Controling inflation

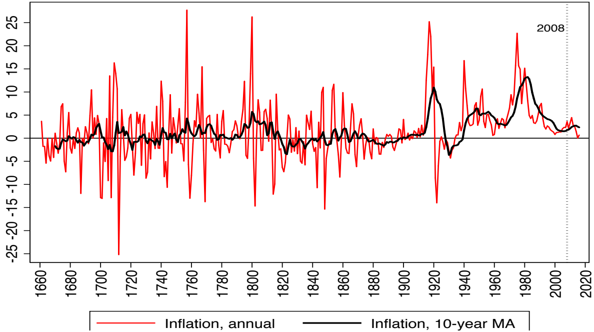
Lecture 16

THE 5 STAGES IN INFLATION

History of inflation

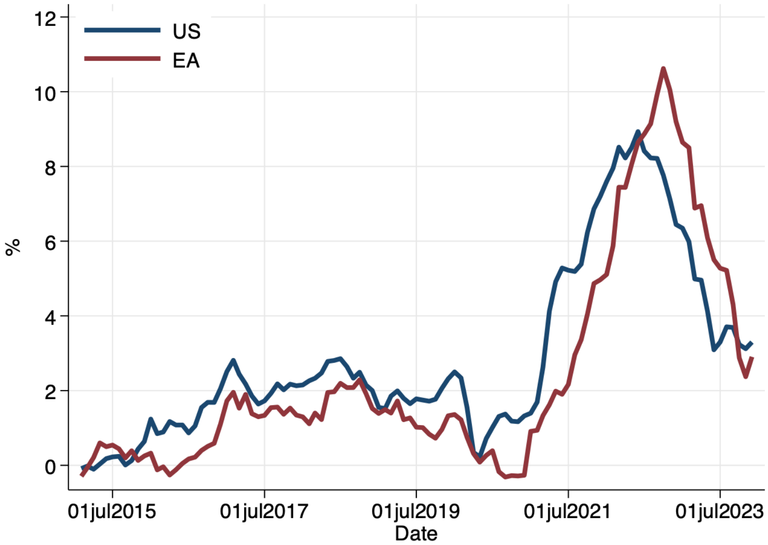
UK: 1660-2016
Average: 1.5%, Std. Dev: 6.5%

Gold Standard	WW1 & WW2	Bretton Woods	Up to EMS crisis	Great Mod.	Post GFC
(1717-1913)	(1914-1945)	(1946-1973)	(1974-1993)	(1994-2008)	(2008-2016)
μ=0.5%	μ=3.6%	μ=4.8%	μ=8.7%	μ=1.9%	μ=2.2%
σ=6%	σ=8.8%	σ=2.7%	σ=5.6%	σ=0.7%	σ=1.4%



Gold standard; interwar high volatile; Bretton Woods; volatile 70s-80s; the Great stability.

THE RECENT PAST (CPI-US, HICP-EZ): BAD LUCK OR POLICY?



QUESTIONS

- Determinacy: can policy deliver a unique prive level?
- Effectiveness: can policy minimize deviations between actual and target inflation?
- Explain how inflation was controlled for twenty years.
  Reference: Laura Castillo-Martinez and Ricardo Reis, 2025, "How do central banks control inflation? A guide for the perplexed", mimeo

Classical economy: savers and investment

$$\mathbb{E}_t \left[M_{t+1}(1 + R_t) \right] = 1$$

- From micro: M_{t+1} is the MRS, how many units of a good the private agents would require next period in exchange for one unit of good now; $1 + R_t$: the opportunity cost of consuming one more unit today in terms of foregone consumption tomorrow.

- From macro: Euler equation. Smooth out marginal utility of consumption over time. But tilt them according to the interest rate.

- From finance: a no-arbitrage condition, risk and time adjusted net return on any investment is zero. SDF is adjustment factor (with risk neutrality: $M_{t+1} = \beta$)

Classical economy: consumers and goods market

$$\Re_t(i) = \frac{P_t(i)}{P_t(0)} \quad \text{for } i = 1, ..., I$$

- Households equate static marginal rates of substitution and relative prices across goods within the same period.

- $\Re(i)$ be how many units of good i consumers would trade for one unit of good 0,

- $P(i)$ is the nominal price of good i.

Firms, workers, and the labor market

$$\tilde{P}_t(i) = Z_t(i) P_t F(Y_t(i), Q_t)$$

- Together maximize surplus from production

- Desired price: $\tilde{P}_t(i)$

- Markup $Z_t(i)$

- Real marginal cost of production are function $F(Y_t(i), Q_t)$ that depends on how much is produced $Y_t(i)$ and on the real cost of inputs, Q_t:

Price index and inflation

- Denominate prices in a unit of account, say EUR.
  - . Common unit of account across goods: since care about relative prices only, easier.

- A price: value of good in terms of a unit of account.

- The price level: how much must you must give to get the overall set of goods in the economy.

$$P_t = \mathbb{P}\left(\{P_t(i)\}_{i=0,\ldots I}\right)$$

Linearly homogeneous so that it doubles when all prices double.

- Inflation is the loss of real value of the unit of account.

Market clearing conditions

Stick to closed economy with no savings

$$C_t(i) = Y_t(i)$$

- Consumption CAPM

$$M_{t+1} = \beta \mathbb{U}'(Y_{t+1}) / \mathbb{U}'(Y_t)$$

- Marginal rates of substitution:

$$\Re(i) \equiv \frac{\partial \mathbb{C}(.) / \partial C_t(i)}{\partial \mathbb{C}(.) / \partial C_t(0)}$$

- Disutility from supplying labor

$$Q_t = \mathbb{V}'(Y_t) / \mathbb{U}'(Y_t)$$

INDETERMINACY

- Goal is to study $\{P_t\}_{t=0}^{\infty}$ and inflation: $\pi_t = \log(P_t) - \log(P_{t+1})$ or $\Pi_t = P_t/P_{t-1}$, so $\pi_t = \log(\Pi_t)$

- If actual and desired prices are the same, $\tilde{P}_t(i) = P_t(i)$, then $\Re_t(i)$ and M_t are both exogenous with respect to P_t. The real quantities and relative prices are pinned down.

- Nothing in classical supply-demand economics pins down the price level or inflation, like nothing determines whether measurements should be in inches or centimeters.

- Marshallian economics pins down relative prices by marginal rate of substitution and marginal rates of transformation. Not absolute prices. Classical dichotomy.

Nominal rigidities

- There are many ways to break the classical dichotomy. For instance, nominal rigidities that drive a wedge between desired and actual prices.

New Keynesian model, log linearizing the economy around the classical equilibrium, and with firms having Calvo-prices.

$$
\begin{aligned}
y_t &= \mathbb{E}_t(y_{t+1}) - \theta r_t \\
\pi_t &= \beta \mathbb{E}_t(\pi_{t+1}) + \kappa\alpha(y_t - y_t^n) + z_t
\end{aligned}
$$

- There are now two equations in three unknowns, r_t, y_t and π_t.

- Price stickiness of firms and workers and aggregate demand makes inflation indeterminacy be a real indeterminacy as well.

Modern monetary system

- Make payments electronically: call to subtract from my cell in the bank's spreadsheet and add to your cell. Since many payments are to buy goods, again same unit of account in the bank spreadsheet.

- When multiple banks, need upper layer spreadsheet. A bank for the banks to perform clearing or settlement.

- The owner of the top spreadsheet is the central bank. The units in the spreadsheet are called the EUR. It determines the unit of account everywhere else.

Central banks and inflation

- Reserves or bank deposits: are the balance in the cell of each bank at the central banks (so are liabilities of central bank). So reserves are the unit of account, price is how many units of reserves must give away to obtain the good.

- Minimal central bank: clearing house / spreadsheet where payments take place using reserves as a digital mean of payment.

- Central bank controls spreadsheet: amount of reserves, V_t, and a rate of remuneration, I_t^v, by which multiply every entry overnight.

Nominal bonds and no arbitrage

- Say that there is a piece of paper that promises to give you 1 nominal unit, that is a +1 entry in your cell.

- This piece of paper costs Q_t nominal units today.

- The return on the bond is $1 + I_t = 1/Q_t$.

- Principle of no arbitrage: I could freely buy bonds with reserves, and vice-versa, and if the return was different, I could make infinite profits by going long on the high-interest rate one, and short in the low-interest rate one.

- By no arbitrage between nominal bonds and reserves delivers $I_t = I_t^v$.

- Central bank has enormous power in affecting nominal interest rates in the economy.

STILL INDETERMINACY OF THE UNIT OF ACCOUNT

- By definition of the price level, it costs Q_t/P_t in real goods to buy a bond. In turn, its real payoff, in units of the consumption good then is $(1+I_t)/P_{t+1}$.

- Pricing equation for this bond:

$$\mathbb{E}_t\left[M_{t+1}\left(\frac{P_t}{Q_t P_{t+1}}\right)\right] = \mathbb{E}_t\left[M_{t+1}\left(\frac{1+I_t}{\Pi_{t+1}}\right)\right] = 1.$$

  Reserves promise a nominal interest rate I_t. Real return depends on inflation. Indifference towards holding them must result from equating this expected return times the MRS between consumption today and tomorrow to one.

- Still, for every I_t (or Q_t) there is a different Π_{t+1}. Indeterminacy of equilibrium $\{P_t\}_{t=0}^{\infty}$

WHAT DOES CENTRAL BANK WANT TO DO?

- Policy aim: keep $\{P_t\}_{t=0}^{\infty}$ close to target $\{P_t^*\}_{t=0}^{\infty}$. Target is exogenous w.r.t P_t.

- Policy rules: choose policy tool $I_t^v = f(P_t, X_t^i)$, where X_t^i is an exogenous component.

- Log-linearize around steady state point where the real interest rate and inflation are equal to constants, β and $\bar{P}_t = P_0\bar{\Pi}$ to get $p_t = \log P_t - \log \bar{P}_t$. Notation: $\mathbb{E}_t(p_{t+j})$ is the public's expectation at t of what the price level will be at date $t+j$, while $\hat{p}_{t+j}$ is the central bank's expectation at t

- The effectiveness of a policy is assessed by how small the sequence of deviations between the log price level and its target is:

$$\varepsilon_t \equiv p_t - p_t^*.$$

- The most effective rule, X_t^*: so that errors expected by central bank are $\hat{\varepsilon}_t = 0$.

The Fisher equation

- Combine the two Euler equations to get the Fisher equation

$$\mathbb{E}_t \left[M_{t+1} \left(1 + R_t - \frac{1 + I_t}{\Pi_{t+1}} \right) \right] = 0.$$

- Economic force: banks can choose to hold reserves or real investments. Say P_t was too low, relative to future fixed P_{t+1}, so higher Π_{t+1}.

- Real returns on nominal reserves is lower. Banks would want to hold zero reserves and invest all of their resources in real terms

- Values of reserves must fall. Because reserves are unit of account, real value is $1/P_t$

- As P_t rises back into equilibrium, lower Π_{t+1}, more demand for reserves, market for reserves clears, banks indifferent between real investment and reserves.

Interest rate peg

Central bank chooses: $I_t = I_t^v = X^i{}_t$.

- From Fisher equation:

$$\mathbb{E}_t\left(\frac{M_{t+1}}{\Pi_{t+1}}\right) = \frac{1}{1+X_t^i}$$

- If there is no uncertainty, choosing X_t^i pins down a single Π_{t+1} at each date. Central bank can pin long run inflation.

- But no other condition to pin down P_0. Units indeterminacy.

- And with uncertainty, only expected time-risk adjusted inflation is pinned down. Actual inflation itself is not determinate.

Payment on reserves

- Central bank promises to remunerate reserve holders with a payment in real goods. The nominal return on reserves in euros would then be $1 + I_t^v{}_{,t+1} = (1 + X^i{}_t)P_{t+1}$.

- Rearrange Fisher equation:

$$\mathbb{E}_t \left[M_{t+1} \left(1 + R_t - \frac{1 + I^v_{t,t+1}}{\Pi_{t+1}} \right) \right] = 0 \Rightarrow \mathbb{E}_t \left[M_{t+1} \left(1 + R_t - \frac{(1 + X^i_t) P_{t+1} P_t}{P_{t+1}} \right) \right] = 0$$

$$\Rightarrow 1 - \mathbb{E}_t \left[M_{t+1} (1 + X^i_t) P_t \right] = 0 \Rightarrow 1 - (1 + X^i_t) P_t / (1 + R_t) = 0$$

$$\Rightarrow P_t = \frac{1 + R_t}{1 + X^i_t}$$

- Since X^i_t is exogenously chosen by policy, and R_t is exogenously pinned down by real forces, then the above equation delivers a determinate price level.

Intuition for payment on reserves

- No central bank does this, but instructive to understand intuition. If the central bank promises a real payment on reserves, then arbitrage pins down how many goods reserves are worth today.

- Since real bonds and reserves both deliver the same payment tomorrow, they must be worth the same today. Since reserves are denominated in euros, not goods, this pins down the price level.

Interest rate feedback rules

- Feedback rules: $I_t = f(P^t, X^i{}_t)$.

$$i_t = x_t^i + \phi \pi_t.$$

- Combine with log-linearized Fisher equation $i_t = r_t + \mathbb{E}_t(\pi_{t+1})$ to get:

$$\phi(\pi_t - \pi_t^*) = r_t + \mathbb{E}_t(\pi_{t+1}^*) - \phi\pi_t^* - x_t^i + \mathbb{E}_t(\pi_{t+1} - \pi_{t+1}^*)$$

- Iterate forward, Taylor principle sets $\phi > 1$ needed for sums to be well defined.

$$\pi_t = \pi_t^* + \sum_{j=0}^{T-t} \phi^{-j-1} \mathbb{E}_t \left[r_{t+j} + \pi_{t+1+j}^* - \phi\pi_{t+j}^* - x_{t+j} \right] + \phi^{-T+t} \mathbb{E}_t \left(\pi_{T+1} - \pi_{T+1}^* \right).$$

- Impose: $\lim_{T\to\infty} \phi^{-T} \mathbb{E}_t \left(\pi_{t+T} - \pi_{t+T}^* \right) = 0$, argue can't expect inflation to explode

$$\pi_t = \pi_t^* + \sum_{j=0}^{\infty} \phi^{-j-1} \mathbb{E}_t \left(r_{t+j} + \pi_{t+1+j}^* - \phi\pi_{t+j}^* - x_{t+j}^i \right)$$

OTHER FEEDBACK RULES: WICKSELLIAN

Table 1: Determinacy conditions

Rule	*Condition*
Benchmark: $x_t^i + \phi\pi_t$	$\phi > 1$
Inertial: $x_t^i + \phi\pi_t + \chi i_{t\;\;1}$	$\phi + \chi > 1$
Forecast targeting: $x_t^i + \phi\pi_t + \chi\, \mathbb{E}_t(\pi_{t+1})$	$\phi + \chi > 1$
Core inflation: $x_t^i + \phi(1-\chi)\sum_{j=0}^{\infty} \chi^j \pi_{t-j}$	$\phi > 1$
Wicksellian: $x_t^i + \phi p_t$	$\phi > 0$

Mathematics and economic logic of all these cases are similar to the ones in the analysis of the Taylor rule.

- Example with Wicksellian rule

$$\phi p_t + x_t^i = i_t = r_t + \mathbb{E}_t(\pi_{t+1})$$

- Assume $r_t = p_t^* = 0$, difference equation with $\phi > 0$:

$$(1+\phi)p_t = -x_t^i + \mathbb{E}_t(p_{t+1})$$

- Iterate forward and impose terminal condition $\lim_{T\to\infty}(1+\phi)^{-T}\,\mathbb{E}_t\left(p_{t+T}\right) = 0$ to get:

$$p_t = -\sum_{j=0}^{\infty}(1+\phi)^{-j-1}\,\mathbb{E}_t\left(x_{t+j}^i\right)$$

Nominal Rigidities

- Defining the output gap as $\tilde{y}_t \equiv y_t - y_t^n$, there are three relevant equations:

$$\begin{aligned}
\pi_t &= \beta\,\mathbb{E}_t(\pi_{t+1}) + \kappa\alpha\tilde{y}_t + z_t \\
\tilde{y}_t &= \mathbb{E}_t(\tilde{y}_{t+1}) - (i_t - \mathbb{E}_t(\pi_{t+1}) - r_t^n) \\
i_t &= x_t^i + \phi\pi_t + \phi_y\tilde{y}_t.
\end{aligned}$$

- Can solve system to get generalized Taylor principle for determinacy

$$\phi > 1 - \frac{\phi_y(1-\beta)}{\kappa\alpha}$$

- Real indeterminacy has an aggregate demand channel as well: changes in the return of financial assets affect households' desire to save, while nominal rigidities make output demand determined. Therefore changes in the interest rate now also affect inflation through changes in consumption.

The Taylor rule historically

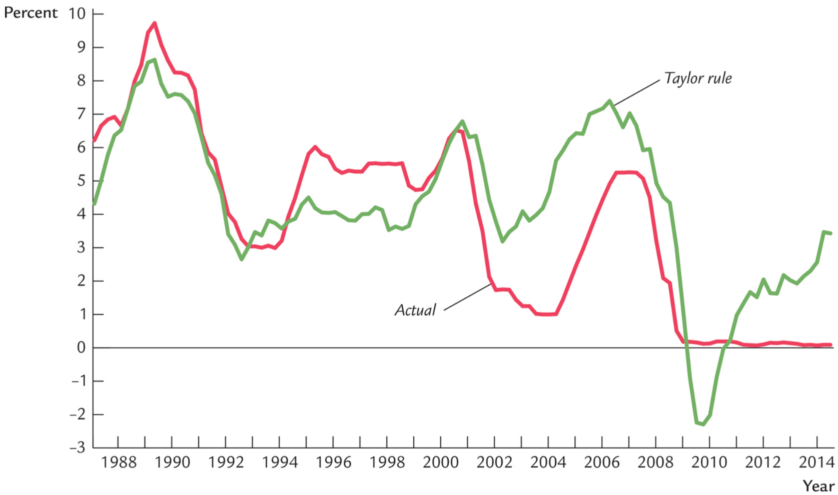
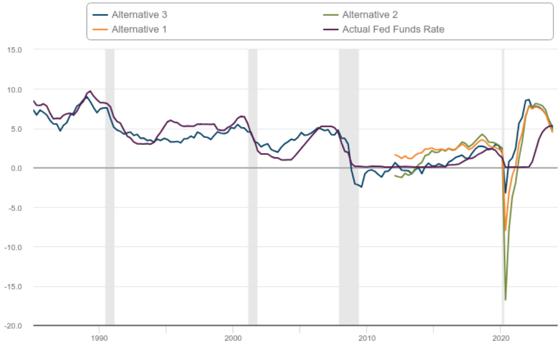


Figure 15.1 The Federal Funds Rate: Actual and Suggested
Mankiw: Macroeconomics, Ninth Edition

THE BIG 2021-22 DEVIATION

Actual Fed Funds Rate and Taylor Rule Prescriptions



	Alternative 1	Alternative 2	Alternative 3
Inflation Target Measures:	2PercentInflation	2PercentInflation	2PercentInflation
Natural Real Interest Rate Measures:	RstarFOMCMedian	RstarFOMCMedian	LWRstar1side
Resource Gap Measures:	U3gapFOMC	U3gapFOMC	CBOGDPgap
Inflation Measures:	CorePCEInflation	CorePCEInflation	CorePCEInflation
Weight on Gap:	0.5	1	0.5
Interest Rate Smoothing:	0	0	0

Source: Atlanta Fed

Federal Reserve Bank *of* Atlanta

Exported on: Sunday, January 21, 2024

Intuition behind determinacy -

The mere presence of $\phi > 1$ solves indeterminacy. How?

- Imagine that inflation is higher at date t by one log unit. Taylor rule raises the nominal interest rate by ϕ.

- Fisher equation increases expected inflation between t and $t+1$ by ϕ.

- But this in turn leads the central bank to raise i_{t+1} by ϕ^2, which raises expected inflation between $t+1$ and $t+2$ by that amount.

- The process continues so inflation keeps on rising exponentially. Inflation in T periods is larger by ϕ^T. Terminal condition rules these deviations out.

- But where does the terminal condition come from in the first place?

The elusive terminal condition

$$\lim_{T\to\infty} \phi^{-T}\, \mathbb{E}_t \left(\pi_{t+T} - \pi^*_{t+T}\right) = 0$$

- Equivalently, the random variable $\mathbb{E}_t \left(\pi_{t+T} - \pi^*_{t+T}\right)$ grows slower than ϕ.

- This is not an optimality condition. The unit of account may be exploding, but agents don't care as real outcomes continue to be finite.

- Behavioral argument: People would never believe explosive paths for inflation, $\mathbb{E}_t \left(\pi_{t+T} - \pi^*\right)$ is bounded Limited planning horizons, limited GE understanding, ... -

Economy blows up: With nominal rigidities, explosion in welfare, violate TVC.

At infinity the utility value of the wealth musy be zero otherwise she would be better off consuming more and saving less. But with explosion, prices would not stay sticky, so subtle and unclear.

- Coherence bad argument: The derivations relied on log-linearization, bounded ...

Escape clauses

- The central bank commits to a feedback rule only while inflation does not go on an explosive path. If inflation exceeds a pre-announced threshold, the central bank would switch to a different policy approach.

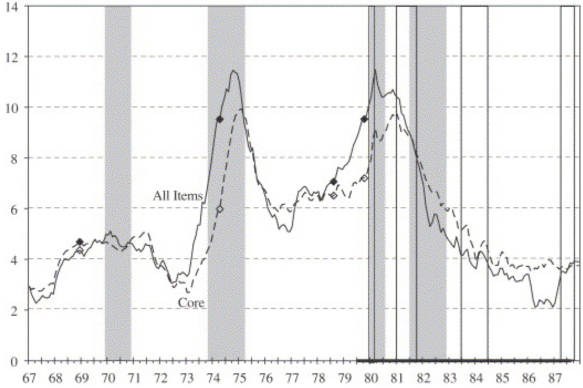
- Recall solution for inflation with a Taylor rule:

$$\pi_t = \pi^* - \sum_{j=0}^{T-t} \phi^{-j-1} \mathbb{E}_t \left[\hat{x}_{t+j}\right] + (1+\phi)^{-T+t} \mathbb{E}_t \left(\hat{\pi}_{T+1} - \pi^*\right)$$

  Switch pins down last term. Inflation is uniquely pinned down as well.

- Realistically, if inflation was rising without bound, no central bank would stick to following blindly a Taylor rule that tells it to raise interest rates more and more, even as it sees inflation rising faster and faster. ECB's monetary pillar perhaps.

ESCAPE CLAUSE: DO A PAUL VOLCKER?



Escape clause: January 2022?

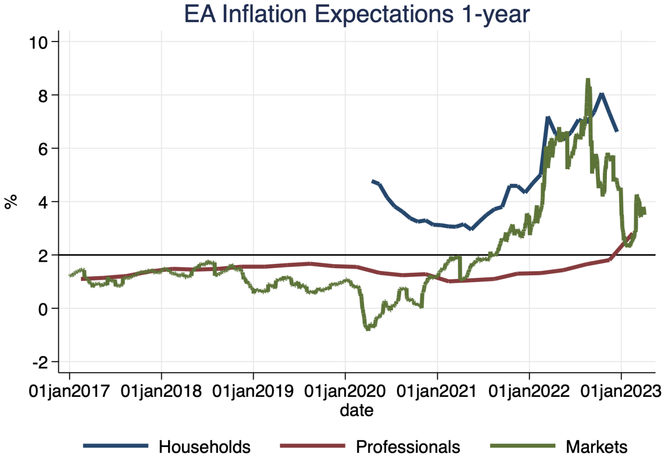
‘No more Mr Nice Guy’: Fed chair signals tougher stance on inflation

Jay Powell refuses to rule out string of aggressive rate rises to bring US prices under control

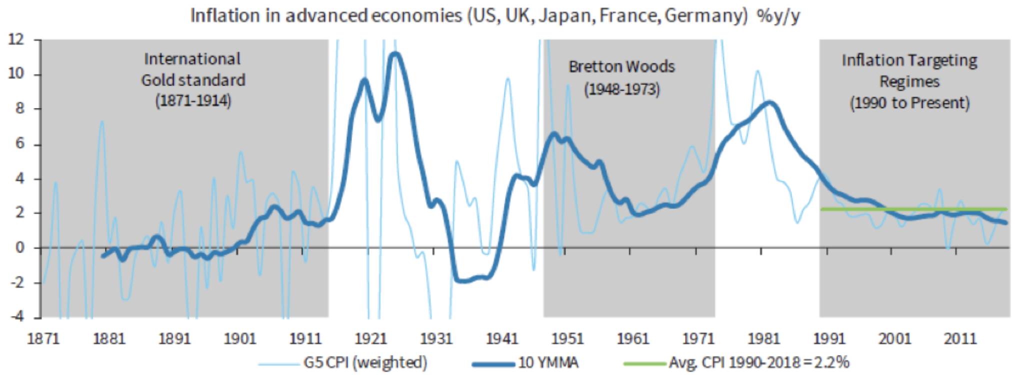
Jay Powell repeatedly dodged questions about the central bank’s thinking now that inflation appears to be persistent © Financial Times

Colby Smith in Washington JANUARY 27 2022

69



SUCCESS: CONQUEST OF INFLATION



Source: Jordà-Schularick-Taylor Macrohistory Database, Haver Analytics, Barclays Research