Lisbon School of Economics & Management
Universidade de Lisboa

STATISTICS I

Economics / Finance/ Management

2nd Year/2nd Semester

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LESSON 6

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https://doity.com.br/estatistica-aplicada-a-nutricao

https://basiccode.com.br/produto/informatica-basica/

Roadmap:

- Probability
- Random variable and two dimensional random variables:
    - Distribution
    - Joint distribution
    - Marginal distribution
    - Conditional distribution functions
- Expectations and parameters for a random variable and two dimensional random variables
- Discrete Distributions
- Continuous Distributions

Bibliography: Miller & Miller, John E. (2014) Freund's Mathematical Statistics with applications, 8th Edition, Pearson Education, [MM]

Expected Value, Variance and Quantiles for a Random Variable: Exercises

Chapter 3

1

10. Find the expected value, the median and the mode, of the random variable Y whose probability density is given by

$$f_Y(y) = \begin{cases} (y+1)/8 & \text{for } 2 \leq y \leq 4 \\ 0 & \text{elsewhere} \end{cases}$$

Exercise 10

$$f_Y(y) = \begin{cases} \dfrac{y+1}{8} & (2 \leq y \leq 4) \\ 0 & (\text{otherwise}) \end{cases}$$

$$E(Y) = \int_2^4 y \frac{y+1}{8} \, dy = \frac{1}{8}\int_2^4 y^2 + y \, dy = \frac{1}{8}\left[\frac{y^3}{3} + \frac{y^2}{2}\right]_2^4 =$$

$$= \frac{1}{8}\left(\frac{4^3}{3} + \frac{4^2}{2} - \left(\frac{2^3}{3} + \frac{2^2}{2}\right)\right) = \frac{1}{8}\left(\frac{64}{3} + \frac{16}{2} - \frac{8}{3} - \frac{4}{2}\right) =$$

$$= \frac{1}{8}\left(\frac{56}{3} + \frac{12}{2}\right) = \frac{1}{8}\left(\frac{112}{6} + \frac{36}{6}\right) = \frac{1}{8}\left(\frac{148}{6}\right) =$$

$$= \frac{148}{48} = \frac{74}{24} = \frac{37}{12}$$

Exercise 10

Let $m = me(X)$:

$F_Y(m) = \frac{1}{2}$ $\Leftrightarrow$

$$\Leftrightarrow \int_2^m f_Y(y)\,dy = \int_2^m \frac{y+1}{8}\,dy = \frac{1}{8}\left[\frac{y^2}{2} + y\right]_2^m = \frac{1}{8}\left(\frac{m^2}{2} + m - 4\right) =$$

$$= \frac{m^2}{16} + \frac{m}{8} - \frac{1}{2} = \frac{1}{2} \Leftrightarrow$$

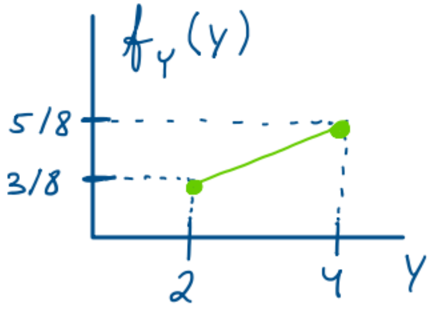
$$\Leftrightarrow \frac{1}{16}m^2 + \frac{1}{8}m - 1 = 0 \Leftrightarrow m = -1 + \sqrt{17}$$

Auxiliary calculation:

$$m = \frac{-\frac{1}{8} \pm \sqrt{\left(\frac{1}{8}\right)^2 + \frac{4}{16}}}{\frac{2}{16}} = \frac{-\frac{1}{8} \pm \sqrt{\frac{17}{64}}}{\frac{1}{8}} = \frac{-\frac{1}{8} \pm \frac{1}{8}\sqrt{17}}{\frac{1}{8}} = -1 \pm \sqrt{17}$$

Only $m = -1 + \sqrt{17}$ works because $m > 2$.

Exercise 10



$mo(Y) = \arg\max_{y \in \mathbb{R}} f_Y(y) = 4$ because $f_Y(y)$ is strictly increasing in the interval $[2, 4]$.

11. Let X be a random variable that has the probability function $f_X(x) = 1/2$ for for $x = -2$ and $x = 2$.

   (a) Find $E(X)$, $E(X^2)$ and σ_X^2.

   (b) Calculate the the mode and median.

   (c) Calculate first and third quartiles.

   (d) Compute the standard deviation.

   (e) Compute $Var(2X - 2)$

Exercise 11 a)

$$f_X(x) = \frac{1}{2} \quad (x = -2, 2)$$

a)

$$E(X) = \sum_{x \in D_X} x \, f_X(x) = -2 \times \frac{1}{2} + 2 \times \frac{1}{2} = 0$$

$$E(X^2) = \sum_{x \in D_X} x^2 f_X(x) = (-2)^2 \times \frac{1}{2} + 2^2 \times \frac{1}{2} = 4$$

$$\sigma_X^2 = E(X^2) - \underbrace{E(X)}_{0}{}^2 = 4$$

Exercise 11 b)

There is no Mode

$$F_X(x) = \begin{cases} 0 & (x < -2) \\ \frac{1}{2} & (-2 \leq x < 2) \\ 1 & (x \geq 2) \end{cases}$$

$$F_X(-2) = 0.5$$

$$me(X) = \min\{x \in \mathbb{R} : F_X(x) \geq 0.5\} = -2 = Q_{0.5}$$

Exercise 11 c), d) and e)

c)

$$Q_{0.25} = \min \{ x \in \mathbb{R} : F_X(x) \geq 0.25) = -2$$

$$Q_{0.75} = \min \{ x \in \mathbb{R} : F_X(x) \geq 0.75 = 2$$

d)

$$\sigma_X = \sqrt{\sigma_X^2} = \sqrt{4} = 2$$

e)

$$Var(2X - 2) = 4 \, Var(X) = 4 \times 4 = 16$$

12. Let X be a random variable that has probability density function

$$f_X(x) = \begin{cases} x & \text{for } 0 < x < 1 \\ 2 - x & \text{for } 1 \leq x < 2 \\ 0 & \text{elsewhere} \end{cases}.$$

(a) Find the expected value, the median and the mode of the random variable X.

(b) Compute the variance of $g(X) = 2X + 3$.

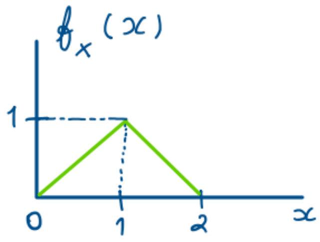
Exercise 12 a)

a)

$$\epsilon(X) = \int_{-\infty}^{+\infty} x \, f_X(x)\, dx = \int_0^1 x \cdot x \, dx + \int_1^2 x(2-x)\, dx = \int_0^1 x^2 dx + \int_1^2 2x - x^2 \, dx =$$

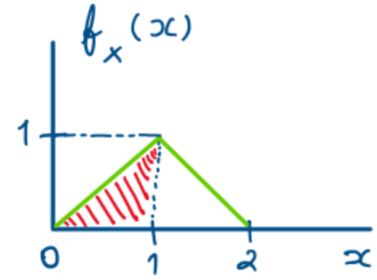
$$= \left[\frac{x^3}{3}\right]_0^1 + \left[x^2 - \frac{x^3}{3}\right]_1^2 = \frac{1}{3} + \left(2^2 - \frac{2^3}{3} - \left(1 - \frac{1}{3}\right)\right) =$$

$$= \frac{1}{3} + 4 - \frac{8}{3} - 1 + \frac{1}{3} = -2 + 3 = 1$$



$$mo(X) = \underset{x \in \mathbb{R}}{\arg\max} \; f_X(x) = 1$$

Exercise 12 a)



$$\blacksquare = 0.5 \Rightarrow me(X) = 1$$

$$F_X(1) = \int_0^1 f_X(x)\,dx = \int_0^1 x\,dx = \left[\frac{x^2}{2}\right]_0^1 = \frac{1}{2}$$

Exercise 12 b)

b)

$$Var(g(X)) = Var(2X+3) = 4\,Var(X) = 4 \times \frac{1}{6} = \frac{2}{3}$$

Auxiliary calculations:

$$Var(X) = E(X^2) - E(X)^2 = \frac{7}{6} - 1 = \frac{1}{6}$$

$$E(X^2) = \int_{-\infty}^{+\infty} x^2 f_X(x)\,dx = \int_0^1 x^2 * x\,dx + \int_1^2 x^2(2-x)\,dx =$$

$$= \int_0^1 x^3\,dx + \int_1^2 2x^2 - x^3\,dx = \frac{1}{4}\left[x^4\right]_0^1 + \left[\frac{2}{3}x^3 - \frac{x^4}{4}\right]_1^2 =$$

$$= \frac{1}{4} + \frac{2}{3}2^3 - \frac{2^4}{4} - \left(\frac{2}{3} - \frac{1}{4}\right) = \frac{1}{4} + \frac{16}{3} - \frac{16}{4} - \frac{2}{3} + \frac{1}{4} =$$

$$= -\frac{14}{4} + \frac{14}{3} = -\frac{42}{12} + \frac{56}{12} = \frac{14}{12} = \frac{7}{6}$$

13. Let X be a random variable that has probability density function

$$f(x) = \begin{cases} \frac{1}{x\log(3)} & \text{for } 1 \leq x \leq 3 \\ 0 & \text{elsewhere} \end{cases}.$$

(a) Find $E(X)$, the median and the mode of X.

(b) Find $E(X^2)$ and $E(X^3)$.

(c) Use the results of part (a) and (b) to determine $E(X^3 + 2X^2 - 3X + 1)$.

Exercise 13 a)

$$f(x) = \begin{cases} \frac{1}{x\log(3)} & \text{for } 1 \le x \le 3 \\ 0 & \text{elsewhere} \end{cases}$$

log stands for "natural logarithm" (base e)

a)

$$E(X) = \int_1^3 x \, f_X(x)\, dx = \int_1^3 x \frac{1}{x\log(3)}\, dx =$$

$$= \int_1^3 \frac{1}{\log(3)}\, dx = \frac{1}{\log(3)} [x]_1^3 \, dx = \frac{2}{\log(3)}$$

$mo(X) = 1$ because $f_X(x)$ is strictly decreasing in $[1, 3]$ and $f_X(x) = 0$ outside that interval. Therefore, $x = 1$ is the absolute maximizer of $f_X(x)$.

Exercise 13 a)

$$\int_1^{me(X)} f_X(x)\,dx = \int_1^{me(X)} \frac{1}{x\log(3)}\,dx = \frac{1}{\log(3)}\int_1^{me(X)} \frac{1}{x}\,dx$$

$$= \frac{1}{\log(3)}\Big[\log(x)\Big]_1^{me(x)} = \frac{\log(me(X)) - \overset{0}{\log(1)}}{\log(3)} = \frac{1}{2} \Leftrightarrow$$

$$\Leftrightarrow \log(me(X)) = \frac{\log(3)}{2} \Leftrightarrow$$

$$\Leftrightarrow me(X) = \exp\left\{\frac{\log(3)}{2}\right\} = \sqrt{3} \simeq 1.732$$

It would be enough to use the calculator and present an approximated result in decimal form.

Auxiliary calculation:

$$\frac{\log(3)}{2} = \log\left(3^{\frac{1}{2}}\right) = \log(\sqrt{3})$$

so: $\exp\{\log(3)/2\} = \exp\{\log(\sqrt{3})\} = \sqrt{3}$

Exercise 13 b)

$$E(X^2) = \int_1^3 x^2 f_X(x)\,dx = \int_1^3 x^2 \frac{1}{x\log(3)}\,dx =$$

$$= \int_1^3 \frac{x}{\log(3)}\,dx = \frac{1}{2\log(3)}\left[x^2\right]_1^3 dx =$$

$$= \frac{8}{2\log(3)} = \frac{4}{\log(3)}$$

$$E(X^3) = \int_1^3 x^3 f_X(x)\,dx = \int_1^3 x^3 \frac{1}{x\log(3)}\,dx =$$

$$= \int_1^3 \frac{x^2}{\log(3)}\,dx = \frac{1}{3\log(3)}\left[x^3\right]_1^3 dx =$$

$$= \frac{26}{3\log(3)}$$

Exercise 13 c)

c)

$$E(X^3+2X^2-3X+1)=E(X^3)+2E(X)-3E(X)+1=$$

$$=\frac{26}{3\log(3)}+2\frac{4}{\log(3)}-3\frac{2}{\log(3)}+1=$$

$$=1+\frac{26}{3\log(3)}+\frac{24}{3\log(3)}-\frac{18}{3\log(3)}=1+\frac{32}{3\log(3)}$$

Exercise 13 d)

Supplementary Exercise

d)

$$\sigma_X^2 = E(X^2) - E(X)^2 =$$

$$= \frac{4}{\log(3)} - \left(\frac{2}{\log(3)}\right)^2 = \frac{4\log(3) - 4}{\log(3)^2}$$

$$\sigma_X = \sqrt{\frac{4\log(3) - 4}{\log(3)^2}} = \frac{2\sqrt{\log(3) - 4}}{\log(3)} \simeq 0.57167745$$

Para dar igual às soluções:

$$\sigma_X^2 = \frac{4\log(3) - 4}{\log(3)^2} = \frac{4(\log(3) - 1)}{\log(3)^2}$$

$$\sigma_X = \sqrt{\frac{4(\log(3) - 1)}{\log(3)^2}} = \frac{2\sqrt{\log(3) - 1}}{\log(3)}$$ → Resultado das soluções está errado

14. Let X be a random variable such that

$$f_X(x) = \begin{cases} \frac{1}{b-a}, & a < x < b \\ 0 & \end{cases}$$

(a) Find the moment generating function of X.

(b) Calculate the first and third quantiles.

Exercise 14 a)

$\underline{t \neq 0:}$

$$M_X(t) = E(e^{tX}) = \int_{-\infty}^{+\infty} e^{tx} f_X(x)\,dx = \int_a^b e^{tX} \frac{1}{b-a}\,dx =$$

$$= \frac{1}{b-a}\left[\frac{e^{tx}}{t}\right]_a^b = \frac{e^{tb} - e^{ta}}{t(b-a)} \qquad (t \neq 0)$$

$\underline{t = 0:}$

$$M_X(t) = E(e^{0X}) = E(1) = 1 \qquad (t = 0)$$

Conclusion:

$$M_X(t) = \begin{cases} \dfrac{e^{tb\,-}\;e^{ta}}{t(b-a)} & (t \neq 0) \\ 1 & (t = 0) \end{cases}$$

$\rightarrow$ The solution has the wrong sign

Exercise 14 b)

$$Q_{0.25} : \int_{-\infty}^{Q_{0.25}} f_X(x)\,dx = \frac{1}{4} \iff$$

$$\iff \int_{a}^{Q_{0.25}} \frac{1}{b-a}\,dx = \frac{1}{4} \iff$$

$$\iff \frac{1}{b-a}\left[x\right]_{a}^{Q_{0.25}} = \frac{1}{4} \iff$$

$$\iff Q_{0.25} - a = \frac{b-a}{4} \iff$$

$$\iff Q_{0.25} = a + \frac{1}{4}(b-a)$$

Exercise 14 b)

$$Q_{0.75}: \int_{-\infty}^{Q_{0.75}} f_X(x)\,dx = \frac{3}{4} \Leftrightarrow$$

$$\Leftrightarrow \int_{a}^{Q_{0.75}} \frac{1}{b-a}\,dx = \frac{3}{4} \Leftrightarrow$$

$$\Leftrightarrow \frac{1}{b-a}\left[x\right]_{a}^{Q_{0.75}} = \frac{3}{4} \Leftrightarrow$$

$$\Leftrightarrow Q_{0.75} - a = \frac{3(b-a)}{4} \Leftrightarrow$$

$$\Leftrightarrow Q_{0.75} = a + \frac{3}{4}(b-a)$$

15. Find the moment-generating function of the discrete random variable X that has the probability distribution given by

$$f(x) = 2\left(\frac{1}{3}\right)^x, \quad x = 1, 2, \ldots$$

Use it to find the values of μ_1' and μ_2'.

Exercise 15

$$f_X(x) = 2\left(\frac{1}{3}\right)^x \quad (x = 1, 2, \dots)$$

$$M_X(t) = E(e^{tX}) = \sum_{x=1}^{+\infty} e^{tx} f_X(x) =$$

$$= \sum_{x=1}^{+\infty} 2e^{tx}\left(\frac{1}{3}\right)^x =$$

$$= 2\sum_{x=1}^{+\infty}\left(\frac{e^t}{3}\right)^x =$$

geometric series ($r = \frac{e^t}{3}$)
converges if $|r|<1$

$$= 2\,\frac{\frac{e^t}{3}}{1-\frac{e^t}{3}} =$$

$$= 2\,\frac{\frac{e^t}{3}}{\frac{3-e^t}{3}} = \frac{2e^t}{3-e^t} \quad (0 < t < \ln(3))$$

Exercise 15

Auxiliary calculation:

$$\left|\frac{e^t}{3}\right| < 1 \iff \frac{|e^t|}{3} < 1 \iff 0 < e^t < 3 \iff$$

$$\iff 0 < t < \ln(3) \iff$$

$$\iff 0 < t < \ln(3)$$

$$\mu_1' = M_X'(0) = \left.\frac{2e^t(3-e^t) - 2e^t(-e^t)}{(3-e^t)^2}\right|_{t=0} =$$

$$= \left.\frac{2e^t(3-e^t+e^t)}{(3-e^t)^2}\right|_{t=0} =$$

$$= \left.\frac{6e^t}{(3-e^t)^2}\right|_{t=0} = \frac{6e^0}{(3-e^0)^2} = \frac{6}{2^2} = \frac{6}{4} = \frac{3}{2}$$

Exercise 15

$$\mu_2' = M_x''(0) = \left(\frac{6e^t}{(3-e^t)^2}\right)'\Bigg|_{t=0} = \frac{6e^t(3-e^t)^2 - 6e^t 2(3-e^t)(-e^t)}{(3-e^t)^4}\Bigg|_{t=0} =$$

$$= \frac{6e^t(3-e^t)^2 + 12e^t(3-e^t)(e^t)}{(3-e^t)^4}\Bigg|_{t=0} =$$

$$= \frac{6e^0(3-e^0)^2 + 12e^0(3-e^0)e^0}{(3-e^0)^4} =$$

$$= \frac{6 \times 2^2 + 12 \times 2}{2^4} = \frac{24+24}{16} = \frac{48}{16} = 3$$

16. Derive the moment generating function of the random variable has the probability density function $f(x) = e^{-|x|}/2$ for $x \in \mathbb{R}$ and use it to find σ_X^2.

Exercise 16

Ex 16

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$f_X(x) = e^{-|x|}/2 \quad (x \in \mathbb{R})$

Note: $|x| = \begin{cases} x & \text{for } x \geq 0 \\ -x & \text{for } x < 0 \end{cases}$

- for $x > 0$ we have $|x| = x$, therefore $-|x| = -x$ and consequently $e^{-|x|} = e^{-x}$
- for $x < 0$ we have $|x| = -x$, therefore $-|x| = x$ and consequently $e^{-|x|} = e^{x}$

$$M_X(t) = E(e^{tX}) = \int_{-\infty}^{+\infty} e^{tx} \frac{e^{-|x|}}{2} dx =$$

$$= \int_{-\infty}^{0} \frac{e^{tx} e^{x}}{2} dx + \int_{0}^{+\infty} \frac{e^{tx} e^{-x}}{2} dx =$$

$$= \frac{1}{2} \int_{-\infty}^{0} e^{x(1+t)} dx + \frac{1}{2} \int_{0}^{+\infty} e^{x(t-1)} dx =$$

converges only if $\underbrace{x}_{x>0}(t-1) < 0 \Leftrightarrow t-1<0 \Leftrightarrow t<1$

$$= \frac{1}{2} \lim_{a \to -\infty} \left[\frac{e^{x(1+t)}}{1+t}\right]_a^0 + \frac{1}{2} \lim_{b \to +\infty} \left[\frac{e^{x(t-1)}}{t-1}\right]_0^b =$$

$$= \frac{1}{2} \lim_{a \to -\infty} \left(\frac{1}{1+t} - \frac{e^{a(1+t)}}{1+t}\right) + \frac{1}{2} \boxed{\lim_{b \to +\infty} \left(\frac{e^{b(t-1)}}{t-1} - \frac{1}{t-1}\right)} =$$

Auxiliary calculation:

$$\lim_{b \to +\infty} \frac{e^{b(t-1)}}{t-1} = 0 \text{ because } \left\{ \begin{array}{l} b>0 \\ t-1<0 \end{array} \right. \to b(t-1) < 0$$

$$= \frac{1}{2}\left(\frac{1}{1+t} - 0\right) + \frac{1}{2}\left(0 - \frac{1}{t-1}\right) =$$

$$= \frac{1}{2(1+t)} - \frac{1}{2(t-1)} =$$

$$= \frac{2(t-1) - 2(t+1)}{4(t+1)(t-1)} = \frac{t-1-t-1}{2(t+1)(t-1)} = -\frac{2}{2(t+1)(t-1)} =$$

$$\underset{(*)}{=} -\frac{1}{t^2-1} = \frac{1}{1-t^2} \quad (t<1)$$

Exercise 16

(*) Note: $a^2 - b^2 = (a-b)(a+b)$

$$E(X) = M'_X(0) = \frac{d}{dt}\left(\frac{1}{1-t^2}\right)\Bigg|_{t=0} = \frac{2t}{(1-t^2)^2}\Bigg|_{t=0} = 0$$

$$E(X^2) = M''_X(0) = \frac{d}{dt}\left(\frac{2t}{1-t^2}\right)\Bigg|_{t=0} = \frac{2(1-t^2) - 2t(-2t)}{(1-t^2)^2}\Bigg|_{t=0} = 2$$

$$\sigma_X^2 = E(X^2) - E(X)^2 = 2 - 0^2 = 2$$

17. Let X and Y be two independent random variables such that the moment generating function of X is given by

$$M_X(t) = 0.2 + 0.5e^t + 0.3e^{2t}$$

and the probability function of Y is given by

$$f_Y(y) = \begin{cases} 0.3, & y = -1 \\ 0.5, & y = 1 \\ 0.2, & y = 3 \\ 0, & \text{otherwise} \end{cases}$$

a) Compute the cumulative distribution function of Y.

b) Compute the moment generating function of Y.

c) Compute the mode and the median of Y.

d) Compute the coefficient of variation of X.

e) Let Z be the random variable given by $Z = aY + b$. Find a and b such that $M_X(t) = M_Z(t)$.

f) Compute the moment generating function of $W = X + Y$.

Exercise 17 a)

$X \perp Y$

$M_X(t) = 0.2 + 0.5e^{t} + 0.3e^{2t}$

$$f_Y(y) = \begin{cases} 0.3, & y = -1 \\ 0.5, & y = 1 \\ 0.2, & y = 3 \\ 0, & \text{otherwise} \end{cases}$$

$D_Y = \{-1, 1, 3\}$

a)

$$F_Y(y) = \begin{cases} 0 & (y < -1) \\ 0.3 & (-1 \leq y < 1) \\ 0.8 & (1 \leq y < 3) \\ 1 & (y \geq 3) \end{cases}$$

Exercise 17 b) and c)

b)

$$M_Y(t) = E(e^{tY}) = \sum_{y \in D_Y} e^{ty} f_Y(y) =$$

$$= 0.3e^{-t} + 0.5e^{t} + 0.2e^{3t}$$

c)

$$Mo(Y) = \underset{y \in D_Y}{\operatorname{argmax}} f_Y(y) = 1$$

$$me(Y) = \min\{y \in D_Y : F_Y(y) \geq 0.5\} = 1$$

Exercise 17 d)

$$E(X) = \mu_1' = M'(t)\Big|_{t=0} = 0.5e^t + 0.6e^{2t}\Big|_{t=0} =$$

$$= 0.5e^0 + 0.6e^0 = 0.5 + 0.6 = 1.1$$

$$E(X^2) = \mu_2' = M''(t)\Big|_{t=0} = \left(0.5e^t + 0.6e^{2t}\right)'\Big|_{t=0} =$$

$$= 0.5e^t + 1.2e^{2t}\Big|_{t=0} = 0.5e^0 + 1.2e^0 =$$

$$= 1.7$$

$$\sigma_x^2 = E(X^2) - E(X)^2 = 1.7 - 1.1^2 = 0.49$$

$$\sigma_x = \sqrt{0.49}$$

$$\rho_x = \frac{\sigma_x}{\mu_x} = \frac{\sqrt{0.49}}{1.1} \simeq 0.6364$$

Exercise 17 e)

e)

$$Z = aY + b$$

$$M_Z(t) = E(e^{tZ}) = E(e^{t(aY+b)}) =$$

$$= E(e^{atY} e^{tb}) = e^{tb} E(e^{(at)Y}) =$$

$$= e^{tb} M_Y(at) =$$

$$= e^{tb}(0.3e^{-at} + 0.5e^{at} + 0.2e^{3at})$$

$$= \underline{0.3}e^{tb-at} + \underline{0.5}e^{tb+at} + \underline{0.2}e^{tb+3at}$$

Exercise 17 e)

$$M_Z(t) = M_X(t) = 0.2 + 0.5e^{t} + 0.3e^{2t} \Leftrightarrow$$

$$\Leftrightarrow \begin{cases} tb - at = 2t \\ tb + at = t \\ 2t = 0 \end{cases} \Leftrightarrow \begin{cases} b - a = 2 \\ b + a = 1 \\ \_\_\_\_ \end{cases} \Leftrightarrow \begin{cases} a = b - 2 \\ \_\_\_\_ \\ \_\_\_\_ \end{cases}$$

$$\Leftrightarrow \begin{cases} \_\_\_\_ \\ b + b - 2 = 1 \\ \_\_\_\_ \end{cases} \Leftrightarrow \begin{cases} \_\_\_\_ \\ 2b = 3 \\ \_\_\_\_ \end{cases} \Leftrightarrow \begin{cases} \_\_\_\_ \\ b = \frac{3}{2} \\ \_\_\_\_ \end{cases} \Leftrightarrow$$

$$\Leftrightarrow \begin{cases} a = \frac{3}{2} - 2 = -\frac{1}{2} \\ b = \frac{3}{2} \end{cases}$$

Exercise 17 e)

f)

$$W = X + Y$$

$$M_W(t) = M_X(t)\,M_Y(t) =$$

$$= (0.2 + 0.5e^{t} + 0.3e^{2t})(0.3e^{-t} + 0.5e^{t} + 0.2e^{3t})$$

Thanks!

Questions?