Lisbon School of Economics & Management
Universidade de Lisboa

STATISTICS I

Bachelor's degrees in Economics and Finance

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Pratical Class 4

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https://doity.com.br/estatistica-aplicada-a-nutricao

https://basiccode.com.br/produto/informatica-basica/

Exercises: 13, 15, 16, 17, 18, 20, 21

1 Continuous Random Variable: Exercises

Probability Density Funcion (PDF) and Cumulative Distribution Function (CDF)

13. The probability density of the random variable Z is given by

$$f_Z(z) = \begin{cases} kze^{-z^2} & z > 0 \\ 0 & z \leq 0 \end{cases}$$

Find k.

Exercise 13

$$f_Z(z) = \begin{cases} K z e^{-z^2} & (z > 0) \\ 0 & (z \leq 0) \end{cases}$$

$f_Z(z)$ is a probability density function, so it must verify $\int_{-\infty}^{+\infty} f_Z(z)\, dz = 1$.

$$\int_{-\infty}^{+\infty} f_Z(z)\, dz = \int_{-\infty}^{0} 0\, dz + \int_{0}^{+\infty} K z \cdot e^{-z^2}\, dz = 1 \Leftrightarrow$$

$$\Leftrightarrow 0 + K \lim_{b \to +\infty} \left[-\frac{e^{-z^2}}{2} \right]_0^b = \frac{K}{2} \lim_{b \to +\infty} \left[-e^{-z^2} \right]_0^b = \frac{K}{2} \lim_{b \to +\infty} \left(-e^{-b^2} - (-e^0) \right) = 1$$

$$\Leftrightarrow \frac{K}{2} \left(1 - \underbrace{\lim_{b \to +\infty} e^{-b^2}}_{0} \right) = 1 \Leftrightarrow$$

$$\Leftrightarrow \frac{K}{2} = 1 \Leftrightarrow K = 2$$

Auxiliar calculations:

$$(e^u)' = u' e^u$$

$$\int u' e^u = e^u$$

$$\int z e^{-z^2} dz = -\frac{1}{2} \int \underbrace{-2z}_{u'} e^{\overset{u}{-z^2}} dz =$$

$$= -\frac{1}{2} e^{-z^2}$$

14. Find the cumulative distribution function of the random variable X whose probability density is given by

$$f_X(x) = \begin{cases} \frac{1}{3}, & 0 < x < 1 \\ \frac{1}{3}, & 2 < x < 4 \\ 0, & \text{elsewhere.} \end{cases}$$

Also sketch the graphs of the probability density and distribution functions.

Exercise 14

I found F(x) using only graphical inspection and the formula for the area of rectangles. But here are the auxiliary calculations to obtain F(x) via integration:

$$x < 0 \quad \Rightarrow \quad F_x(x) = 0$$

$$0 < x < 1 \quad \Rightarrow \quad F_x(x) = \int_0^x f_x(u)\,du = \int_0^x \frac{1}{3}\,du = \left[\frac{1}{3}u\right]_0^x = \frac{x}{3} - 0 = \frac{x}{3}$$

$$1 < x < 2 \quad \Rightarrow \quad F_x(x) = \int_0^x f_x(u)\,du = \int_0^1 f_x(x)\,dx + \int_1^x f_x(u)\,du =$$

$$= \int_0^1 \frac{1}{3}\,dx + \int_1^x 0\,dx = \frac{1}{3}[x]_0^1 + [K]_1^x = \qquad (K \in \mathbb{R})$$

$$= \frac{1}{3} - 0 + K - K = \frac{1}{3}$$

Exercise 14

$$2 < x < 4 \Rightarrow F_X(x) = \int_0^x f_X(u)\,du = \int_0^1 f_X(x)\,dx + \int_1^2 f_X(x)\,dx + \int_2^x f_X(u)\,du =$$

$$= \int_0^1 \frac{1}{3}\,dx + \int_1^2 0\,dx + \int_2^x \frac{1}{3}\,du =$$

$$= \frac{1}{3} + 0 + \frac{1}{3}[u]_2^x = \frac{1}{3} + \frac{1}{3}(x-2)$$

$$x > 4 \Rightarrow F_X(x) = \int_0^x f_X(u)\,du = \int_0^1 f_X(x)\,dx + \int_1^2 f_X(x)\,dx + \int_2^4 f_X(x)\,dx + \int_4^x f_X(u)\,du =$$

$$= \int_0^1 \frac{1}{3}\,dx + \int_1^2 0\,dx + \int_2^4 \frac{1}{3}\,dx + \int_4^x 0\,dx$$

$$= \frac{1}{3} + 0 + \frac{1}{3}[x]_2^4 + [K]_4^x =$$

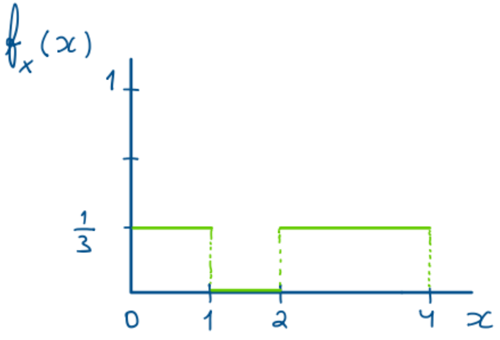
$$= \frac{1}{3} + \frac{1}{3}(4-2) + K - K = \qquad (K \in \mathbb{R})$$

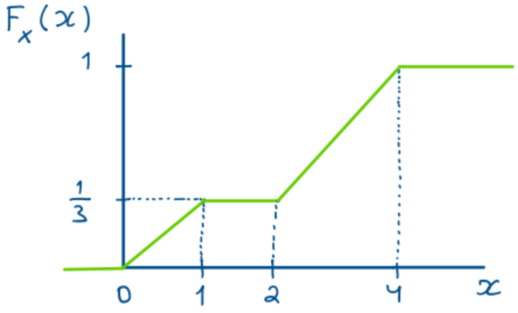
$$= \frac{1}{3} + \frac{2}{3} = 1$$

Exercise 14

$$f_X(x) = \begin{cases} \frac{1}{3}, & 0 < x < 1 \\ \frac{1}{3}, & 2 < x < 4 \\ 0, & \text{elsewhere.} \end{cases}$$

$$F_x(x) = \begin{cases} 0 & (x < 0) \\ \frac{x}{3} & (0 \leq x < 1) \\ \frac{1}{3} & (1 \leq x < 2) \\ \frac{1}{3} + \frac{x-2}{3} = \frac{x-1}{3} & (2 \leq x < 4) \\ 1 & (x \geq 4) \end{cases}$$





15. Find the cumulative distribution function of the random variable X whose probability density is given by

$$f_X(x) = \begin{cases} \frac{x}{2}, & 0 < x \leq 1 \\ \frac{1}{2}, & 1 < x \leq 2 \\ \frac{3-x}{2}, & 2 < x < 3 \\ 0, & \text{elsewhere.} \end{cases}$$

Also sketch the graphs of these probability density and distribution functions.

Exercise 15

$$f_X(x) = \begin{cases} \frac{x}{2}, & 0 < x \leq 1 \\ \frac{1}{2}, & 1 < x \leq 2 \\ \frac{3-x}{2}, & 2 < x < 3 \\ 0, & \text{elsewhere.} \end{cases}$$

Auxiliary calculations:

$x < 0$: $F_X(x) = 0$

$0 < x < 1$: $F_X(x) = \int_0^x \frac{u}{2} \, du = \frac{1}{2} \left[\frac{u^2}{2} \right]_0^x = \frac{x^2}{4}$

$1 < x < 2$: $F_X(x) = \int_0^1 \frac{x}{2} \, dx + \int_1^x \frac{1}{2} \, du = \left[\frac{x^2}{4} \right]_0^1 + \left[\frac{u}{2} \right]_1^x = \frac{1}{4} + \frac{x}{2} - \frac{1}{2} = \frac{x}{2} - \frac{1}{4}$

$2 < x < 3$: $F_X(x) = \int_0^1 \frac{x}{2} \, dx + \int_1^2 \frac{1}{2} \, dx + \int_2^x \frac{3-u}{2} \, du =$

$= \frac{1}{4} + \left[\frac{x}{2} \right]_1^2 + \int_2^x \frac{3-u}{2} \, du = \frac{1}{4} + \frac{1}{2} + \frac{1}{2} \left[3u - \frac{u^2}{2} \right]_2^x =$

$= \frac{3}{4} + \frac{1}{2} \left(3x - \frac{x^2}{2} - 4 \right) =$

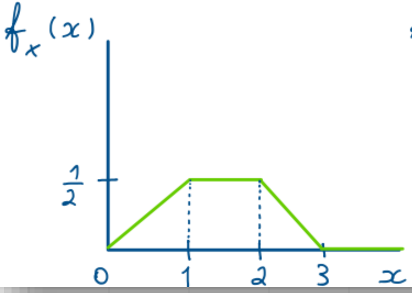
$= \frac{3}{4} + \frac{3}{2} x - \frac{x^2}{4} - 2 =$

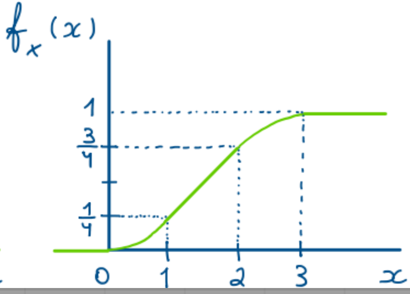
$= -\frac{x^2}{4} + \frac{3}{2} x - \frac{5}{4}$

Exercise 15

Conclusion:

$$F_X(x) = \begin{cases} 0 & (x < 0) \\ \dfrac{x^2}{4} & (0 \leq x < 1) \\ \dfrac{x}{2} - \dfrac{1}{4} & (1 \leq x < 2) \\ -\dfrac{x^2}{4} + \dfrac{3}{2}x - \dfrac{5}{4} & (2 \leq x < 3) \\ 1 & (x \geq 3) \end{cases}$$





16. The cumulative distribution function of the random variable X is given by

$$F_X(x) = \begin{cases} 1 - (1 + x)e^{-x} & \text{for } x > 0 \\ 0 & \text{elsewhere} \end{cases}$$

Find $P(X \leq 2)$, $P(1 < X < 3)$, and $P(X > 4)$.

Exercise 16

$$F_X(x) = \begin{cases} 1-(1+x)e^{-x} & (x>0) \\ 0 & (\text{elsewhere}) \end{cases}$$

$$P(X \leq 2) = F_X(2) = 1-(1+2)e^{-2} = 1-3e^{-2}$$

$$P(1 < X < 3) = F_X(3) - F_X(2) =$$

$$= 1-(1+3)e^{-3} - (1-(1+1)e^{-1}) =$$

$$= 1-4e^{-3} - (1-2e^{-1}) =$$

$$= -4e^{-3} + 2e^{-1}$$

$$P(X>4) = 1 - F_X(4) = 1-(1-(1+4)e^{-4}) =$$

$$= 5e^{-4}$$

17. The distribution function of the random variable X is given by

$$F_X(x) = \begin{cases} 0, & x < -1 \\ \frac{x+1}{2}, & -1 \leq x < 1 \\ 1, & x \geq 1. \end{cases}$$

Find

a) the probability function f_X;

b) $P(-\frac{1}{2} \leq X \leq -\frac{1}{2})$;

c) $P(2 < X < 3)$.

Exercise 17 a)

$$F_X(x) = \begin{cases} 0 & (x < -1) \\ \frac{x+1}{2} & (-1 \leq x < 1) \\ 1 & (x \geqslant 1) \end{cases}$$

a)

$f_X(x) = F_X'(x)$ for the values of x such that $F_X'(x)$ exists

For $x < 1$ we have $F_X'(x) = 0$

For $-1 < x < 1$ we have $F'_X(x) = \frac{1}{2}$

For $x > 1$ we have $F'_X(x) = 0$

Conclusion:

$$f_X(x) = \begin{cases} \frac{1}{2} & (-1 < x < 1) \\ 0 & (\text{elsewhere}) \end{cases}$$

Exercise 17 b) and c)

b)

$$P\left(-\frac{1}{2} \leqslant X \leqslant -\frac{1}{2}\right) = F_X\left(-\frac{1}{2}\right) - F_X\left(-\frac{1}{2}\right) = 0$$

$$= P\left(X = -\frac{1}{2}\right) = 0$$

c)

$$P(2 < X < 3) = F_X(3) - F_X(2) = 1 - 1 = 0$$

18. The cumulative distribution function of the random variable Z is given by

$$F_Z(z) = \begin{cases} 0 & \text{for } z < -2 \\ \frac{z+4}{8} & \text{for } -2 \leq z < 2 \\ 1 & \text{for } z \geq 2 \end{cases}$$

a) Sketch the graph of the distribution function F_Z;

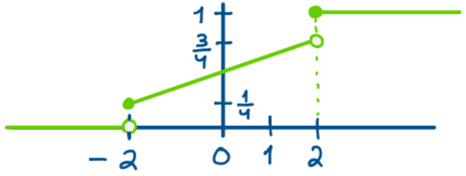
b) Is Z a continuous random variable? Why?

c) Compute $P(Z = -2)$, $P(Z = 2)$, $P(-2 < Z < 1)$, and $P(0 \leq Z \leq 2)$.

Exercise 18 a) and b)

$$F_Z(z) = \begin{cases} 0 & (z < -2) \\ (z+4)/8 & (-2 \leq z < 2) \\ 1 & (z \geq 2) \end{cases}$$

a)



b)

No, because $F_Z(z)$ is not continuous for all z.

$D_Z = \{-2, 2\}$ are discontinuity points.

Z is a mixed random variable.

Exercise 18 c)

c)

$$P(Z=-2) = F_Z(-2) - F_Z(-2^-) = \frac{1}{4} - 0 = \frac{1}{4}$$

$$P(Z=2) = F_Z(2) - F_Z(2^-) = 1 - \frac{3}{4} = \frac{1}{4}$$

$$P(-2 < Z < 1) = F_Z(1^-) - F_Z(-2) = F(1) - F_Z(-2)$$

$$= \frac{5}{8} - \frac{1}{4} = \frac{3}{8}$$

$$P(0 \leqslant Z \leqslant 2) = F_Z(2) - F_Z(0) = 1 - \frac{4}{8} = \frac{1}{2}$$

19. Let X be a random variable with cumulative distribution function given by

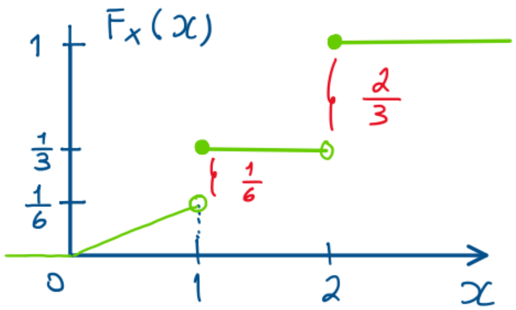
$$F_X(x) = \begin{cases} 0, & x < 0 \\ \frac{1}{6}x, & 0 \le x < 1 \\ \frac{1}{3}, & 1 \le x < 2 \\ 1, & x \ge 2 \end{cases}.$$

a) Prove that X is a mixed random variable;

b) Calculate the probabilities: $P(X < 1/2)$, $P(X < 3/2)$, $P(1/2 < X < 2)$, $P(X = 1)$, $P(X > 1)$, $P(X = 2)$.

Exercise 19

$$F_X(x) = \begin{cases} 0 & (x < 0) \\ \frac{1}{6}x & (0 \leq x < 1) \\ \frac{1}{3} & (1 \leq x < 2) \\ 1 & (x \geq 2) \end{cases}$$



$$f_X(1) = F_X(1) - F_X(1^-) = \frac{1}{3} - \frac{1}{6} = \frac{1}{6} > 0$$

$$f_X(2) = F_X(2) - F_X(2^-) = 1 - \frac{1}{3} = \frac{2}{3} > 0$$

$D_X = \{1, 2\} \neq \emptyset \Rightarrow X$ is not a continuous r.v.

$P(X \in D_X) = \frac{1}{6} + \frac{2}{3} = \frac{5}{6} > 0 \Rightarrow X$ is not discrete

20. If the probability density of X is given by

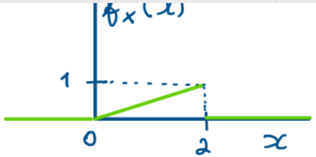
$$f_X(x) = \begin{cases} \frac{x}{2}, & 0 < x < 2 \\ 0, & \text{elsewhere} \end{cases}.$$

a) Compute the cumulative distribution function of $Y = X^3$.

b) Compute the probability density function of $Y = X^3$.

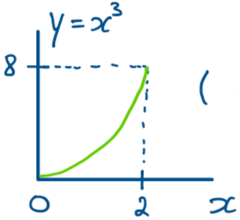
Exercise 20 a)

$$f_X(x) = \begin{cases} \frac{x}{2} & (0 < x < 2) \\ 0 & (\text{elsewhere}) \end{cases}$$



a)

$$Y = X^3$$



$$(0 < x < 2 \rightarrow 0 < y < 8)$$

$$F_X(x) = \begin{cases} 0 & (x < 0) \\ \int_0^x f_X(u)\, du = \int_0^x \frac{u}{2}\, du = \frac{1}{2}\left[\frac{u^2}{2}\right]_0^x = \frac{x^2}{4} & (0 \leq x < 2) \\ 1 & (x \geq 2) \end{cases}$$

Exercise 20 a)

For $0 < y < 8$:

$x > 0$

$$F_Y(y) = P(Y \leq y) = P(X^3 \leq y) =$$

$$= P(X \leq \sqrt[3]{y}) = F_X(\sqrt[3]{y}) = \frac{(\sqrt[3]{y})^2}{4} = \frac{y^{\frac{2}{3}}}{4}$$

The complete c.d.f. of Y is :

$$F_Y(y) = \begin{cases} 0 & (y < 0) \\ \frac{y^{\frac{2}{3}}}{4} & (0 \leq y < 8) \\ 1 & (y \geq 8) \end{cases}$$

Exercise 20 b)

b)

$$f_Y(y) = \begin{cases} F_Y'(y) = \dfrac{\frac{2}{3} y^{-\frac{1}{3}}}{4} = \dfrac{1}{6\sqrt[3]{y}} & (0 < y < 8) \\ 0 & (\text{otherwise}) \end{cases}$$

21. Let X be a discrete random variable such that

$$P(X = x) = \frac{1}{7}, \quad \text{with } x \in \{-3, -2, -1, 0, 1, 2, 3\}.$$

Determine the probability function of $Y = X^2 - 3X$.

Exercise 21

$P(X = x) = \frac{1}{7} \qquad (x = -3, -2, -1, 0, 1, 2, 3)$

$Y = X^2 - 3X$

x	-3	-2	-1	0	1	2	3
$y = x^2 - 3x$	18	10	4	0	-2	-2	0

$Y = \psi(X)$ Y is a function of X

$f_Y(y) = \sum_{x \in D_X : \psi(x) = y} f_X(x) \qquad (y \in D_Y)$

$D_Y = \{-2, 0, 4, 10, 18\}$

Exercise 21

$$f_Y(-2) = f_X(1) + f_X(2) = \frac{2}{7}$$

$$f_Y(0) = f_X(0) + f_X(3) = \frac{2}{7}$$

$$f_Y(4) = f_X(-1) = \frac{1}{7}$$

$$f_Y(10) = f_X(-2) = \frac{1}{7}$$

$$f_Y(18) = f_X(-3) = \frac{1}{7}$$

Conclusion:
$$f_Y(y) = \begin{cases} \frac{2}{7} & (y = -2, 0) \\ \frac{1}{7} & (y = 4, 10, 18) \\ 0 & (\text{otherwise}) \end{cases}$$

22. If the probability density of X is given by

$$f_X(x) = \begin{cases} \frac{3x^2}{2}, & -1 < x < 1 \\ 0, & \text{elsewhere} \end{cases}.$$

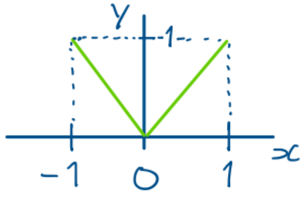
Determine the cumulative distribution function of $Y = |X|$ and $Z = X^2$.

Exercise 22

$$f_X(x) = \begin{cases} \frac{3x^2}{2}, & -1 < x < 1 \\ 0, & \text{elsewhere} \end{cases}$$

$$F_X(x) = \begin{cases} 0 & (x < -1) \\ \int_{-1}^{x} \frac{3u^2}{2}\,du = \frac{3}{2}\left[\frac{u^3}{3}\right]_{-1}^{x} = \frac{3}{6}\left(x^3 - (-1)^3\right) = \frac{x^3+1}{2} & (-1 \leq x < 1) \\ 1 & (x \geq 1) \end{cases}$$

$$Y = |X| \qquad (-1 < x < 1 \quad \Rightarrow \quad 0 \leq y < 1)$$



$$F_Y(y) = P(Y \leq y) = P(|X| \leq y) = P(-y \leq x \leq y)$$

$$= F_X(y) - F_X(-y) = \frac{y^3}{2} - \frac{(-y)^3}{2} =$$

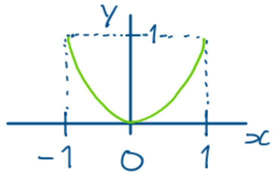
$$= \frac{y^3}{2} + \frac{y^3}{2} = y^3 \qquad (0 < y < 1)$$

Exercise 22

$$F_Y(y) = \begin{cases} 0 & (y < 1) \\ y^3 & (0 \leqslant y \leqslant 1) \\ 1 & (y \geqslant 1) \end{cases}$$

$$f_Y(y) = F_Y'(y) = \begin{cases} 3y^2 & (0 < y < 1) \\ 0 & (\text{elsewhere}) \end{cases}$$

$$Z = X^2 \quad (-1 < x < 1 \rightarrow 0 < z < 1)$$



$$F_Z(z) = P(Z \leqslant z) = P(X^2 \leqslant z) =$$

$$= P(-\sqrt{z} \leqslant X \leqslant \sqrt{z}) = F_X(\sqrt{z}) - F_X(-\sqrt{z}) =$$

$$= \frac{(\sqrt{z})^3}{2} - \frac{(-\sqrt{z})^3}{2} = \frac{z^{\frac{3}{2}}}{2} + \frac{z^{\frac{3}{2}}}{2} =$$

$$= z^{\frac{3}{2}} \quad (0 < z < 1)$$

Exercise 22

$$F_Z(z) = \begin{cases} 0 & (z < 0) \\ z^{\frac{3}{2}} & (0 \leq z < 1) \\ 1 & (z \geqslant 1) \end{cases}$$

$$f_Z(z) = F_Z'(z) = \begin{cases} \frac{3}{2}\sqrt{z} & (0 < z < 1) \\ 0 & (\text{elsewhere}) \end{cases}$$

23. A company has received €1 million to invest in one of two projects. With probability 1/3 the firm invests in project 1, and, with probability 2/3, the firm invests in project 2. Let X_1 be a discrete random variable that represents the return of project 1, in millions of €, and X_2 a continuous random variable that represents the return , of project 2, in millions of €. The cumulative distribution function of X_1 is given by

$$F_{X_1}(x) = \begin{cases} 0, & x < 0 \\ 1/3, & 0 \le x < 1 \\ 1/2, & 1 \le x < 2 \\ 7/10, & 2 \le x < 3 \\ 1, & x \ge 3 \end{cases},$$

and the probability density function of X_2 is given by

$$f_{X_2}(x) = \begin{cases} 1/3, & 0 < x < 1 \\ \frac{4}{45}x, & 1 \le x < 4 \\ 0, & \text{otherwise} \end{cases}$$

(a) Compute the probability function of X_1.

(b) Compute the cumulative distribution function of X_2

(c) If the firm invests in project 1, what is the probability that it gets back more than 2 million? Compute the same probability in case the firm invests in project 2.

(d) Compute $P(X_2 > 1 | X_2 < 3)$.

(e) Let X be the amount, in millions of €, received by the company after its investment.

   i. Find the cumulative distribution function of X. Classify the random variable.

   ii. Compute the probability that the firm receives at least 3 million.

   iii. Compute the probability that the firm has a negative profit (profit = amount received minus amount invested).

Exercise 23 a)

$X_1 \equiv$ Return of project 1 (in millions of €) Discrete R.V.

$X_2 \equiv$ " " " 2 (" " " ") Continuous R.V.

$$F_{X_1}(x) = \begin{cases} 0, & x < 0 \\ 1/3, & 0 \leq x < 1 \\ 1/2, & 1 \leq x < 2 \\ 7/10, & 2 \leq x < 3 \\ 1, & x \geq 3 \end{cases},$$

a)

$D_{X_1} = \} 0, 1, 2, 3 \{$

$f_{X_1}(x) = F_{X_1}(x) - F_X(x^-), \quad x \in D_X$

$f_X(0) = \frac{1}{3} - 0 = \frac{1}{3}$

$f_X(1) = \frac{1}{2} - \frac{1}{3} = \frac{3}{6} - \frac{2}{6} = \frac{1}{6}$

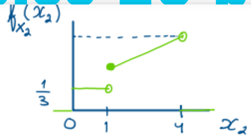
$f_X(2) = \frac{7}{10} - \frac{1}{2} = \frac{7}{10} - \frac{5}{10} = \frac{2}{10} = \frac{1}{5}$

$f_X(3) = 1 - \frac{7}{10} = \frac{3}{10}$

$$f_X(x) = \begin{cases} \frac{1}{3} & (x = 0) \\ \frac{1}{6} & (x = 1) \\ \frac{1}{5} & (x = 2) \\ \frac{3}{10} & (x = 3) \\ 0 & (\text{other } x) \end{cases}$$

Exercise 23 b), c) and d)

$$f_{X_2}(x) = \begin{cases} 1/3, & 0 < x < 1 \\ \frac{4}{45}x, & 1 \le x < 4 \\ 0, & \text{otherwise} \end{cases}$$



$$F_{X_2}(x) = \begin{cases} 0 & (x < 0) \\ \int_0^x \frac{1}{3}\,du = \left[\frac{u}{3}\right]_0^x = \frac{x}{3} & (0 \le x < 1) \\ \frac{1}{3} + \int_1^x \frac{4}{45}u\,du = \frac{1}{3} + \frac{4}{45}\left[\frac{u^2}{2}\right]_1^x = \frac{1}{3} + \frac{2}{45}(x^2 - 1) = \frac{1}{3} + \frac{2x^2}{45} - \frac{2}{45} = \frac{39}{135} + \frac{2x^2}{45} = \frac{13}{45} + \frac{2x^2}{45} & (1 \le x < 4 \\ 1 & (x \ge 4) \end{cases}$$

c)

$$P(X_1 > 2) = 1 - F_{X_1}(2) = 1 - \frac{7}{10} = \frac{3}{10}$$

$$P(X_2 > 2) = 1 - F_{X_2}(2) = 1 - \left(\frac{13}{45} + \frac{2 \times 2^2}{45}\right) = 1 - \frac{21}{45} = \frac{24}{45}$$

d)

$$F_{X_2}(2) = \frac{13}{45} + \frac{2 \times 2^2}{45} = \frac{13 + 8}{45} = \frac{21}{45}$$

$$F_{X_2}(3) = \frac{13}{45} + \frac{2 \times 3^2}{45} = \frac{13 + 18}{45} = \frac{31}{45}$$

$$P(X_2 > 2 \mid X_2 < 3) = \frac{P(2 < X_2 < 3)}{P(X_2 < 3)} = \frac{F_{X_2}(3) - F_{X_2}(2)}{F_{X_2}(3)}$$

$$= \frac{\frac{31}{45} - \frac{21}{45}}{\frac{31}{45}} = \frac{10}{31}$$

Exercise 23 e)

e)

X = Amount received after investment (in millions of €)

(i)

$I_1 \equiv \{$ The firm invests in project 1 $\}$

$I_2 \equiv \{$ " " " " " 2 $\}$

I_1 and I_2 are a partition of the sample space

$$F_X(x) = P(X \leq x) = P(X \leq x \cap I_1) + P(X \leq x \cap I_2) =$$

$$= P(X \leq x \mid I_1)\, P(I_1) + P(X \leq x \mid I_2)\, P(I_2) =$$

$$= P(X_1 \leq x)\, \frac{1}{3} + P(X_2 \leq x)\, \frac{2}{3} =$$

$$= \underbrace{\frac{1}{3}}_{\lambda:\ 0<\lambda<1} \underbrace{F_{X_1}(x)}_{\text{Discrete r.v.}} + \underbrace{\frac{2}{3}}_{1-\lambda} \underbrace{F_{X_2}(x)}_{\text{Continuous r.v.}}$$

Exercise 23 e)

Therefore, X is a mixed random variable.

To find the expression for $F_X(x)$ let's substitute the expressions of $F_{X_1}(x)$ and $F_{X_2}(x)$:

$$F_X(x) = \frac{1}{3} \times \begin{cases} 0 & (x<0) \\ \frac{1}{3} & (0 \leq x < 1) \\ \frac{1}{2} & (1 \leq x < 2) \\ \frac{7}{10} & (2 \leq x < 3) \\ 1 & (x \geq 3) \end{cases} + \frac{2}{3} \begin{cases} 0 & (x<0) \\ \frac{x}{3} & (0 \leq x < 1) \\ \frac{13}{45} + \frac{2}{45}x^2 & (1 \leq x < 4) \\ 1 & (x \geq 4) \end{cases} =$$

$$= \begin{cases} 0 & (x<0) \\ \frac{1}{9} & (0 \leq x < 1) \\ \frac{1}{6} & (1 \leq x < 2) \\ \frac{7}{30} & (2 \leq x < 3) \\ \frac{1}{3} & (x \geq 3) \end{cases} + \begin{cases} 0 & (x<0) \\ \frac{2}{9}x & (0 \leq x < 1) \\ \frac{26}{135} + \frac{4}{135}x^2 & (1 \leq x < 4) \\ \frac{2}{3} & (x \geq 4) \end{cases} =$$

$$= \begin{cases} 0 & (x<0) \\ \frac{1}{9} + \frac{2}{9}x & (0 \leq x < 1) \\ \frac{1}{6} + \frac{26}{135} + \frac{4}{135}x^2 = \frac{97}{270} + \frac{4}{135}x^2 & (1 \leq x < 2) \\ \frac{7}{30} + \frac{26}{135} + \frac{4}{135}x^2 = \frac{23}{54} + \frac{4}{135}x^2 & (2 \leq x < 3) \\ \frac{1}{3} + \frac{26}{135} + \frac{4}{135}x^2 = \frac{71}{135} + \frac{4}{135}x^2 & (3 \leq x < 4) \\ 1 & (x \geq 4) \end{cases}$$

Exercise 23 e)

(ii)

$$P(X \geq 3) = 1 - P(X<3) = 1 - F_X(3^-)$$

$$= 1 - \left(\frac{23}{54} + \frac{4}{135}3^2\right) = 1 - \frac{187}{270} = \frac{83}{270}$$

(iii)

$$P(X-1<0) = P(X<1) = F_X(1^-) =$$

$$= \frac{1}{9} + \frac{2}{9}(1) = \frac{3}{9} = \frac{1}{3}$$

Thanks!

Questions?