained by making equation (1) equal to equation (6). $\mathrm{WACC}_{\mathrm{BT}}$ represents the discount rate that ensures that the value of the company obtained using the two expressions is the same[7]:

$$E_0 + D_0 = \mathrm{PV}[\mathrm{WACC}_{\mathrm{BT}t}; \mathrm{CCF}_t] = \mathrm{PV}[\mathrm{WACC}_t; \mathrm{FCF}_t]$$

The expression that relates the CCF with the ECF and the FCF is equation (8):

$$\begin{aligned} &\mathrm{CCF}_t = \mathrm{ECF}_t + \mathrm{CFd}_t = \mathrm{ECF}_t - \Delta D_t + I_t = \mathrm{FCF}_t + I_t T \\ &\Delta D_t = D_t - D_{t-1}; \quad I_t = D_{t-1}\mathrm{Kd}_t \end{aligned} \tag{8}$$

Method 4. Adjusted present value (APV)

The APV equation (9) indicates that the value of the debt (D) plus that of the shareholders' equity (E) is equal to the value of the unlevered company's shareholders' equity, Vu, plus the present value of the value of the tax shield (VTS):

$$E_0 + D_0 = \mathrm{Vu}_0 + \mathrm{VTS}_0 \tag{9}$$

We can see in Appendixes 1 and 2 that there are several theories for calculating the VTS.

If Ku is the required return to equity in the debt-free company (also called the required return to assets), Vu is given by equation (10):

$$\mathrm{Vu}_0 = \mathrm{PV}_0[\mathrm{Ku}_t; \mathrm{FCF}_t] \tag{10}$$

Consequently:

$$\mathrm{VTS}_0 = E_0 + D_0 - \mathrm{Vu}_0 = \mathrm{PV}_0[\mathrm{WACC}_t; \mathrm{FCF}_t] - \mathrm{PV}_0[\mathrm{Ku}_t; \mathrm{FCF}_t]$$

We can talk of a fifth method (using the business risk-adjusted free cash flow), although this is not actually a new method but is derived from the previous methods:

Method 5. Using the business risk-adjusted free cash flow and Ku (required return to assets)

Equation (11) indicates that the value of the debt (D) plus that of the shareholders' equity (E) is the present value of the expected business risk-adjusted free cash flows (FCF\\Ku) that will be generated by the company, discounted at the required return to assets (Ku):

$$E_0 + D_0 = \mathrm{PV}_0[\mathrm{Ku}_t; \mathrm{FCF}_t\backslash\backslash\mathrm{Ku}] \tag{11}$$

The definition of the business risk-adjusted free cash flows[8] (FCF\\Ku) is equation (12):

$$\mathrm{FCF}_t \backslash\backslash \mathrm{Ku} = \mathrm{FCF}_t - (E_{t-1} + D_{t-1})[\mathrm{WACC}_t - \mathrm{Ku}_t] \tag{12}$$

Likewise, we can talk of a sixth method (using the business risk-adjusted equity cash flow), although this is not actually a new method but is derived from the previous methods:

Method 6. Using the business risk-adjusted equity cash flow and Ku (required return to assets)

Equation (13) indicates that the value of the equity (E) is the present value of the expected business risk-adjusted equity cash flows (ECF\\Ku) discounted at the required return to assets (Ku):

$$E_0 = \mathrm{PV}_0[\mathrm{Ku}_t; \mathrm{ECF}_t \backslash\backslash \mathrm{Ku}] \tag{13}$$

The definition of the business risk-adjusted equity cash flows [9] (ECF\\Ku) is equation (14):

$$\mathrm{ECF}_t \backslash\backslash \mathrm{Ku} = \mathrm{ECF}_t - E_{t-1}[\mathrm{Ke}_t - \mathrm{Ku}_t] \tag{14}$$

Method 7. Using the economic profit and Ke (required return to equity)

Equation (15) indicates that the value of the equity (E) is the equity's book value plus the present value of the expected economic profit (EP) discounted at the required return to equity (Ke):

$$E_0 = \mathrm{Ebv}_0 + \mathrm{PV}_0[\mathrm{Ke}_t; \mathrm{EP}_t] \tag{15}$$

The term economic profit (EP) is used to define the accounting net income or profit after tax (PAT) less the equity's book value (Ebv_{t-1}) multiplied by the required return to equity:

$$\mathrm{EP}_t = \mathrm{PAT}_t - \mathrm{KeEbv}_{t-1} \tag{16}$$

Method 8. Using the EVA (economic value added) and the WACC (weighted average cost of capital)

Equation (17) indicates that the value of the debt (D) plus that of the shareholders' equity (E) is the book value of the shareholders' equity and the debt ($\mathrm{Ebv}_0 + N_0$) plus the present value of the expected EVA, discounted at the weighted average cost of capital (WACC):

$$E_0 + D_0 = (\mathrm{Ebv}_0 + N_0) + \mathrm{PV}_0[\mathrm{WACC}_t; \mathrm{EVA}_t] \tag{17}$$

The EVA is the Net Operating Profit After Tax (NOPAT) less the company's book value ($D_{t-1} + \mathrm{Ebv}_{t-1}$) multiplied by the WACC. The NOPAT is the profit of the unlevered company (debt-free):

$$\mathrm{EVA}_t = \mathrm{NOPAT}_t - (D_{t-1} + \mathrm{Ebv}_{t-1})\mathrm{WACC}_t \tag{18}$$

Method 9. Using the risk-free-adjusted free cash flows discounted at the risk-free rate
Equation (19) indicates that the value of the debt (D) plus that of the shareholders' equity (E) is the present value of the expected risk-free-adjusted free cash flows (FCF\\ R_F) that will be generated by the company, discounted at the risk-free rate (R_F):

$$E_0 + D_0 = \mathrm{PV}_0[R_{Ft}; \mathrm{FCF}_t \backslash\backslash R_F] \tag{19}$$

The definition of the risk-free-adjusted free cash flows [10] (FCF\\R_F) is equation (20):

$$\mathrm{FCF}_t \backslash\backslash R_F = \mathrm{FCF}_t - (E_{t-1} + D_{t-1})[\mathrm{WACC}_t - R_{Ft}] \tag{20}$$

Likewise, we can talk of a tenth method (using the risk-free-adjusted equity cash flow), although this is not actually a new method but is derived from the previous methods:

Method 10. Using the risk-free-adjusted equity cash flows discounted at the risk-free rate
Equation (21) indicates that the value of the equity (E) is the present value of the expected risk-free-adjusted equity cash flows (ECF\\R_F) discounted at the risk-free rate (R_F):

$$E_0 = \mathrm{PV}_0[R_{Ft}; \mathrm{ECF}_t \backslash\backslash R_F] \tag{21}$$

The definition of the risk-free-adjusted equity cash flows [11] (ECF\\R_F) is equation (22):

$$\mathrm{ECF}_t \backslash\backslash R_F = \mathrm{ECF}_t - E_{t-1}[\mathrm{Ke}_t - R_{Ft}] \tag{22}$$

We could also talk of an 11th method; using the business risk-adjusted capital cash flow and Ku (required return to assets), but the business risk-adjusted capital cash flow is identical to the business risk-adjusted free cash flow (CCF\\Ku = FCF\\Ku). Therefore, this method would be identical to Method 5.

We could also talk of a 12th method; using the risk-free-adjusted capital cash flow and R_F (risk-free rate), but the risk-free-adjusted capital cash flow is identical to the risk-free-adjusted free cash flow (CCF\\$R_F$ = FCF\\$R_F$). Therefore, this method would be identical to Method 9.

3. An example: valuation of the company Toro Inc.

The company Toro Inc. has the balance sheet and income statement forecasts for the next few years shown in Table I. After year 3, the balance sheet and the income statement are expected to grow at an annual rate of 2 per cent.

Using the balance sheet and income statement forecasts in Table I, we can readily obtain the cash flows given in Table II. Obviously, the cash flows grow at a rate of 2 per cent after year 4.

The unlevered beta (βu) is one. The risk-free rate is 6 per cent. The cost of debt is 8 per cent. The corporate tax rate is 35 per cent. The market risk premium is 4 per cent. Consequently, using the CAPM, the required return to assets is 10 per cent[12]. With these parameters, the valuation of this company's equity, using the above equations, is given in Table III. The required return to equity (Ke) appears in the second line of the table[13]. Equation (3) enables the value of the equity to be obtained by discounting the equity cash flows at the required return to equity (Ke)[14]. Likewise, equation (4)

	0	1	2	3	4	5
Working capital requirements (WCR)	400	430	515	550	561.00	572.22
Gross fixed assets	1,600	1,800	2,300	2,600	2,913.00	3,232.26
Accumulated depreciation		200	450	720	995.40	1,276.31
Net fixed assets	1,600	1,600	1,850	1,880	1,917.60	1,955.95
Total assets	2,000	2,030	2,365	2,430	2,478.60	2,528
Debt (*N*)	1,500	1,500	1,500	1,500	1,530.00	1,560.60
Equity (book value)	500	530	865	930	948.60	967.57
Total liabilities	2,000	2,030	2,365	2,430	2,478.60	2,528
Income statement						
Margin		420	680	740	765.00	780
Interest payments		120	120	120	120.00	122
Profit before tax (PBT)		300	560	620	645.00	658
Taxes		105	196	217	225.75	230.27
PAT (profit after tax = net income)		195	364	403	419.25	427.64

Table I. Balance sheet and income statement forecasts for Toro Inc.

	1	2	3	4	5
PAT (profit after tax)	195	364	403	419.25	427.64
+ depreciation	200	250.00	270.00	275.40	280.91
+ increase of debt	0	0.00	0.00	30.00	30.60
− increase of working capital requirements	−30	−85	−35	−11	−11.22
− investment in fixed assets	−200	−500.00	−300.00	−313.00	−319.26
ECF	165.00	29.00	338.00	400.65	408.66
FCF	243.00	107.00	416.00	448.65	457.62
CFd	120.00	120.00	120.00	90.00	91.80
CCF	285.00	149.00	458.00	490.65	500.46

Table II. Cash flow forecasts for Toro Inc.

enables the value of the debt to be obtained by discounting the debt cash flows at the required return to debt (Kd)[15]. Another way to calculate the value of the equity is using equation (1). The present value of the free cash flows discounted at the WACC (equation (2)) gives us the value of the company, which is the value of the debt plus that of the equity[16]. By subtracting the value of the debt from this quantity, we obtain the value of the equity. Another way of calculating the value of the equity is using equation (6). The present value of the capital cash flows discounted at the $WACC_{BT}$ (equation (7)) gives us the value of the company, which is the value of the debt plus that of the equity. By subtracting the value of the debt from this quantity, we obtain the value of the equity. The fourth method for calculating the value of the equity is using the APV, equation (9). The value of the company is the sum of the value of the unlevered company (equation (10)) plus the present value of the VTS[17].

The business risk-adjusted equity cash flow and free cash flow (ECF\\Ku and FCF\\Ku) are also calculated using equations (14) and (12). Equation (13) enables us to obtain the value of the equity by discounting the business risk-adjusted equity cash

Equation		0	1	2	3	4	5
	Ku	10.00%	10.00%	10.00%	10.00%	10.00%	10.00%
	Ke	10.49%	10.46%	10.42%	10.41%	10.41%	10.41%
(1)	$E+D=$ PV(WACC; FCF)	5,458.96	5,709.36	6,120.80	6,264.38	6,389.66	6,517.46
(2)	WACC	9.04%	9.08%	9.14%	9.16%	9.16%	9.16%
	$(1)-D=E$	3,958.96	4,209.36	4,620.80	4,764.38	4,859.66	4,956.86
(3)	$E=$ PV(Ke; ECF)	3,958.96	4,209.36	4,620.80	4,764.38	4,859.66	4,956.86
(4)	$D=$ PV(CFd; Kd)	1,500.00	1,500.00	1,500.00	1,500.00	1,530.00	1,560.60
(6)	$D+E=$ PV($\mathrm{WACC_{BT}}$; CCF)	5,458.96	5,709.36	6,120.80	6,264.38	6,389.66	6,517.46
(7)	$\mathrm{WACC_{BT}}$	9.81%	9.82%	9.83%	9.83%	9.83%	9.83%
	$(6)-D=E$	3,958.96	4,209.36	4,620.80	4,764.38	4,859.66	4,956.86
	VTS = PV(Ku; DT Ku)	623.61	633.47	644.32	656.25	669.38	682.76
(10)	Vu = PV(Ku; FCF)	4,835.35	5,075.89	5,476.48	5,608.12	5,720.29	5,834.69
(9)	VTS + Vu	5,458.96	5,709.36	6,120.80	6,264.37	6,389.66	6,517.46
	$(9)-D=E$	3,958.96	4,209.36	4,620.80	4,764.37	4,859.66	4,956.86
(11)	$D+E=$ PV(Ku; FCF\\Ku)	5,458.96	5,709.36	6,120.80	6,264.37	6,389.66	6,517.46
(12)	FCF\\Ku		295.50	159.50	468.50	501.15	511.17
	$(11)-D=E$	3,958.96	4,209.36	4,620.80	4,764.38	4,859.66	4,956.86
(13)	$E=$ PV(Ku; ECF\\Ku)	3,958.96	4,209.36	4,620.80	4,764.38	4,859.66	4,956.86
(14)	ECF\\Ku		145.50	9.50	318.50	381.15	388.77
(16)	EP		142.54	308.54	312.85	322.44	328.89
	PV(Ke; EP)	3,458.96	3,679.36	3,755.80	3,834.38	3,911.06	3,989.28
(15)	PV(Ke; EP) + Ebv $=E$	3,958.96	4,209.36	4,620.80	4,764.38	4,859.66	4,956.86
(18)	EVA		92.23	257.67	264.79	274.62	280.11
	PV(WACC; EVA)	3,458.96	3,679.36	3,755.80	3,834.38	3,911.06	3,989.28
(17)	$E=$ PV(WACC; EVA) + Ebv $+N-D$	3,958.96	4,209.36	4,620.80	4,764.38	4,859.66	4,956.86
(19)	$D+E=$ PV(R_F; FCF\\R_F)	5,458.96	5,709.36	6,120.80	6,264.38	6,389.66	6,517.46
(20)	FCF\\R_F		77.14	−68.87	223.67	250.58	255.59
	$(19)-D=E$	3,958.96	4,209.36	4,620.80	4,764.38	4,859.66	4,956.86
(21)	$E=$ PV(R_F; ECF\\R_F)	3,958.96	4,209.36	4,620.80	4,764.38	4,859.66	4,956.86
(22)	ECF\\R_F		−12.86	−158.87	133.67	190.58	194.39

Table III.
Valuation of Toro Inc.
No-cost-of-leverage

flows at the required return to assets (Ku). Another way to calculate the value of the equity is using equation (11). The present value of the business risk-adjusted free cash flows discounted at the required return to assets (Ku) gives us the value of the company, which is the value of the debt plus that of the equity. By subtracting the value of the debt from this quantity, we obtain the value of the equity.

The economic profit (EP) is calculated using equation (16). Equation (15) indicates that the value of the equity (E) is the equity's book value plus the present value of the expected economic profit (EP) discounted at the required return to equity (Ke).

The EVA is calculated using equation (18). Equation (17) indicates that the equity value (E) is the present value of the expected EVA discounted at the WACC, plus the book value of the equity and the debt ($\mathrm{Ebv}_0 + N_0$) minus the value of the debt (D).

The risk-free-adjusted equity cash flow and free cash flow (ECF\\$R_F$ and FCF\\$R_F$) are also calculated using equations (22) and (20). Equation (21) enables us to obtain the value of the equity by discounting the risk-free-adjusted equity cash flows at the risk-free rate (R_F). Another way to calculate the value of the equity is using equation (19). The present value of the risk-free-adjusted free cash flows discounted at the required return to assets (R_F) gives us the value of the company, which is the value of the debt plus that of the equity. By subtracting the value of the debt from this quantity, we obtain the value of the equity.

Table III shows that the result obtained with all ten valuations is the same. The value of the equity today is 3,958.96. As we have already mentioned, these valuations have been performed according to the No-cost-of-leverage theory. The valuations performed using other theories are discussed further on.

Tables IV to XI contain the most salient results of the valuation performed on the company Toro Inc. according to Damodaran (1994), Practitioners method, Harris and Pringle (1985), Myers (1974), Miles and Ezzell (1980), Miller (1977), With-cost-of-leverage theory, and Modigliani and Miller (1963).

Table XII is a compendium of the valuations of Toro Inc. performed according to the nine theories. It can be seen that Modigliani and Miller gives the highest equity value (4,080.75) and Miller the lowest (3,335.35). Note that Modigliani and Miller and Myers yield a higher equity value than the No-cost-of-leverage theory. This result is inconsistent, as discussed in Fernández (2002).

Table XIII is the valuation of Toro Inc. if the growth after year 3 were 5.6 per cent instead of 2 per cent. Modigliani and Miller and Myers provide a required return to equity (Ke) lower than the required return to unlevered equity (Ku = 10 per cent), which is an inconsistent result because it does not make any economic sense.

4. How is the company valued when it reports losses in one or more years?

In such cases, we must calculate the tax rate that the company will pay, and this is the rate that must be used to perform all the calculations. It is as if the tax rate were the rate obtained after subtracting the taxes that the company must pay.

Example. The company Campa S.A. reports a loss in year 1. The tax rate is 35 per cent. In year 1, it will not pay any tax as it has suffered losses amounting to 220 million. In year

	0	1	2	3	4	5
VTS = PV[Ku; DTKu − D(Kd − R_F) (1 − T)]	391.98	398.18	405.00	412.50	420.75	429.16
β_L	1.261581	1.245340	1.222528	1.215678	1.215678	1.215678
Ke	11.05	10.98	10.89	10.86	10.86	10.86
E	3,727.34	3,974.07	4,381.48	4,520.62	4,611.04	4,703.26
WACC	9.369	9.397	9.439	9.452	9.452	9.452
$WACC_{BT}$	10.172	10.164	10.153	10.149	10.149	10.149
$E + D$	5,227.34	5,474.07	5,881.48	6,020.63	6,141.04	6,263.86
EVA		85.63	251.24	257.77	267.57	272.92
EP		139.77	305.80	308.80	318.23	324.59
ECF\\Ku		126.00	−10.00	299.00	361.65	368.88
FCF\\Ku		276.00	140.00	449.00	481.65	491.28
ECF\\R_F		−23.09	−168.96	123.74	180.83	184.44
FCF\\R_F		66.91	−78.96	213.74	240.83	245.64

Table IV. Valuation of Toro Inc. according to Damodaran (1994)

	0	1	2	3	4	5
VTS = PV[Ku; TDKd− D(Kd−R_F)]	142.54	144.79	147.27	150.00	153.00	156.06
β_L	1.431296	1.403152	1.363747	1.352268	1.352268	1.352268
Ke	11.73	11.61	11.45	11.41	11.41	11.41
E	3,477.89	3,720.68	4,123.75	4,258.13	4,343.29	4,430.15
WACC	9.759	9.770	9.787	9.792	9.792	9.792
$WACC_{BT}$	10.603	10.575	10.533	10.521	10.521	10.521
$E+D$	4,977.89	5,220.68	5,623.75	5,758.13	5,873.29	5,990.75
EVA		77.82	243.67	249.55	259.31	264.50
EP		136.37	302.45	303.91	313.15	319.41
ECF\\Ku		105.00	−31.00	278.00	340.65	347.46
FCF\\Ku		255.00	119.00	428.00	460.65	469.86
ECF\\R_F		−34.12	−179.83	113.05	170.33	173.73
FCF\\R_F		55.88	−89.83	203.05	230.33	234.93

Table V.
Valuation of Toro Inc. according to the Practitioners method

	0	1	2	3	4	5
VTS = PV[Ku; TDKd]	498.89	506.78	515.45	525.00	535.50	546.21
β_L	1.195606	1.183704	1.166966	1.161878	1.161878	1.161878
Ke	10.78	10.73	10.67	10.65	10.65	10.65
E	3,834.24	4,082.67	4,491.93	4,633.12	4,725.79	4,820.30
WACC	9.213	9.248	9.299	9.315	9.315	9.315
$WACC_{BT}$ = Ku	10.000	10.000	10.000	10.000	10.000	10.000
$E+D$	5,334.24	5,582.67	5,991.93	6,133.12	6,255.79	6,380.90
EVA		88.75	254.27	261.08	270.89	276.31
EP		141.09	307.11	310.72	320.23	326.63
ECF\\Ku		135.00	−1.00	308.00	370.65	378.06
FCF\\Ku		285.00	149.00	458.00	490.65	500.46
ECF\\R_F		−18.37	−164.31	128.32	185.33	189.03
FCF\\R_F		71.63	−74.31	218.32	245.33	250.23

Table VI.
Valuation of Toro Inc. according to Harris and Pringle (1985) and Ruback (1995)

2, it will pay corporate tax amounting to 35 per cent of that year's profit less the previous year's losses (350-220). The resulting tax is 45.5, that is, 13 per cent of the EBT for year 2. Consequently, the effective tax rate is zero in year 1, 13 per cent in year 2, and 35 per cent in the other years.

5. Conclusion

The paper shows the ten most commonly used methods for valuing companies by discounted cash flows always give the same value. This result is logical, since all the

	0	1	2	3	4	5
VTS = PV (Kd; DKdT)	663.92	675.03	687.04	700.00	714.00	728.28
β_L	1.104529	1.097034	1.087162	1.083193	1.083193	1.083193
Ke	10.42	10.39	10.35	10.33	10.33	10.33
E	3,999.27	4,250.92	4,663.51	4,808.13	4,904.29	5,002.37
WACC	8.995	9.035	9.096	9.112	9.112	9.112
$WACC_{BT}$	9.759	9.765	9.777	9.778	9.778	9.778
$E+D$	5,499.27	5,750.92	6,163.51	6,308.12	6,434.29	6,562.97
EVA		93.10	258.59	265.89	275.82	281.34
EP		142.91	308.94	313.48	323.16	329.62
ECF\\Ku		148.28	12.50	321.74	384.65	392.34
FCF\\Ku		298.28	162.50	471.74	504.65	514.74
ECF\\R_F		−11.69	−157.54	135.20	192.33	196.17
FCF\\R_F		78.31	−67.54	225.20	252.33	257.37

Table VII. Valuation of Toro Inc. according to Myers (1974)

	0	1	2	3	4	5
VTS = PV [Ku; TDKd] (1 + Ku)/(1 + Kd)	508.13	516.16	525.00	534.72	545.42	556.33
β_L	1.190077	1.178530	1.162292	1.157351	1.157351	1.157351
Ke	10.76%	10.71%	10.65%	10.63%	10.63%	10.63 %
E	3,843.5	4,092.1	4,501.5	4,642.8	4,735.7	4,830.4
WACC	9.199%	9.235%	9.287%	9.304%	9.304%	9.304%
$WACC_{BT}$	9.985%	9.986%	9.987%	9.987%	9.987%	9.987%
$E+D$	5,343.48	5,592.05	6,001.48	6,142.85	6,265.70	6,391.02
EVA		89.01	254.53	261.36	271.17	276.60
EP		141.20	307.22	310.88	320.40	326.80
ECF\\Ku		135.78	−0.22	308.78	371.43	378.86
FCF\\Ku		285.78	149.78	458.78	491.43	501.26
ECF\\R_F		−17.96	−163.90	128.72	185.71	189.43
FCF\\R_F		72.04	−73.90	218.72	245.71	250.63

Table VIII. Valuation of Toro Inc. according to Miles and Ezzell

methods analyze the same reality under the same hypotheses; they differ only in the cash flows taken as the starting point for the valuation. The ten methods analyzed are:

(1) free cash flow discounted at the WACC;

(2) equity cash flows discounted at the required return to equity;

(3) capital cash flows discounted at the WACC before tax;

(4) APV (adjusted present value);

Table IX.
Valuation of Toro Inc. according to Miller

	0	1	2	3	4	5
VTS = 0	0	0	0	0	0	0
β_L	1.539673	1.503371	1.452662	1.438156	1.438156	1.438156
Ke	12.16%	12.01%	11.81%	11.75%	11.75%	11.75%
E = Vu	3,335.35	3,575.89	3,976.48	4,108.13	4,190.29	4,274.09
WACC = Ku	10.000%	10.000%	10.000%	10.000%	10.000%	10.000%
$\mathrm{WACC}_{\mathrm{BT}}$	10.869%	10.827%	10.767%	10.749%	10.749%	10.749%
$E + D$	4,835.35	5,075.89	5,476.48	5,608.13	5,720.29	5,834.69
EVA		73.00	239.00	244.50	254.25	259.34
EP		134.21	300.33	300.84	309.95	316.15
ECF\\Ku		93.00	−43.00	266.00	328.65	335.22
FCF\\Ku		243.00	107.00	416.00	448.65	457.62
ECF\\R_F		−40.41	−186.04	106.94	164.33	167.61
FCF\\R_F		49.59	−96.04	196.94	224.33	228.81

Table X.
Valuation of Toro Inc. according to the With-cost-of-leverage theory

	0	1	2	3	4	5
VTS = PV[Ku; $D(\mathrm{Ku}T + R_F - \mathrm{Kd})$]	267.26	271.49	276.14	281.25	286.88	292.61
β_L	1.343501	1.321648	1.290998	1.281931	1.281931	1.281931
Ke	11.37%	11.29%	11.16%	11.13%	11.13%	11.13%
E	3,602.61	3,847.38	4,252.61	4,389.38	4,477.16	4,566.71
WACC	9.559%	9.579%	9.609%	9.618%	9.618%	9.618%
$\mathrm{WACC}_{\mathrm{BT}}$	10.382%	10.365%	10.339%	10.331%	10.331%	10.331%
$E + D$	5,102.61	5,347.38	5,752.61	5,889.38	6,007.16	6,127.31
EVA		81.82	247.54	253.75	263.53	268.80
EP		138.13	304.18	306.43	315.76	322.08
ECF\\Ku		115.50	−20.50	288.50	351.15	358.17
FCF\\Ku		265.50	129.50	438.50	471.15	480.57
ECF\\R_F		−28.60	−174.40	118.40	175.58	179.09
FCF\\R_F		61.40	−84.40	208.40	235.58	240.29

(5) the business's risk-adjusted free cash flows discounted at the required return to assets;

(6) the business's risk-adjusted equity cash flows discounted at the required return to assets;

(7) economic profit discounted at the required return to equity;

(8) EVA discounted at the WACC;

(9) the risk-free rate-adjusted free cash flows discounted at the risk-free rate; and

(10) the risk-free rate-adjusted equity cash flows discounted at the required return to assets.

	0	1	2	3	4	5
VTS = PV[R_F; $DR_F T$]	745.40	758.62	772.64	787.50	803.25	819.31
β_L	1.065454	1.058571	1.050506	1.045959	1.045959	1.045959
Ke	10.26%	10.23%	10.20%	10.18%	10.18%	10.18%
E	4,080.75	4,334.51	4,749.12	4,895.62	4,993.54	5,093.41
WACC	8.901%	8.940%	9.001%	9.015%	9.015%	9.015%
$WACC_{BT}$	9.654%	9.660%	9.673%	9.672%	9.672%	9.672%
$E + D$	5,580.75	5,834.51	6,249.12	6,395.62	6,523.54	6,654.01
EVA		94.97	260.52	268.12	278.19	283.75
EP		143.69	309.76	314.75	324.54	331.03
ECF\\Ku		154.32	18.84	328.41	391.65	399.48
FCF\\Ku		304.32	168.84	478.41	511.65	521.88
ECF\\R_F		−8.91	−154.54	138.44	195.83	199.74
FCF\\R_F		81.09	−64.54	228.44	255.83	260.94

Table XI. Valuation of Toro Inc. according to Modigliani and Miller

(Value in $t = 0$)	Equity value (E)	Value of tax shield (VTS)	Leverage cost	Ke $t = 0$ (%)	Ke $t = 4$ (%)
No-cost-of-leverage	3,958.96	623.61	0.00	10.49	10.41
Damodaran	3,727.34	391.98	231.63	11.05	10.86
Practitioners	3,477.89	142.54	481.07	11.73	11.41
Harris and Pringle	3,834.24	498.89	124.72	10.78	10.65
Myers	3,999.27	663.92	−40.31	10.42	10.33
Miles and Ezzell	3,843.48	508.13	115.48	10.76	10.63
Miller	3,335.35	0.00	623.61	12.16	11.75
With-cost-of-leverage	3,602.61	267.26	356.35	11.37	11.13
Modigliani and Miller	4,080.75	745.40	−121.79	10.26	10.18

Table XII. Valuation of Toro Inc. according to the nine theories

(Value in $t = 0$)	Equity value (E)	Value of tax shield (VTS)	Leverage cost	Ke $t = 0$ (%)	Ke $t = 4$ (%)
No-cost-of-leverage	6,615.67	1,027.01	0.00	10.29	10.23
Damodaran	6,234.21	645.55	381.46	10.63	10.50
Practitioners	5,823.40	234.75	792.27	11.03	10.81
Harris and Pringle	6,410.27	821.61	205.40	10.47	10.37
Myers	7,086.10	1,497.44	−470.43	10.00	9.94
Miles and Ezzell	6,425.48	836.83	190.19	10.45	10.36
Miller	5,588.66	0.00	1,027.01	11.29	11.01
With-cost-of-leverage	6,028.81	440.15	586.87	10.82	10.65
Modigliani and Miller	12,284.86	6,696.20	−5,669.19	8.15	8.17

Table XIII. Valuation of Toro Inc. according to the nine theories if growth after year 3 is 5.6 per cent instead of 2 per cent

The paper also analyzes nine different theories on the calculation of the VTS, which implies nine different theories on the relationship between the levered and the unlevered beta, and nine different theories on the relationship between the required return to equity and the required return to assets. The nine theories analyzed are:

(1) No-cost-of-leverage;
(2) Modigliani and Miller (1963);
(3) Myers (1974);
(4) Miller (1977);
(5) Miles and Ezzell (1980);
(6) Harris and Pringle (1985);
(7) Damodaran (1994);
(8) With-cost-of-leverage; and
(9) Practitioners method.

The disagreements among the various theories on the valuation of the firm arise from the calculation of the VTS. Using a simple example, we show that Modigliani and Miller (1963) and Myers (1974) provide inconsistent results.

The paper contains the most important valuation equations according to these theories (Appendix 2, Table AII to Table AV) and also shows how the valuation equations change if the debt's market value is not equal to its book value (Appendix 3, Table AVI and Table AVII).

Notes

1. Instead of the relationship obtained from No-cost-of-leverage: $\beta_L = \beta u + D(1-T)(\beta u - \beta d)/E$.
2. The tax shield of a given year is DKdT. D is the value of debt, Kd is the required return to debt, and T is the corporate tax rate. DKd are the interest paid in a given year. The formulas used in the paper are valid if the interest rate on the debt matches the required return to debt (Kd), or to put it another way, if the debt's market value is identical to its book value. The formulas for when this is not the case are given in Appendix 3.
3. In actual fact, "market values" are the values obtained when the valuation is performed using formula (1). Consequently, the valuation is an iterative process: the free cash flows are discounted at the WACC to calculate the company's value ($D+E$) but, in order to obtain the WACC, we need to know the company's value ($D+E$).
4. Obviously, the free cash flow is the hypothetical equity cash flow when the company has no debt.
5. Indeed, one way of defining the WACC is: the WACC is the rate at which the FCF must be discounted so that equation (2) gives the same result as that given by the sum of equations (3) and (4).
6. Arditti and Levy (1977) suggested that the firm's value could be calculated by discounting the capital cash flows instead of the free cash flow.
7. One way of defining the $\mathrm{WACC_{BT}}$ is: the $\mathrm{WACC_{BT}}$ is the rate at which the CCF must be discounted so that equation (6) gives the same result as that given by the sum of equations (3) and (4).
8. Expression (12) is obtained by making equation (11) equal to equation (1).

9. Expression (14) is obtained by making equation (13) equal to equation (3).
10. Expression (20) is obtained by making equation (19) equal to equation (1).
11. Expression (22) is obtained by making equation (21) equal to equation (3).
12. In this example, we use the CAPM: $\mathrm{Ku} = R_\mathrm{F} + \beta \mathrm{u} P_\mathrm{M} = 6$ per cent + 4 per cent = 10 per cent.
13. The required return to equity (Ke) has been calculated according to the No-cost-of-leverage theory (see Appendix 1).
14. The relationship between the value of the equity in two consecutive years is: $E_t = E_{t-1}(1 + \mathrm{Ke}_t) - \mathrm{ECF}_t$.
15. The value of the debt is equal to the nominal value (book value) given in Table I because we have considered that the required return to debt is equal to its cost (8 per cent).
16. The relationship between the company's value in two consecutive years is: $(D+E)_t = (D+E)_{t-1}\,(1 + \mathrm{WACC}_t) - \mathrm{FCF}_t$.
17. As the required return to equity (Ke) has been calculated according to the No-cost-of-leverage theory, we must also calculate the VTS according to the No-cost-of-leverage theory, namely: $\mathrm{VTS} = \mathrm{PV}(\mathrm{Ku};\, DT\mathrm{Ku})$.
18. See Damodaran (1994, p. 31).
19. One of the many places where it appears is Ruback (1995, p. 5).
20. This formula can be completed with another parameter φ that takes into account that the cost of leverage is not strictly proportional to debt. φ should be lower for small leverage and higher for high leverage. Introducing this parameter, the value of tax shields is $\mathrm{VTS} = \mathrm{PV}\,[\mathrm{Ku};\, DT\,\mathrm{Ku} - \varphi D(\mathrm{Kd} - R_\mathrm{F})]$.

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Further reading

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Appendix 1. A brief overview of the most significant papers on the discounted cash flow valuation of firms

There is a considerable body of literature on the discounted cash flow valuation of firms. We will now discuss the most salient papers, concentrating particularly on those that proposed different expressions for the present value of the tax savings due to the payment of interest or VTS. The main problem with most papers is that they consider the VTS as the present value of the tax savings due to the payment of interest. Fernández (2004a, b) argues and proves that the VTS is the difference between two present values: the present value of taxes paid by the unlevered firm and the present value of taxes paid by the levered firm.

Modigliani and Miller (1958) studied the effect of leverage on the firm's value. Their proposition 1 (1958, equation 3) states that, in the absence of taxes, the firm's value is independent of its debt, i.e.

$$E + D = \mathrm{Vu}, \text{if } T = 0 \tag{23}$$

E is the equity value, D is the debt value, Vu is the value of the unlevered company, and T is the tax rate.

In the presence of taxes and for the case of a perpetuity, they calculate the VTS by discounting the present value of the tax savings due to interest payments on a risk-free debt (TDR_F) at the risk-free rate (R_F). Their first proposition, with taxes, is transformed into Modigliani and Miller (1963, p. 436, equation 3):

$$E + D = \mathrm{Vu} + \mathrm{PV}[R_F; DTR_F] = \mathrm{Vu} + DT \tag{24}$$

DT is the VTS for perpetuity. This result is only correct for perpetuities. As Fernández (2004a) demonstrates, discounting the tax savings due to interest payments on a risk-free debt at the risk-free rate provides inconsistent results for growing companies. We have seen this in Table XIII.

Myers (1974) introduced the APV. According to Myers, the value of the levered firm is equal to the value of the firm with no debt (Vu) plus the present value of the tax saving due to the payment of interest (VTS). Myers proposes calculating the VTS by discounting the tax savings (DTKd) at the cost of debt (Kd). The argument is that the risk of the tax saving arising from the use of debt is the same as the risk of the debt. Therefore, according to Myers (1974):

$$\mathrm{VTS} = \mathrm{PV}[\mathrm{Kd}; DT\mathrm{Kd}] \tag{25}$$

Luehrman (1997) recommends valuing companies using the APV and calculates the VTS in the same way as Myers. This theory yields inconsistent results for growing companies, as shown in Fernández (2004a). Fernández (2004b) shows that this theory yields consistent results only if the company will not increase its debt.

Miller (1977) assumes no advantages of debt financing: "I argue that even in a world in which interest payments are fully deductible in computing corporate income taxes, the value of the firm, in equilibrium, will still be independent of its capital structure". According to Miller (1977), the value of the firm is independent of its capital structure, that is:

$$\mathrm{VTS} = 0 \tag{26}$$

According to Miles and Ezzell (1980), a firm that wishes to keep a constant D/E ratio must be valued in a different manner from a firm that has a preset level of debt. For a firm with a fixed debt target $[D/(D+E)]$, they claim that the correct rate for discounting the tax saving due to debt ($\mathrm{Kd}TD_{t-1}$) is Kd for the tax saving during the first year, and Ku for the tax saving during the following years. The expression of Ke is their equation (22):

$$\mathrm{Ke} = \frac{\mathrm{Ku} + D(\mathrm{Ku} - \mathrm{Kd})[1 + \mathrm{Kd}(1 - T)]}{[(1 + \mathrm{Kd})E]} \tag{27}$$

Although Miles and Ezzell do not mention what the VTS should be, equation (27) relating the required return to equity with the required return for the unlevered company implies that:

$$\mathrm{VTS} = \frac{\mathrm{PV}[\mathrm{Ku}; TD\mathrm{Kd}](1 + \mathrm{Ku})}{(1 + \mathrm{Kd})} \tag{28}$$

Lewellen and Emery (1986) also claim that the most logically consistent method is Miles and Ezzell.

Harris and Pringle (1985) propose that the present value of the tax saving due to the payment of interest (VTS) should be calculated by discounting the tax saving due to the debt ($\mathrm{Kd}TD$) at the rate Ku. Their argument is that the interest tax shields have the same systematic risk as the firm's underlying cash flows and, therefore, should be discounted at the required return to assets (Ku).

Therefore, according to Harris and Pringle (1985):

$$\mathrm{VTS} = \mathrm{PV}[\mathrm{Ku}; D\mathrm{Kd}T] \tag{29}$$

Harris and Pringle (1985, p. 242) say "the MM position is considered too extreme by some because it implies that interest tax shields are no more risky than the interest payments themselves. The Miller position is too extreme for some because it implies that debt cannot benefit the firm at all. Thus, if the truth about the VTS lies somewhere between the MM and Miller positions, a supporter of either Harris and Pringle or Miles and Ezzell can take comfort in the fact that both produce a result for unlevered returns between those of MM and Miller. A virtue of Harris and Pringle compared to Miles and Ezzell is its simplicity and straightforward intuitive explanation". Ruback (1995) reaches equations that are identical to those of Harris and Pringle (1985). Kaplan and Ruback (1995) also calculate the VTS "discounting interest tax shields at the discount rate for an all-equity firm". Tham and Vélez-Pareja (2001), following an arbitrage argument, also claim that the appropriate discount rate for the tax shield is Ku, the required return to unlevered equity. Fernández (2002) shows that Harris and Pringle (1985) provide inconsistent results.

Damodaran (1994, p. 31) argues that if all the business risk is borne by the equity, then the equation relating the levered beta (β_L) to the asset beta (βu) is:

$$\beta_L = \beta u + \left(\frac{D}{E}\right)\beta u(1-T) \tag{30}$$

It is important to note that equation (30) is exactly equation (22) assuming that $\beta d = 0$. One interpretation of this assumption is that "all of the firm's risk is borne by the stockholders (i.e. the beta of the debt is zero)"[18]. However, we think that it is difficult to justify that the debt has no risk (unless the cost of debt is the risk-free rate) and that the return on the debt is uncorrelated with the return on assets of the firm. We rather interpret equation (30) as an attempt to introduce some leverage cost in the valuation: for a given risk of the assets (βu), by using equation (30) we obtain a higher β_L (and consequently a higher Ke and a lower equity value) than with equation (22). Equation (30) appears in many finance books and is used by some consultants and investment banks.

Although Damodaran does not mention what the VTS should be, his equation (30) relating the levered beta to the asset beta implies that the VTS is:

$$\text{VTS} = \text{PV}[\text{Ku}; DT\text{Ku} - D(\text{Kd} - R_F)(1-T)] \tag{31}$$

Another way of calculating the levered beta with respect to the asset beta is the following:

$$\beta_L = \beta u\left(\frac{1+D}{E}\right) \tag{32}$$

We will call this method the Practitioners' method, because consultants and investment banks often use it[19]. It is obvious that according to this equation, given the same value for βu, a higher β_L (and a higher Ke and a lower equity value) is obtained than according to equation (22) and (30).

One should notice that equation (32) is equal to equation (30) eliminating the $(1-T)$ term. We interpret equation (32) as an attempt to introduce still higher leverage cost in the valuation: for a given risk of the assets (βu), by using equation (32) we obtain a higher β_L (and consequently a higher Ke and a lower equity value) than with equation (30).

Equation (32) relating the levered beta with the asset beta implies that the VTS is:

$$\text{VTS} = \text{PV}[\text{Ku}; DT\text{Kd} - D(\text{Kd} - R_F)] \tag{33}$$

By comparing equations (33) to (31) it can be seen that (33) provides a VTS, that is, PV[Ku; $DT(\text{Ku} - R_F)$] lower than equation (31). We interpret this difference as additional leverage cost (on top of the leverage cost of Damodaran) introduced in the valuation.

Inselbag and Kaufold (1997) argue that if the firm targets the dollar values of debt outstanding, the VTS is given by Myers (1974) equation. However, if the firm targets a constant debt/value ratio, the VTS is given by Miles and Ezzell (1980) equation.

Copeland *et al.* (2000) treat the APV in their Appendix A. They only mention perpetuities and only propose two ways of calculating the VTS: Harris and Pringle (1985) and Myers (1974). They conclude "we leave it to the reader's judgment to decide which approach best fits his or her situation". They also claim that "the finance literature does not provide a clear answer about which discount rate for the tax benefit of interest is theoretically correct". It is quite interesting to note that Copeland *et al.* (2000, p. 483) only suggest Inselbag and Kaufold (1997) as additional reading on APV.

We will consider two additional theories to calculate the VTS. We label these two theories No-costs-of-leverage and With-costs-of-leverage.

We label the first theory the No-costs-of-leverage equation because, as may be seen in Fernández (2004a), it is the only equation that provides consistent results when there are no leverage costs. According to this theory, the VTS is the present value of DTKu (not the interest tax shield) discounted at the unlevered cost of equity (Ku):

$$\mathrm{PV}[\mathrm{Ku}; DT\mathrm{Ku}] \tag{34}$$

Equation (34) is the result of considering that the VTS is the difference between two present values: the present value of taxes paid by the unlevered firm and the present value of taxes paid by the levered firm. It can be seen in Fernández (2004a).

Comparing equations (31) to (34), it can be seen that equation (31) provides a VTS, that is, $\mathrm{PV}[\mathrm{Ku}; D(\mathrm{Kd} - R_F)(1 - T)]$ lower than equation (34). We interpret this difference as leverage cost introduced in the valuation by Damodaran.

Comparing equations (33) to (34), it can be seen that equation (33) provides a VTS, that is, $\mathrm{PV}[\mathrm{Ku}; DT(\mathrm{Ku} - \mathrm{Kd}) + D(\mathrm{Kd} - R_F)]$ lower than equation (34). We interpret this difference as leverage cost introduced in the valuation by the Practitioners' method.

Fernández (2004b) shows that only two of them are correct:

(1) If the company expects to increase its debt, the VTS is the present value of $D\mathrm{Ku}T$ discounted at the required return to unlevered equity (Ku): $\mathrm{VTS} = \mathrm{PV}\,[D\mathrm{Ku}T; \mathrm{Ku}]$. See Fernández (2004a).

(2) If the company will not increase its debt, the VTS is: $\mathrm{PV}[DT\mathrm{Kd}; \mathrm{Kd}]$. See Myers (1974).

With-costs-of-leverage. This theory provides another way of quantifying the VTS:

$$\mathrm{VTS} = \mathrm{PV}[\mathrm{Ku}; D\mathrm{Ku}T - D(\mathrm{Kd} - R_F)] \tag{35}$$

One way of interpreting equation (35) is that the leverage costs (with respect to equation (34)) are proportional to the amount of debt and to the difference between the required return on debt and the risk-free rate[20].

By comparing equations (35) to (34), it can be seen that equation (40) provides a VTS, that is, $\mathrm{PV}[\mathrm{Ku}; D(\mathrm{Kd} - R_F)]$ lower than equation (34). We interpret this difference as leverage cost introduced in the valuation.

Table AI provides a synthesis of the nine theories about the VTS applied to level perpetuities.

	Theories	Equation	VTS
1	No-costs-of-leverage	(34)	DT
2	Damodaran	(31)	$\frac{DT - [D(\mathrm{Kd} - R_F)(1 - T)]}{\mathrm{Ku}}$
3	Practitioners	(33)	$\frac{D[R_F - \mathrm{Kd}(1 - T)]}{\mathrm{Ku}}$
4	Harris and Pringle	(29)	$\frac{TD\mathrm{Kd}}{\mathrm{Ku}}$
5	Myers	(25)	DT
6	Miles and Ezzell	(28)	$\frac{TD\mathrm{Kd}(1 + \mathrm{Ku})}{[(1 + \mathrm{Kd})\mathrm{Ku}]}$
7	Miller (1977)	(26)	0
8	With-costs-of-leverage	(35)	$\frac{D(\mathrm{Ku}T + R_F - \mathrm{Kd})}{\mathrm{Ku}}$
9	Modigliani and Miller	(24)	DT

Table AI. Perpetuities: value of tax shields (VTS) according to the nine theories

Appendix 2. Valuation equations according to the main theories when the debt's market value (D) is equal to its nominal value (N)

Equations common to all methods:

$$\mathrm{WACC}_t = \frac{E_{t-1}\mathrm{Ke}_t + D_{t-1}\mathrm{Kd}_t(1-T)}{E_{t-1}+D_{t-1}} \qquad \mathrm{WACC}_{\mathrm{BT}t} = \frac{E_{t-1}\mathrm{Ke}_t + D_{t-1}\mathrm{Kd}_t}{E_{t-1}+D_{t-1}}$$

Relationships between cash flows:

$$\mathrm{ECF}_t = \mathrm{FCF}_t + (D_t - D_{t-1}) - D_{t-1}\mathrm{Kd}_t(1-T) \quad \mathrm{CCF}_t = \mathrm{FCF}_t + D_{t-1}\mathrm{Kd}_t T$$
$$\mathrm{CCF}_t = \mathrm{ECF}_t - (D_t - D_{t-1}) + D_{t-1}\mathrm{Kd}_t$$

Cash flows\\Ku:

$$\mathrm{ECF}\backslash\backslash\mathrm{Ku} = \mathrm{ECF}_t - E_{t-1}(\mathrm{Ke}_t - \mathrm{Ku}_t) \qquad \mathrm{FCF}\backslash\backslash\mathrm{Ku} = \mathrm{FCF}_t - (E_{t-1}+D_{t-1})(\mathrm{WACC}_t - \mathrm{Ku}_t) = \mathrm{CCF}\backslash\backslash\mathrm{Ku} = \mathrm{CCF}_t - (E_{t-1}+D_{t-1})(\mathrm{WACC}_{\mathrm{BT}t} - \mathrm{Ku}_t)$$

Cash flows\\ R_{F}:

$$\mathrm{ECF}\backslash\backslash R_{\mathrm{F}} = \mathrm{ECF}_t - E_{t-1}(\mathrm{Ke}_t - R_{\mathrm{F}t})$$
$$\mathrm{FCF}\backslash\backslash R_{\mathrm{F}} = \mathrm{FCF}_t - (E_{t-1}+D_{t-1})(\mathrm{WACC}_t - R_{\mathrm{F}t}) = \mathrm{CCF}\backslash\backslash R_{\mathrm{F}} = \mathrm{CCF}_t - (E_{t-1}+D_{t-1})(\mathrm{WACC}_{\mathrm{BT}t} - R_{\mathrm{F}t})$$
$$\mathrm{ECF}\backslash\backslash R_{\mathrm{F}} = \mathrm{ECF}\backslash\backslash\mathrm{Ku} - E_{t-1}(\mathrm{Ku}_t - R_{\mathrm{F}t})$$
$$\mathrm{FCF}\backslash\backslash R_{\mathrm{F}} = \mathrm{FCF}\backslash\backslash\mathrm{Ku} - (E_{t-1}+D_{t-1})(\mathrm{Ku}_t - R_{\mathrm{F}t})$$
$$\mathrm{FCF}\backslash\backslash\mathrm{Ku} - \mathrm{ECF}\backslash\backslash\mathrm{Ku} = D_{t-1}\mathrm{Ku}_t - (D_t - D_{t-1})$$
$$\mathrm{FCF}\backslash\backslash R_{\mathrm{F}} - \mathrm{ECF}\backslash\backslash R_{\mathrm{F}} = D_{t-1}R_{\mathrm{F}t} - (D_t - D_{t-1})$$

	No-cost-of-leverage	Damodaran (1994)
Ke	$\mathrm{Ke} = \mathrm{Ku} + \frac{D(1-T)}{E}(\mathrm{Ku} - \mathrm{Kd})$	$\mathrm{Ke} = \mathrm{Ku} + \frac{D(1-T)}{E}(\mathrm{Ku} - R_{\mathrm{F}})$
β_{L}	$\beta_{\mathrm{L}} = \beta\mathrm{u} + \frac{D(1-T)}{E}(\beta\mathrm{u} - \beta\mathrm{d})$	$\beta_{\mathrm{L}} = \beta\mathrm{u} + \frac{D(1-T)}{E}\beta\mathrm{u}$
WACC	$\mathrm{Ku}\left(1 - \frac{DT}{E+D}\right)$	$\mathrm{Ku}\left(1 - \frac{DT}{E+D}\right) + D\frac{(\mathrm{Kd} - R_{\mathrm{F}})(1-T)}{E+D}$
$\mathrm{WACC}_{\mathrm{BT}}$	$\mathrm{Ku} - \frac{DT(\mathrm{Ku} - \mathrm{Kd})}{E+D}$	$\mathrm{Ku} - D\frac{T(\mathrm{Ku} - R_{\mathrm{F}}) - (\mathrm{Kd} - R_{\mathrm{F}})}{E+D}$
VTS	PV[Ku; DTKu]	PV[Ku; $DT\mathrm{Ku} - D(\mathrm{Kd} - R_{\mathrm{F}})(1-T)$]
ECF_t\\Ku	$\mathrm{ECF}_t - D_{t-1}(\mathrm{Ku}_t - \mathrm{Kd}_t)(1-T)$	$\mathrm{ECF}_t - D_{t-1}(\mathrm{Ku} - R_{\mathrm{F}})(1-T)$
FCF_t\\Ku	$\mathrm{FCF}_t + D_{t-1}\mathrm{Ku}_t T$	$\mathrm{FCF}_t + D_{t-1}\mathrm{Ku}T - D_{t-1}(\mathrm{Kd} - R_{\mathrm{F}})(1-T)$
ECF_t\\R_{F}	$\mathrm{ECF}_t - D_{t-1}(\mathrm{Ku}_t - \mathrm{Kd}_t)(1-T) - E_{t-1}(\mathrm{Ku}_t - R_{\mathrm{F}t})$	$\mathrm{ECF}_t - D_{t-1}(\mathrm{Ku} - R_{\mathrm{F}})(1-T) - E_{t-1}(\mathrm{Ku}_t - \mathrm{R}_{\mathrm{F}t})$
FCF_t\\R_{F}	$\mathrm{FCF}_t + D_{t-1}\mathrm{Ku}_t T - (E_{t-1}+D_{t-1})(\mathrm{Ku}_t - R_{\mathrm{F}t})$	$\mathrm{FCF}_t + D_{t-1}\mathrm{Ku}T - D_{t-1}(\mathrm{Kd} - R_{\mathrm{F}})(1-T) - (E_{t-1}+D_{t-1})(\mathrm{Ku}_t - \mathrm{R}_{\mathrm{F}t})$

Table AII.

Appendix 3. Valuation equations according to the main theories when the debt's market value (D) is not equal to its nominal or book value (N)

This Appendix contains the expressions of the basic methods for valuing companies by discounted cash flows when the debt's market value (D) is not equal to its nominal value (N). If the debt's market value (D) is not equal to its nominal value (N), it is because the required return to debt (Kd) is different from the cost of the debt (r).

The interest paid in a period t is: $I_t = N_{t-1} r_t$. The increase in debt in period t is: $\Delta N_t = N_t - N_{t-1}$. Consequently, the debt cash flow in period t is: $\text{CFd} = I_t - \Delta N_t = N_{t-1} r_t - (N_t - N_{t-1})$.

Consequently, the value of the debt at $t = 0$ is:

$$D_0 = \sum_{t=1}^{\infty} \frac{N_{t-1} r_t - (N_t - N_{t-1})}{\prod_1^t (1 + \text{Kd}_t)}$$

It is easy to show that the relationship between the debt's market value (D) and its nominal value (N) is:

$$D_t - D_{t-1} = N_t - N_{t-1} + D_{t-1}\text{Kd}_t - N_{t-1} r_t$$

Consequently:

$$\Delta D_t = \Delta N_t + D_{t-1}\text{Kd}_t - N_{t-1} r_t$$

The fact that the debt's market value (D) is not equal to its nominal value (N) affects several equations given in section 1 of this paper. Equations (1, 3, 4, 6, 7, 9, 10) continue to be valid, but the other equations change.

The expression of the WACC in this case is:

$$\text{WACC} = \frac{E\text{Ke} + D\text{Kd} - NrT}{E + D} \tag{36}$$

The expression relating the ECF to the FCF is:

$$\text{ECF}_t = \text{FCF}_t + (N_t - N_{t-1}) - N_{t-1} r_t (1 - T) \tag{37}$$

The expression relating the CCF to the ECF and the FCF is:

$$\text{CCF}_t = \text{ECF}_t + \text{CFd}_t = \text{ECF}_t - (N_t - N_{t-1}) + N_{t-1} r_t = \text{FCF}_t + N_{t-1} r_t T \tag{38}$$

Equations common to all the methods: WACC and WACC_{BT}:

$$\text{WACC}_t = \frac{E_{t-1}\text{Ke}_t + D_{t-1}\text{Kd}_t - N_{t-1}\text{r}_t T}{(E_{t-1} + D_{t-1})} \qquad \text{WACC}_{\text{BT}t} = \frac{E_{t-1}\text{Ke}_t + D_{t-1}\text{Kd}_t}{(E_{t-1} + D_{t-1})}$$

$$\text{WACC}_{\text{BT}t} - \text{WACC}_t = \frac{N_{t-1} r_t T}{(E_{t-1} + D_{t-1})}$$

Relationships between the cash flows:

$$\text{ECF}_t = \text{FCF}_t + (N_t - N_{t-1}) - N_{t-1} r_t (1 - T) \qquad \text{CCF}_t = \text{FCF}_t + N_{t-1} r_t T$$

$$\text{CCF}_t = \text{ECF}_t - (N_t - N_{t-1}) + N_{t-1} r_t$$

	Harris-Pringle (1985); Ruback (1995)	Myers (1974)	Miles and Ezzell (1980)
Ke	$\mathrm{Ke} = \mathrm{Ku} + \frac{D}{E}(\mathrm{Ku} - \mathrm{Kd})$	$\mathrm{Ke} = \mathrm{Ku} + \frac{\mathrm{Vu} - E}{E}(\mathrm{Ku} - \mathrm{Kd})$	$\mathrm{Ke} = \mathrm{Ku} + \frac{D}{E}(\mathrm{Ku} - \mathrm{Kd}) \times \left[1 - \frac{T\mathrm{Kd}}{1 + \mathrm{Kd}}\right]$
β_{L}	$\beta_{\mathrm{L}} = \beta\mathrm{u} + \frac{D}{E}(\beta\mathrm{u} - \beta\mathrm{d})$	$\beta_{\mathrm{L}} = \beta\mathrm{u} + \frac{\mathrm{Vu} - E}{E}(\beta\mathrm{u} - \beta\mathrm{d})$	$\beta_{\mathrm{L}} = \beta\mathrm{u} + \frac{D}{E}\beta\mathrm{u} - \beta\mathrm{d}) \times \left[1 - \frac{T\mathrm{Kd}}{1 + \mathrm{Kd}}\right]$
WACC	$\mathrm{Ku} - \frac{D\mathrm{Kd}T}{E + D}$	$\mathrm{Ku} - \frac{\mathrm{VTS}(\mathrm{Ku} - \mathrm{Kd}) + D\mathrm{Kd}T}{E + D}$	$\mathrm{Ku} - \frac{D\mathrm{Kd}T}{E + D}\frac{1 + \mathrm{Ku}}{1 + \mathrm{d}_0}$
$\mathrm{WACC}_{\mathrm{BT}}$	Ku	$\mathrm{Ku} - \frac{\mathrm{VTS}(\mathrm{Ku} - \mathrm{Kd})}{E + D}$	$\mathrm{Ku} - \frac{D\mathrm{Kd}T}{E + D}\frac{(\mathrm{Ku} - \mathrm{Kd})}{(1 + \mathrm{Kd}_0)}$
VTS	PV[Ku; TDKd]	PV[Kd; TDKd]	PV[Ku; TDKd] $\frac{(1 + \mathrm{Ku})}{(1 + \mathrm{Kd})}$
ECF$_{\mathrm{t}}$\\Ku	$\mathrm{ECF}_t - D_{t-1}(\mathrm{Ku}_t - \mathrm{Kd}_t)$	$\mathrm{ECF}_t - (\mathrm{Vu} - E)\ (\mathrm{Ku}_t - \mathrm{Kd}_t)$	$\mathrm{ECF} - D(\mathrm{Ku} - \mathrm{Kd}) \times \frac{1 + \mathrm{Kd}(1 - T)}{(1 + \mathrm{Kd}_0)}$
FCF$_{\mathrm{t}}$\\Ku	$\mathrm{FCF}_t + TD_{t-1}\mathrm{Kd}_t$	$\mathrm{FCF}_t - TD\mathrm{Kd} + \mathrm{VTS} \times (\mathrm{Ku} - \mathrm{Kd})$	$\mathrm{FCF} + TD\mathrm{Kd}\frac{(1 + \mathrm{Ku})}{(1 + \mathrm{Kd})}$
ECF$_t$\\R_{F}	$\mathrm{ECF}_t - D_{t-1}\ (\mathrm{Ku}_t - \mathrm{Kd}_t) - E_{t-1}(\mathrm{Ku}_t - R_{\mathrm{F}t})$	$\mathrm{ECF}_t - (\mathrm{Vu}{-}E)\ (\mathrm{Ku}_t - \mathrm{Kd}_t) - E_{t-1}\ (\mathrm{Ku}_t - R_{\mathrm{F}t})$	$\mathrm{ECF} - D(\mathrm{Ku} - \mathrm{Kd}) \times \frac{1 + \mathrm{Kd}(1 - T)}{(1 + \mathrm{Kd}_0)} - E_{t-1}(\mathrm{Ku}_t - R_{\mathrm{F}t})$
FCF$_t$\\R_{F}	$\mathrm{FCF}_t + TD_{t-1}\mathrm{Kd}_t - (E_{t-1} + D_{t-1})(\mathrm{Ku}_t - R_{\mathrm{F}t})$	$\mathrm{FCF}_t + TD\mathrm{Kd} + \mathrm{VTS}\ (\mathrm{Ku} - \mathrm{Kd}) - (\mathrm{E}_{t-1} + \mathrm{D}_{t-1}) \times (\mathrm{Ku}_t - R_{\mathrm{F}t})$	$\mathrm{FCF} + TD\mathrm{Kd}\frac{(1 + \mathrm{Ku})}{(1 + \mathrm{Kd})} - (E_{t-1} + D_{t-1}) \times (\mathrm{Ku}_t - R_{\mathrm{F}t})$

Table AIII.

	Miller	With-cost-of-leverage
Ke	$\mathrm{Ke} = \mathrm{Ku} + \frac{D}{E}[\mathrm{Ku} - \mathrm{Kd}(1 - T)]$	$\mathrm{Ke} = \mathrm{Ku} + \frac{D}{E}[\mathrm{Ku}(1 - T) + \mathrm{Kd}T - R_{\mathrm{F}}]$
β_{L}	$\beta_{\mathrm{L}} = \beta\mathrm{u} + \frac{D}{E}(\beta\mathrm{u} - \beta\mathrm{d}) + \frac{D}{E}\frac{T\mathrm{Kd}}{P_{\mathrm{M}}}$	$\beta_{\mathrm{L}} = \beta\mathrm{u} + \frac{D}{E}[(\beta\mathrm{u}(1 - T) - T\beta\mathrm{d})]$
WACC	Ku	$\mathrm{Ku} - \frac{D(\mathrm{Ku}T - \mathrm{Kd} + R_{\mathrm{F}})}{E + D}$
$\mathrm{WACC}_{\mathrm{BT}}$	$\mathrm{Ku} + \frac{D\mathrm{Kd}T}{E + D}$	$\mathrm{Ku} - \frac{D[(\mathrm{Ku} - \mathrm{Kd})T + R_{\mathrm{F}} - \mathrm{Kd})]}{E + D}$
VTS	0	PV[Ku; $D(\mathrm{Ku}T + R_{\mathrm{F}} - \mathrm{Kd})$]
ECF$_t$\\Ku	$\mathrm{ECF}_t - D_{t-1}\ [\mathrm{Ku}_t - \mathrm{Kd}_t(1 - T)]$	$\mathrm{ECF}_t - D_{t-1}\ [\mathrm{Ku}_t(1 - T) + \mathrm{Kd}_tT - R_{\mathrm{F}t}]$
FCF$_t$\\Ku	FCF_t	$\mathrm{FCF}_t + D_{t-1}\ [\mathrm{Ku}_tT - \mathrm{Kd}_t + R_{\mathrm{F}t}]$
ECF$_t$\\R_{F}	$\mathrm{ECF}_t - D_{t-1}\ [\mathrm{Ku}_t - \mathrm{Kd}_t(1 - T)] - E_{t-1}(\mathrm{Ku}_t - R_{\mathrm{F}t})$	$\mathrm{ECF}_t - D_{t-1}\ [\mathrm{Ku}_t(1 - T) + \mathrm{Kd}_tT - R_{\mathrm{F}t}] - E_{t-1}\ (\mathrm{Ku}_t - R_{\mathrm{F}t})$
FCF$_t$\\R_{F}	$\mathrm{FCF}_t - (E_{t-1} + D_{t-1})(\mathrm{Ku}_t - R_{\mathrm{F}t})$	$\mathrm{FCF}_t + D_{t-1}\ [\mathrm{Ku}_tT - \mathrm{Kd}_t + R_{\mathrm{F}t}] - (E_{t-1} + D_{t-1})(\mathrm{Ku}_t - R_{\mathrm{F}t})$

Table AIV.

	Modigliani and Miller	Practitioners
Ke	$\mathrm{Ke} = \mathrm{Ku} + \frac{D}{E}\left[\mathrm{Ku} - \mathrm{Kd}(1-T) - (\mathrm{Ku}-g)\frac{\mathrm{VTS}}{D}\right]^{a}$	$\mathrm{Ke} = \mathrm{Ku} + \frac{D}{E}(\mathrm{Ku} - R_F)$
β_L	$\beta_L = \beta\mathrm{u} + \frac{D}{E}\left[\beta\mathrm{u} - \beta\mathrm{d} + \frac{T\mathrm{Kd}}{P_M} - \frac{\mathrm{VTS}(\mathrm{Ku}-g)}{DP_M}\right]^{a}$	$\beta_L = \beta\mathrm{u} + \frac{D}{E}\beta\mathrm{u}$
WACC	$\frac{D\mathrm{Ku} - (\mathrm{Ku}-g)\mathrm{VTS}^{a}}{(E+D)}$	$\mathrm{Ku} - D\frac{R_F - \mathrm{Kd}(1-T)}{E+D}$
WACC_{BT}	$\frac{D\mathrm{Ku} - (\mathrm{Ku}-g)\mathrm{VTS} + DT\mathrm{Kd}^{a}}{E+D}$	$\mathrm{Ku} + D\frac{\mathrm{Kd} - R_F}{E+D}$
VTS	$\mathrm{PV}[R_F;\ TDR_F]$	$\mathrm{PV}[\mathrm{Ku};\ TD\mathrm{Kd} - D(\mathrm{Kd} - R_F)]$
$\mathrm{ECF}_t\backslash\backslash\mathrm{Ku}$	$\mathrm{ECF}_t - D_{t-1}\frac{[\mathrm{Ku}_t - \mathrm{Kd}_t(1-T) - (\mathrm{Ku}-g)\mathrm{VTS}]^{a}}{D}$	$\mathrm{ECF}_t - D_{t-1}(\mathrm{Ku}_t - R_{Ft})$
$\mathrm{FCF}_t\backslash\backslash\mathrm{Ku}$	$\mathrm{FCF}_t + E_{t-1}\mathrm{Ku} + (\mathrm{Ku}-g)\mathrm{VTS}^{a}$	$\mathrm{FCF}_t + D_{t-1}[R_{Ft} - \mathrm{Kd}_t\,(1-T)]$
$\mathrm{ECF}_t\backslash\backslash R_F$	$\mathrm{ECF}_t - D_{t-1}[\mathrm{Ku}_t - \mathrm{Kd}_t(1-T) - (\mathrm{Ku}-g)\mathrm{VTS}/D] - E_{t-1}\,(\mathrm{Ku}_t - R_{Ft})^{a}$	$\mathrm{ECF}_t - (E_{t-1} + D_{t-1})\,(\mathrm{Ku}_t - R_{Ft})$
$\mathrm{FCF}_t\backslash\backslash R_F$	$\mathrm{FCF}_t + E_{t-1}\,\mathrm{Ku} + (\mathrm{Ku}-g)\mathrm{VTS} - (E_{t-1} + D_{t-1})(\mathrm{Ku}_t - R_{Ft})^{a}$	$\mathrm{FCF}_t + D_{t-1}\,[R_{Ft} - \mathrm{Kd}_t\,(1-T)] - (E_{t-1} + D_{t-1})\,(\mathrm{Ku}_t - R_{Ft})$

Note: Valid only for growing perpetuities

Table AV.

	No-cost-of-leverage	Damodaran (1994)	Practitioners
WACC	$\mathrm{Ku} - \frac{NrT + DT(\mathrm{Ku} - \mathrm{Kd})}{(E+D)}$	$\mathrm{Ku} - \frac{NrT + D[T(\mathrm{Ku} - R_F) - (\mathrm{Kd} - R_F)]}{(E+D)}$	$\mathrm{Ku} - \frac{NrT - D(\mathrm{Kd} - R_F)}{(E+D)}$
VTS	$\mathrm{PV}[\mathrm{Ku};\ DT\mathrm{Ku} + T(Nr - D\mathrm{Kd})]$	$\mathrm{PV}[\mathrm{Ku};\ TNr + DT(\mathrm{Ku} - R_F) - D(\mathrm{Kd} - R_F)]$	$\mathrm{PV}[\mathrm{Ku};\ TNr - D(\mathrm{Kd} - R_F)]$
$\mathrm{FCF}_t\backslash\backslash\mathrm{Ku}$	$\mathrm{FCF}_t + D_{t-1}\,\mathrm{Ku}_t T + T(N_{t-1}r_t - D_{t-1}\mathrm{Kd}_t)$	$\mathrm{FCF}_t + D_{t-1}\,\mathrm{Ku}_t T + T(N_{t-1}\,r_t - D_{t-1}\mathrm{Kd}_t) - D_{t-1}(\mathrm{Kd}_t - R_{Ft})\,(1-T)$	$\mathrm{FCF}_t + T\,(N_{t-1}r_t - D_{t-1}\mathrm{Kd}_t) + D_{t-1}\,[R_{Ft} - \mathrm{Kd}_t(1-T)]$

Table AVI.

	Harris and Pringle (1985); Ruback (1995)	Myers (1974)	Miles and Ezzell (1980)
WACC	$\mathrm{Ku} - \frac{NrT}{(E+D)}$	$\mathrm{Ku} - \frac{\mathrm{VTS}(\mathrm{Ku} - \mathrm{Kd}) + NrT}{(E+D)}$	$\mathrm{Ku} - \frac{NrT}{(E+D)}\frac{1+\mathrm{Ku}}{1+\mathrm{Kd}}$
VTS	$\mathrm{PV}[\mathrm{Ku};\ TNr]$	$\mathrm{PV}[\mathrm{Kd};\ TNr]$	$\frac{\mathrm{PV}[\mathrm{Ku}_t; N_{t-1}r_t T](1+\mathrm{Ku})}{(1+\mathrm{Kd})}$
$\mathrm{FCF}_t\backslash\backslash\mathrm{Ku}$	$\mathrm{FCF}_t + TN_{t-1}r_t$	$\mathrm{FCF}_t + TNr + \mathrm{VTS}(\mathrm{Ku} - \mathrm{Kd})$	$\frac{\mathrm{FCF} + TNr(1+\mathrm{Ku})}{(1+\mathrm{Kd})}$

Table AVII.

Appendix 4. Dictionary

βd, beta of debt
β_L, beta of levered equity
βu, beta of unlevered equity = beta of assets
D, value of debt
E, value of equity
Ebv, book value of equity
ECF, equity cash flow
EP, economic profit
EVA, economic value added
FCF, free cash flow
g, growth rate of the constant growth case
I, interest paid
Ku, cost of unlevered equity (required return to unlevered equity)
Ke, cost of levered equity (required return to levered equity)
Kd, required return to debt = cost of debt
N, book value of the debt
NOPAT, Net Operating Profit After Tax = profit after tax of the unlevered company
PAT, profit after tax
PBT, profit before tax
P_M, market premium $= E(R_M - R_F)$
PV, present value
r, cost of debt
R_F, risk-free rate
T, corporate tax rate
VTS, value of the tax shield
Vu, value of shares in the unlevered company
WACC, weighted average cost of capital
$WACC_{BT}$, weighted average cost of capital before taxes
WCR, working capital requirements = net current assets

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