Consider an economy similar to the one studied in class with 3 periods: 1, 2 and 3, but with a **different timing** of events. The household starts period t with M_{t-1} units of money and B_{t-1} units of bonds. First, the household trades in the asset market and gets a transfer from the government in the asset market. Second, the household exerts labor effort L_t to produce its good. Third, the seller and buyer split up and go to their respective markets where they exchange goods for money. Finally, they jointly consume the consumption good. This timing is known in the literature as the Lucas timing.

The instantaneous utility of the representative household is:

$$u\left(C_t, L_t\right) \tag{1}$$

The household starts period t with nominal wealth $\mathcal{W}_t$ and decides the portfolio of assets: B_t bonds and M_t money,

$$q_t B_t + M_t \leq \mathcal{W}_t + T_t,\ \ t = 1, 2 \tag{2}$$

where T_t is the net transfer that the government is making in cash to the household.

The purchases of consumption have to be made with cash, i.e.

$$P_t C_t \leq M_t,\ \ t = 1, 2 \tag{3}$$

where P_t is the price level.

The wealth level at the beginning of period $t+1$:

$$\mathcal{W}_{t+1} = P_t Z_t L_t + \left(M_t - P_t C_t\right) + B_t,\ t = 1, 2 \tag{4}$$

The budget constraint can be written as:

$$q_{t+1} B_{t+1} + M_{t+1} - T_{t+1} \leq P_t Z_t L_t + \left(M_t - P_t C_t\right) + B_t,\ t = 1, 2$$

The Lagrangian is:

$$
\begin{aligned}
\mathcal{L} = & \max_{\{C_t, L_t, M_t, B_t\}_{t=1,2}} \min_{\{\lambda_t, \mu_t\}_{t=1,2}} \\
& \sum_{t=1,2} \beta^{t-1} u(C_t, L_t) + \beta^2 V(\mathcal{W}_3) \\
& + \sum_{t=1,2} \lambda_t \{M_t - P_t C_t\} \\
& + \sum_{t=1,2} \mu_t \{P_t Z_t L_t + (M_t - P_t C_t) + B_t - q_{t+1} B_{t+1} - M_{t+1} + T_{t+1}\}.
\end{aligned}
$$

1. Show that the first order conditions are:

$$\beta^{t-1} u_c'(C_t, L_t) = (\lambda_t + \mu_t) P_t$$

$$-\beta^{t-1} u_L'(C_t, L_t) = \mu_t P_t Z_t$$

$$\mu_{t-1} = \lambda_t + \mu_t$$

$$\mu_t = q_t \mu_{t-1}$$

2. Show these equations can be manipulated to obtain:

$$\frac{u_c'(C_t, L_t)}{P_t} = \frac{1}{q_t} \frac{\beta u_c'(C_{t+1}, L_{t+1})}{P_{t+1}}$$

and

$$\frac{-u_L'(C_t, L_t)}{u_c'(C_t, L_t)} = q_t Z_t.$$

3. Give the interpretation (intuition) for each equation.

4. Let TT be total time. Define leisure as $\boldsymbol{l}_t = TT - L_t$. What is the marginal rate of substitution between leisure and consumption when $\lambda_t = 0$, (i..e. the cash in advance constraint is not binding)? And what

is the marginal rate of substitution between leisure and consumption when $\lambda_t > 0$? What is the relative price of leisure?

5. Using the timing considered in class (where the goods market opens first) these conditions become

$$\frac{u_c'(C_t, L_t)}{P_t} = \frac{1}{q_{t-1}} \frac{\beta u_c'(C_{t+1}, L_{t+1})}{P_{t+1}}$$

and

$$\frac{-u_L'(C_t, L_t)}{u_c'(C_t, L_t)} = q_{t-1} Z_t.$$

What is the intuition? Hint: In this case the quantity of money that is used in period t for consumption is decided in the asset market of period $t-1$.

6. Consider the Lucas timing. Assume a particular utility function, $u(C_t, L_t) = \frac{1}{1-\sigma}\left(C_t - \frac{(L_t)^{1+\chi}}{1+\chi}\right)^{1-\sigma}$, σ and χ are parameters.

Show that

$$N_t^{\chi} = q_t Z_t,$$

For this particular utility function the labor supply is only a function of the "relevant" productivity. There are no income effects on the labor supply.

7. Assume the central bank follows an interest rate target. Denote the gross nominal interest rate by R_t. Express the equilibrium for consumption and hours as a function of the exogenous variables: Z_t and R_t.

8. What are the effects in consumption and hours of an increase in the interest rate target? And of an increase in productivity?

9. Now, suppose the central bank targets the money supply. In this case, the money supply is exogenous and the nominal interest rate is endogenously determined. What happens to prices if the central bank decides to double the quantity of money? And to production?

10. Suppose 2 countries: A and B. These countries have identical economies, with the same: utility function, production function and market structures. Both are in the steady state. They differ only on the growth rate of money: the growth rate of money in country A is τ^A and in country B is τ^B with $\tau^A > \tau^B$. Compare the equilibria of the countries.