Lecture 17: Currency Demand

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Currency Demand

- Real-life central banks do more than manage their spreadsheet. For one, they issue banknotes and commit to exchange them for reserves one for one at all times.
- Banknotes, or currency, are distinct from reserves in five ways:
  - They can be freely held by anyone in the economy, not just banks.
  - They are physical
  - They are anonymous as people do not have to declare to the government how much currency they have or from whom they got it.
  - For some payments it may be easier to use banknotes than electronic means backed by reserves (and for others the opposite).
  - Banknotes pay no interest
- The first four properties create a demand for the services provided by banknotes separate to the demand for reserves. The fifth property implies that the opportunity cost of using banknotes is the interest rate paid on reserves.

Currency Demand

The households' problem is to maximize expected utility subject to budget constraints. The choice variables are consumption, C_t, currency, H_t, and reserves, V_t

The Lagrangian is:

$$
\begin{aligned}
L &= E_0 \sum_{t=0}^{\infty} \beta^t u\left(C_t, \frac{H_t}{P_t}\right) + \\
&\quad \sum_{t=0}^{\infty} \lambda_t \left[\begin{array}{c} P_t C_t + H_t + V_t \\ -H_{t-1} - V_{t-1}(1 + I_{t-1}) - P_t Y_t \end{array} \right]
\end{aligned}
$$

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The following are first order conditions of the households' problem:

$$\beta^t u_C\left(C_t, \frac{H_t}{P_t}\right) = \lambda_t P_t,$$

$$\beta^t u_{\frac{H}{P}}\left(C_t, \frac{H_t}{P_t}\right)\frac{1}{P_t} = \lambda_t - E_t\lambda_{t+1},$$

$$E_t\lambda_{t+1}(1 + I_t) = \lambda_t.$$

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Rearranging

$$E_t \lambda_{t+1} = \lambda_t \frac{1}{1+I_t},$$

$$\beta^t u_{\frac{H}{P}} \left(C_t, \frac{H_t}{P_t} \right) \frac{1}{P_t} = \lambda_t \left(1 - \frac{1}{1+I_t} \right),$$

$$\beta^t u_{\frac{H}{P}} \left(C_t, \frac{H_t}{P_t} \right) \frac{1}{P_t} = \lambda_t \left(\frac{I_t}{1+I_t} \right),$$

$$\frac{u_{\frac{H}{P}} \left(C_t, \frac{H_t}{P_t} \right)}{u_C \left(C_t, \frac{H_t}{P_t} \right)} = \frac{I_t}{1+I_t} \tag{1}$$

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Assume the period utility function:

$$u\left(C_t, \frac{H_t}{P_t}\right) = \log\left(C_t\right) + \log\left(\frac{H_t}{P_t}\right).$$

Then the consumer's FOC becomes:

$$\frac{P_t C_t}{H_t} = \frac{I_t}{1+I_t}, \tag{2}$$

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Now we do the loglinearization of (2).
Definition: we'll assume each variable X fluctuates around its steady state $\overline{X}$:

$$X = \overline{X} e^{\widehat{X}}$$

where $\widehat{X}$ the log deviation:

$$\widehat{X} \equiv \log \frac{X}{\overline{X}}$$

Thus:

$$\frac{P_t C_t}{H_t} = \frac{\overline{P}_t \overline{C}_t}{\overline{H}_t} e^{\widehat{\frac{P_t C_t}{H_t}}}$$

$$\begin{aligned} \widehat{\frac{P_t C_t}{H_t}} &= \log \frac{P_t C_t}{H_t} - \log \frac{\overline{P}_t \overline{C}_t}{\overline{H}_t} = \log \frac{P_t}{\overline{P}_t} + \log \frac{C_t}{\overline{C}_t} - \log \frac{H_t}{\overline{H}_t} \\ &= \widehat{P}_t + \widehat{C}_t - \widehat{H}_t \end{aligned}$$

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Now we do the loglinearization of $\frac{I_t}{1+I_t}$

$$\widehat{\frac{I_t}{1+I_t}} = \log \frac{I_t}{1+I_t} - \log \frac{\overline{I_t}}{1+\overline{I}_t}$$

Subtract to get log-deviation:

$$\log \frac{I_t}{1+I_t} - \log \frac{\overline{I_t}}{1+\overline{I}_t} = \log \frac{I_t}{\overline{I}_t} - \left[\log\left(1+I_t\right) - \log\left(1+\overline{I}_t\right)\right]$$

Expand $\log\left(1+I_t\right)$ around $\overline{I_t}$

$$\log\left(1+I_t\right) \approx \log\left(1+\overline{I}_t\right) + \frac{1}{1+\overline{I}_t}\left(I_t - \overline{I}_t\right)$$

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Now

$$\left(I_t - \overline{I}_t\right) \approx \overline{I}_t \widehat{I}_t$$

as

$$\widehat{I}_t = \log\left(\frac{I_t}{\overline{I}_t}\right) \approx \log(1) + \frac{1}{\overline{I}_t}\left(I_t - \overline{I}_t\right)$$

So

$$\log(1 + I_t) - \log\left(1 + \overline{I}_t\right) \approx \frac{\overline{I}_t}{1 + \overline{I}_t}\widehat{I}_t$$

and

$$\begin{aligned}
\widehat{\frac{I_t}{1 + I_t}} &= \log\frac{I_t}{1 + I_t} - \log\frac{\overline{I}_t}{1 + \overline{I}_t} \\
&\approx \widehat{I}_t - \frac{\overline{I}_t}{1 + \overline{I}_t}\widehat{I}_t = \frac{1}{1 + \overline{I}_t}\widehat{I}_t
\end{aligned}$$

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Thus the loglinearization of (2) yields

$$\widehat{P}_t + \widehat{C}_t - \widehat{H}_t = \frac{1}{1 + \bar{I}_t}\widehat{I}_t$$

or

$$\widehat{H}_t - \widehat{P}_t = \widehat{Y}_t - \eta \widehat{I}_t$$

where $\eta = \frac{1}{1+\bar{I}_t}$ and $\widehat{Y}_t = \widehat{C}_t$

The real demand for money depends negatively on the interest rate (between t and $t+1$) and positively on the current output

Notation: $x = \widehat{X}$ then

$$h_t - p_t = y_t - \eta i_t$$