

I (5/20 points)

Choose the answer you consider to be more correct to each of the following questions (**right answers get 1,0/20 each and wrong answers a penalty of 0,25/20 each, with a cap for the penalties = 1,0/20**):

1. In an investment in a fixed-rate coupon bond:
 - a. the investor is benefited when the yield-to-maturity increases after the bond purchase;
 - b. the bond price is the present value of the future coupons to be paid.
 - c. when the coupon rate is equal to the yield-to-maturity, the bond is said to be at par.
 - d. the bond price at the maturity date is unknown, as one cannot fully predict the future behavior of interest rates.
2. When investors in fixed-rate bonds expect short-term interest rates to move up:
 - a. the slope of the yield curve will increase, if one assumes the segmentation theory to hold.
 - b. they must protect the value of their bond portfolio by decreasing its duration and implementing duration hedging strategies.
 - c. the bond price will increase exactly according to the bond duration and the magnitude of the interest rate increases, regardless this magnitude.
 - d. prices of zero-coupon bonds will not be impacted, as their cash-flows do not change with interest rates.
3. Duration is:
 - a. measured as the tangent to the yield curve in the point corresponding to the current yield;
 - b. a sensitivity measure of bond prices to interest rate changes, assuming discretely compounded interest rates;
 - c. always lower than the residual maturity;
 - d. the weighted-average maturity of all the cash-flows to be paid by the bond, where the weights are given by the contribution of the present value of each cash-flow to the bond price.
4. The Security Market Line:
 - a. is a representation of the expected return of an asset as a function of the risk, characterized by the standard-deviation of the asset returns
 - b. illustrates the role of the covariance between the asset and the market returns in computing the expected asset return.
 - c. provides an estimate for the asset return only from its β .
 - d. provides an estimate for the asset return equal to 0 when the β is also zero.
5. The price of a 1-year zero-coupon bond, with a redemption value of 1000€ and a yield-to-maturity of 2% is:
 - a. €1000;
 - b. €980;
 - c. €1,020;
 - d. €980,2, assuming continuously compounded interest rates.

II (15/20 points)

Assuming that a stock price is 30€ and is in equilibrium according to the CAPM, with an expected return of 25%, a risk-free of 2% and a market price of risk of 20:

1. Compute the β . **(2,0/20)**
2. Represent the security market line. **(1,5/20)**
3. What would be the impact on the expected return if the stock price started to exhibit the behavior of a representative market index? **(1,5/20)**
4. What would be the dividend to be paid to ensure that the stock is adequately priced, assuming that its growth rate is 6% and the discount factor is 10%. **(1,5/20)**
5. Compute the expected return and the standard-deviation of a portfolio composed by the stock previously mentioned, with a weight of 60% and a basket replicating a representative stock index, assuming that the index's market price of risk is 2 times that of the stock. **(2,0/20)**
6. What is the most efficient portfolio exclusively composed by the asset and the index mentioned, assuming that short sales and borrowing and lending at the risk-free rate are allowed? **(2,5/20)**
7. Consider an investor who bought 100 of the shares referred at the price per share mentioned, paying in cash and by borrowing:
 - 7.1. Assuming an initial margin requirement of 60%, what would be the maximum amount the investor could borrow. **(2,0/20)**
 - 7.2. Assuming that the investor had borrowed the maximum amount allowed according to the initial margin requirement and that the maintenance margin is equal to the former, what would be the consequences for the investor if the stock price increased or decreased by 30% **(2,0/20)**.

Formulas

Bonds:

Price (P):
$$P = \sum_{n=1}^N \frac{C_n}{(1+y)^n} + \frac{M}{(1+y)^N}$$

Duration:
$$D = \frac{\sum_{n=1}^T n \cdot ce^{-yn} + T \cdot FVe^{-yT}}{P} = 1 \cdot \frac{ce^{-y}}{P} + 2 \cdot \frac{ce^{-2y}}{P} + 3 \cdot \frac{ce^{-3y}}{P} + \dots + T \cdot \frac{ce^{-yT}}{P} + T \cdot \frac{FVe^{-yT}}{P}$$

CAPM:

Security market line:
$$\bar{R}_i = R_F + \beta_i(\bar{R}_M - R_F)$$

Beta:
$$\beta_i = \frac{\sigma_{iM}}{\sigma_M^2}$$

Gordon Model:
$$P_0 = \frac{D_1}{k - g}$$

Margins:
$$\text{Margin} = \frac{\text{Market Value of Assets} - \text{Amount borrowed}}{\text{Market Value of Assets}}$$

Portfolio of 2 assets:

Expected return:
$$\bar{R}_p = E(R_p) = E\left(\sum_{i=1}^2 X_i R_i\right)$$

Variance of the return:
$$\sigma_p^2 = X_1^2 \sigma_1^2 + X_2^2 \sigma_2^2 + 2X_1 X_2 \sigma_{12}$$

Equations for the portfolio allocation when short sales and lending and borrowing at the R_F are allowed:

$$\bar{R}_1 - R_F = Z_1 \sigma_1^2 + Z_2 \sigma_{12} + Z_3 \sigma_{13} + \dots + Z_N \sigma_{1N}$$

$$\bar{R}_2 - R_F = Z_1 \sigma_{12} + Z_2 \sigma_2^2 + Z_3 \sigma_{23} + \dots + Z_N \sigma_{2N}$$

$$\bar{R}_3 - R_F = Z_1 \sigma_{13} + Z_2 \sigma_{23} + Z_3 \sigma_3^2 + \dots + Z_N \sigma_{3N}$$

$$\vdots$$

$$\bar{R}_N - R_F = Z_1 \sigma_{1N} + Z_2 \sigma_{2N} + Z_3 \sigma_{3N} + \dots + Z_N \sigma_N^2$$