Homework 1

Problems 1 and 2 are problems 1 and 7 from Cochrane's Chapter 1 book.

Problem 1

(a) The absolute risk aversion coefficient is $ara = -\frac{u''(c)}{u'(c)}$. *Sometimes is defined as* $\frac{u''(c)}{u'(c)}$. We scale by $u'(c)$ because expected utility is only defined up to linear transformations – $a+bu(c)$ gives the same predictions as $u(c)$ – and this measure of the second derivative is invariant to linear transformations. *It is a measure of the intensity of an individual's aversion to risk. The higher it is, the higher the risk premium require to induce full investment in a risky investment.* Show that the utility function with constant absolute risk aversion is $u(c) = -e^{-\alpha c}$.

(b) The coefficient of relative risk aversion in a one-period model (i.e. when consumption equals wealth) is defined as $rra = -\frac{cu''(c)}{u'(c)}$. $rra = c \cdot ara$. *For instance under increasing relative risk aversion, that is when* $\frac{\partial rra}{\partial c} > 0$ *the proportion of an individual's wealth invested in the risky asset decreases as his wealth increases. Under constant relative risk aversion* $\frac{\partial rra}{\partial c} = 0$*, that proportion does not depend on the wealth of the individual.* For power utility $u(c) = c^{-\gamma}$, show that the risk aversion coefficient equals the power.

(c) The elasticity of intertemporal substitution is defined as $\xi^I \equiv -\dfrac{c_2/c_1 d(c_1/c_2)}{dR/R}$. Show that with power utility $u(c) = c^{-\gamma}$, the intertemporal substitution elasticity is equal to $1/\gamma$.

Problem 2

The first order conditions for an infinitely lived consumer who can buy an asset with dividend stream $\{D_t\}$ are

$$p_t = E_t \left\{ \sum_{s=1}^{\infty} \beta^s \frac{u'(c_{t+s})}{u'(c_t)} D_{t+s} \right\} \tag{1}$$

The first order conditions for buying a security with price p_t and payoff

$$x_{t+1} = D_{t+1} + p_{t+1}$$

are

$$p_t = E_t \left\{ \beta \frac{u'(y_{t+1})}{u'(y_t)} \left(D_{t+1} + p_{t+1} \right) \right\} \tag{2}$$

(a) Derive (40) from (39).

(b) Derive (39) from (40). You need an extra condition. Show that this extra condition is a first order condition for maximization. To do this, think about what strategy the consumer could follow to improve utility if the condition did not hold.

Problem 3

Show that if $\log x = \mu + \sigma z$ and $z \sim N(0,1)$ so that $y \equiv \log x \sim N(\mu, \sigma^2)$ then $Ex = E \exp(y) = \exp(\mu + \frac{\sigma^2}{2})$.

Problem 4

Is it true that the pricing of an asset does not depend on the volatility of the asset's return?

Problem 5

What is the relation between the holding return $R^B_{2,t+1} = \frac{B_{1,t+1}}{B_{2,t}}$ and the riskless return $R_{1,t} = \frac{1}{B_{1,t}}$? Which is larger in expected value?

Problem 6

A stochastic process $\{p_t\}$ is a *martingale* if $E_t \{p_{t+1}\} = p_t$. In a short period horizon is the price of a security (approximately) a martingale?