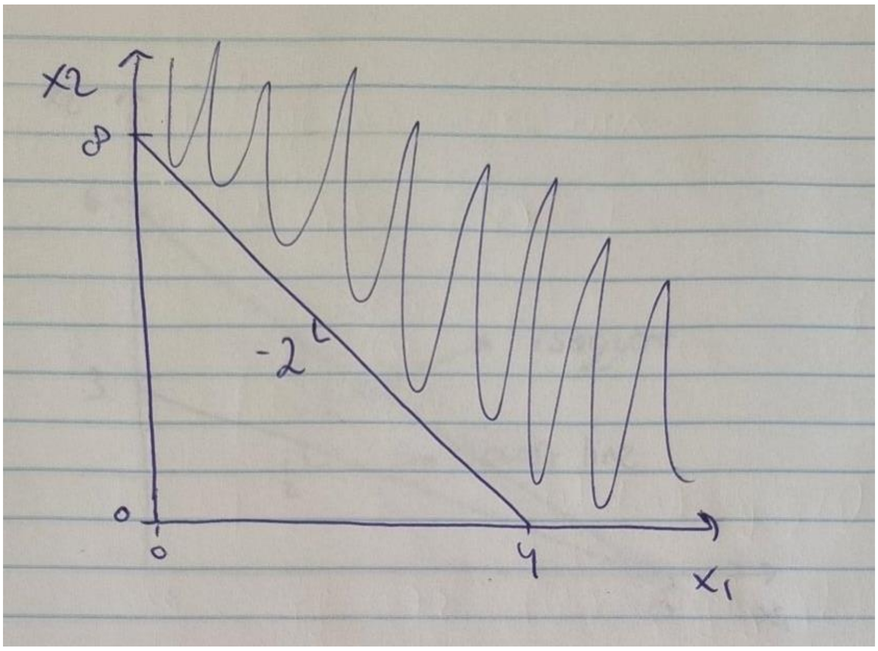
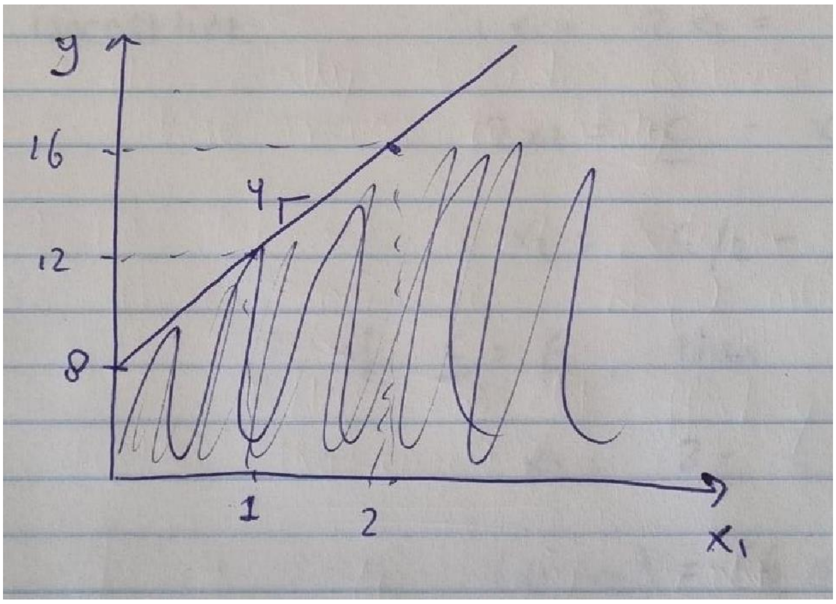
1.1. (1 point)

$$TRS = -\frac{\frac{df}{dx_1}}{\frac{df}{dx_2}} = -\frac{\beta}{\gamma}$$

1.2. (1 point)



1.3. (1 point)



2.1. (1 point)

The WAPM is that $p^t y^t - w^t x^t \geq p^t y^s - w^t x^s$

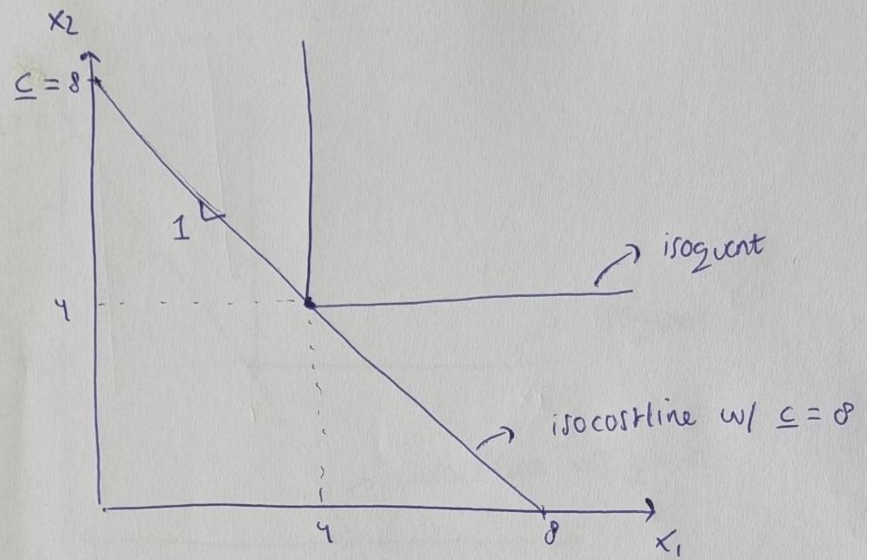
Let t=1 and s=2, then

$p^1 y^1 - w^1 x^1 = 2 * 5 - 1 * 5 = 5$

$p^1 y^2 - w^1 x^2 = 2 * 10 - 1 * 10 = 10$

WAPM does not hold since $p^1 y^1 - w^1 x^1 < p^1 y^2 - w^1 x^2$

2.2. (1 point)



isocostline:

$x_1 + x_2 = \underline{c}$

$x_2 = \underline{c} - x_1$, if $\underline{c} = 8$ then

$x_2 = 8 - x_1$, and $(x_1, x_2) = (4, 4)$ is on the isocost line and allows firm to produce $y = 4$ against lowest costs since it just touches the isoquant.

$x_1 = x_2 = 4$

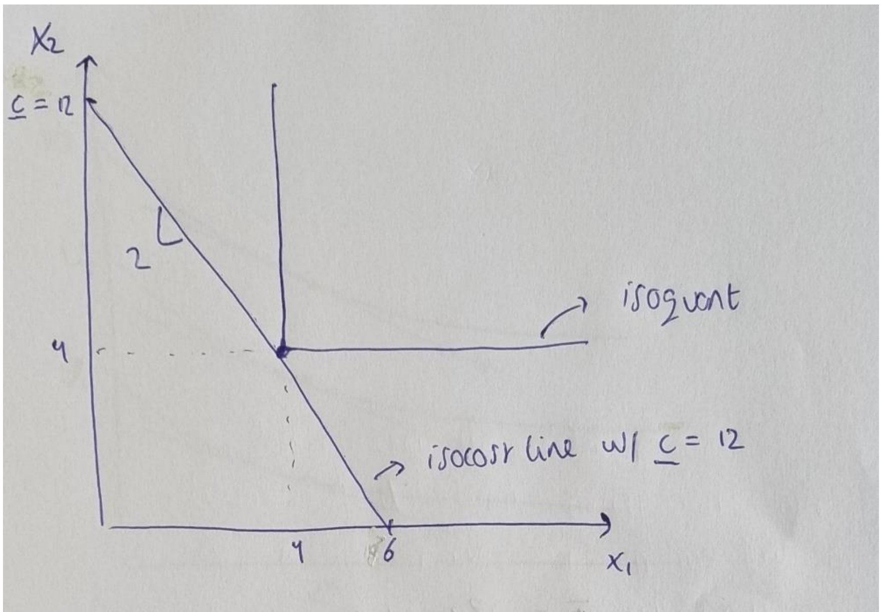
$\underline{c} = 8$

2.3. (1 point)

$x_1 = x_2 = 4$

$\underline{c} = 12$

(the graph below is to provide you with an explanation, but this graph was not necessary to obtain the full point for this question)



$x_2 = \bar{c} - 2x_1$, if $\bar{c} = 12$ then

$x_2 = 12 - 2x_1$, and $(x_1, x_2) = (4, 4)$

is on the isocost line and allows firm

to produce $y = 4$ against lowest costs.

2.4. (1 point)

The substitution effect is zero.

Explanation (1): Cost minimization for firms is analogous to expenditure minimization for consumers. Hence, the conditional factor demand functions for firms are analogous to the Hicksian demand functions for consumers. In question 2.2 and 2.3 we have shown that the conditional factor demand for good 1 does not change with the price of good 1: the firm demands 4 in case the price is both 1 and 2. This also implies that the Hicksian demand for good 1 does not change with the price of good 1. Hence, the substitution effect is zero.

Explanation (2): with this utility function good 1 and good 2 are perfect complements. Hence, to reach u=4 you need 4 of both x1 and x2. Then, even if x1 (or x2) becomes more expensive, you still need to demand 4 of both if you want to reach u=4. Hence, there is no substitution effect.

3.1. (1 point)

1. Write down the Lagrangian for the UMP

2. Take FOCs

3. Solve these FOCs for x1 and x2 to reach:

$$x_1 = \frac{m}{p_1}\frac{\alpha}{\alpha+\beta}$$

$$x_2 = \frac{m}{p_2}\frac{\beta}{\alpha+\beta}$$

3.2. (1 point)

$$\frac{dx_1}{dm} = \frac{1}{p_1}\frac{\alpha}{\alpha+\beta}$$

$$\frac{dx_2}{dm} = \frac{1}{p_2}\frac{\beta}{\alpha+\beta}$$

For the Marshallian demand functions x_1 and x_2 to be less steep (with price on the vertical axis and quantity on the horizontal axis) than the Hicksian demand functions h_1 and h_2 we need the income effect to be positive (so that the goods are normal). Hence, we need that:

$$\frac{dx_1}{dm} = \frac{1}{p_1}\frac{\alpha}{\alpha+\beta} > 0$$

$$\frac{dx_2}{dm} = \frac{1}{p_2}\frac{\beta}{\alpha+\beta} > 0$$

For this we need that $\alpha > 0$ and $\beta > 0$.

3.3. (1 point)

MRS: If a consumer increases consumption for good 1, how much does the consumer need to decrease her consumption for good 2 to keep the same utility level. This describes the slope of the indifference curve.

$$MRS = -\frac{dx_2(x_1)}{dx_1} = -\frac{\frac{du}{dx_1}}{\frac{du}{dx_2}} = -\frac{0.5x_1^{-0.5}x_2^{0.5}}{0.5x_1^{0.5}x_2^{-0.5}} = -\frac{x_2}{x_1}$$

3.4. (1 point)

1. Write down the Lagrangian for the EMP

2. Take FOCs

3. Solve these FOCs for x1 and x2 (denoted by h1 and h2) to reach

$$h_1 = \left(\frac{p_2}{p_1}\right)^{0.5} \bar{u}$$

$$h_2 = \left(\frac{p_1}{p_2}\right)^{0.5} \bar{u}$$

4. Plug these into the budget constraint to reach

$$e(p_1, p_2, \bar{u}) = 2p_1^{0.5}p_2^{0.5}\bar{u}$$

The expenditure function $e(p_1, p_2, \bar{u})$ represents the minimum expenditures required to reach utility level $\bar{u}$ at prices p_1 and p_2.

3.5. (1 point)

To obtain lambda one needs to take the derivative of the expenditure function towards $\bar{u}$:

$$\frac{de(p_1, p_2, \bar{u})}{d\bar{u}} = \lambda(p_1, p_2, \bar{u}) = 2p_1^{0.5}p_2^{0.5}$$

Plugging in for $p_1 = p_2 = 1$ gives us

$$\lambda(p_1, p_2, \bar{u}) = 2 * 1^{0.5} * 1^{0.5} = 2$$

This implies that if we allow the consumer to have one additional util (so we relax the constraint by 1), then the consumer needs to spend 2 additional euros.

4.1. (1 point)

1. Write down the Lagrangian for the UMP

2. Take FOCs

3. Solve these FOCs for x1 to reach:

$$x_1 = \frac{1}{4}\left(\frac{p_2}{p_1}\right)^2$$

1. Write down the Lagrangian for the EMP

2. Take FOCs

3. Solve these FOCs for x1 (denoted by h1) to reach:

$$h_1 = \frac{1}{4}\left(\frac{p_2}{p_1}\right)^2$$

4.2. (1 point)

Since the income effect is zero for quasilinear utility functions, we have that $x_1 = h_1$. The change in consumer surplus is the area to the left of the Marshallian demand curve x_1. The compensating and equivalent variation are the areas to the left of the Hicksian demand curve h_1. Since the Hicksian and Marshalian demand coincide, the compensating and equivalent variation and change in consumer surplus are the same. The equivalent and compensating variation are exact measures of welfare, whereas in general the change in consumer surplus is not. However, in case the income effect is zero all of these are equal and so the change in consumer surplus can be used as an exact measure of welfare.

4.3. (2 points)

Here we calculate the change in consumer surplus. Start with the Marshallian demand function while assuming that $p_2 = 4$ as stated in the exercise, then:

$$x_1(p_1) = \frac{1}{4}\left(\frac{4}{p_1}\right)^2 = 4p_1^{-2}$$

Then we integrate the demand function for a price change from 1 to 2:

$$\int_1^2 x_1(p_1)\, dp_1 = \int_1^2 4p_1^{-2}\, dp_1 = -4p_1^{-1}|_1^2 = -2 - -4 = 2$$

Hence, the change in consumer surplus is 2.

5.1. (2 points)

There are two conditions for the long run equilibrium:

$Y(p) = X(p)$

$\pi_i = 0 \quad \forall\, i$

1. We derive the firms' supply function $y_i(p)$ for each firm i.

$mc_i(y) = \frac{dc_i(y)}{dy} = y,$

and since supply curve is $mc_i(y) = p$,

we have that $y_i(p) = p$.

2. We derive market supply, which is the sum over all firms m.

$Y(p) = \sum_{i=1}^{m} y_i(p) = \sum_{i=1}^{m} p = mp.$

3. We use the first condition to find equilibrium price and firm supply in terms of number of firms m.

$$mp = 40 - 2p$$

$$p = \frac{40}{m+2}$$

$$y_i(p) = p = \frac{40}{m+2}$$

4. We use the second condition to find the number of firms m so that profits are zero.

$$\pi_i = py_i(p) - c_i(y) = 0$$

$$\pi_i = \left(\frac{40}{m+2}\right)^2 - 0.5\left(\frac{40}{m+2}\right)^2 - 2 = 0$$

$$0.5\left(\frac{40}{m+2}\right)^2 = 2$$

$$\left(\frac{40}{m+2}\right)^2 = 4$$

$$\frac{40}{m+2} = 2$$

$$m = 18$$

Hence, in the long run there will be 18 active firms in this perfect competitive market.

6.1. (1 point)

Long run. It is only in the long run that there is entry and exit in a perfect competitive market. Hence, profits are guaranteed to be zero in a perfect competitive market only in the long run.

6.2. (1 point)

No. The monopolist will not be able to ask a mark-up if the elasticity of demand is high (i.e., infinite). This is directly visible from the FOC of profit maximization for the monopolist, which is:

$$\frac{p - mc}{p} = -\frac{1}{e}$$

Where p is the price, mc the marginal cost, and e the elasticity of demand. If demand is completely elastic, then $e \to -\infty$, and so the price of the monopolist is equal to marginal cost.