STATISTICS II

ISEG Lisbon School of Economics & Management
Universidade de Lisboa

Bachelor's degrees in Economics, Finance and Management

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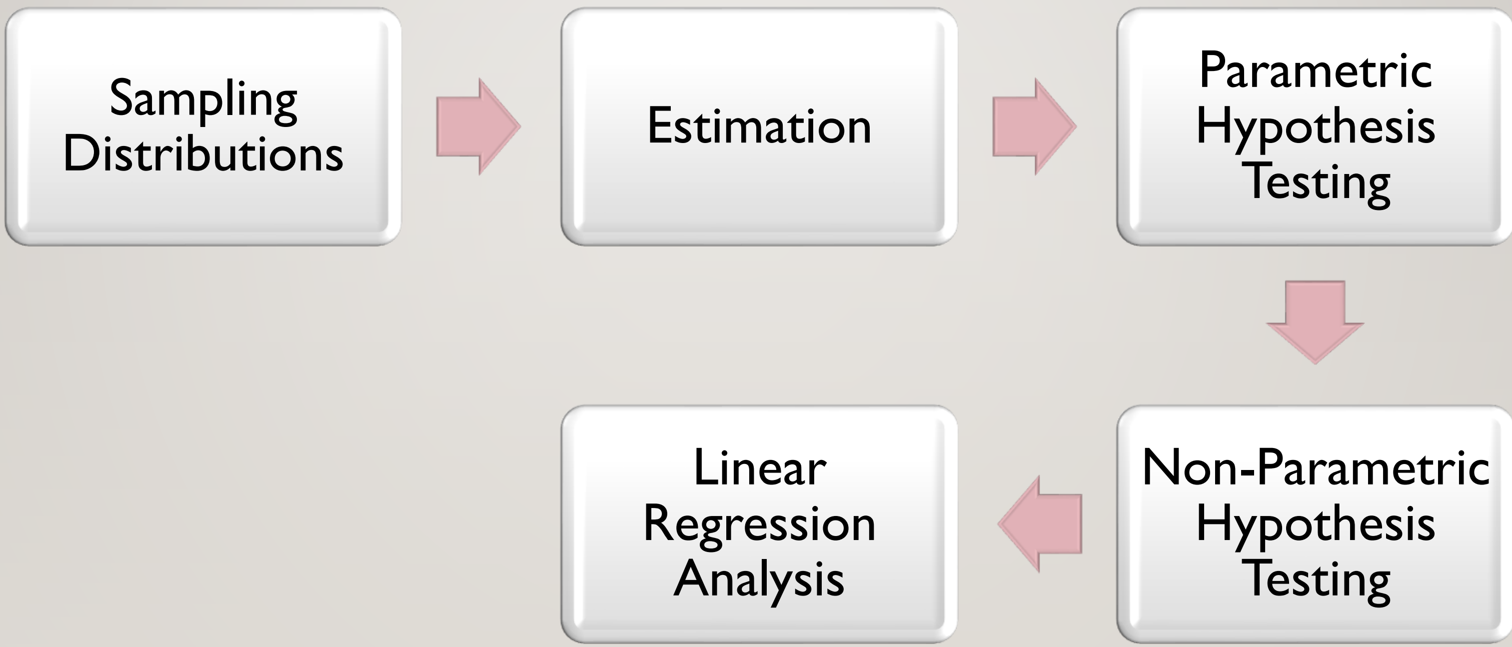
https://doity.com.br/estatistica-aplicada-a-nutricao

https://basiccode.com.br/produto/informatica-basica/

PROGRAM


Sampling Distributions
Estimation
Parametric Hypothesis Testing
Non-Parametric Hypothesis Testing
Linear Regression Analysis

LECTURE 6 HOMEWORK: QUESTIONS AND SOLUTIONS

EXERCISE 7.49

7.49 A manufacturer is concerned about the variability of the levels of impurity contained in consignments of raw material from a supplier. A random sample of 15 consignments showed a standard deviation of 2.36 in the concentration of impurity levels. Assume normality.

a. Find a 95% confidence interval for the population variance.
b. Would a 99% confidence interval for this variance be wider or narrower than that found in part a?

Newbold et al (2013)

EXERCISE 7.49 A): SOLUTION

✅ Answer:

We use the chi-square confidence interval for the population **variance**.

Given $n = 15$, sample standard deviation $s = 2.36$, so $s^2 = 5.5696$. Degrees of freedom $\nu = n - 1 = 14$.

The $100(1-\alpha)\%$ CI for the variance σ^2 is

$$\left(\frac{(\nu)s^2}{\chi^2_{1-\alpha/2,\nu}}, \frac{(\nu)s^2}{\chi^2_{\alpha/2,\nu}} \right).$$

$$CI_{(1-\alpha)}(\sigma^2) = \left(\frac{(n-1)s^2}{\chi^2_{1-\frac{\alpha}{2},n-1}}, \frac{(n-1)s^2}{\chi^2_{\frac{\alpha}{2},n-1}} \right)$$

a) 95% confidence interval ($\alpha = 0.05$)

For $\nu = 14$:

$$\chi^2_{0.975,14} \approx 26.11895, \qquad \chi^2_{0.025,14} \approx 5.62873.$$

Compute numerator $(\nu)s^2 = 14 \times 5.5696 = 77.9744$.

Thus

$$\text{Lower bound for } \sigma^2 = \frac{77.9744}{26.11895} \approx 2.9854,$$

$$\text{Upper bound for } \sigma^2 = \frac{77.9744}{5.62873} \approx 13.8529.$$

$CI_{95\%}$ (σ^2) = (2.99, 13.85)

So a 95% CI for the **variance** is approximately

$$\boxed{2.99 \le \sigma^2 \le 13.85}.$$

For interpretation it is common to also give the CI for the **standard deviation** by taking square roots:

$$\sqrt{2.9854} \approx 1.728, \qquad \sqrt{13.8529} \approx 3.722,$$

so a 95% CI for σ is approximately $\boxed{1.73 \le \sigma \le 3.72}$.

$CI_{95\%}$ (σ) = (1.73, 3.72)

EXERCISE 7.49 B): SOLUTION

✅ Answer:

We use the chi-square confidence interval for the population **variance**.

Given $n = 15$, sample standard deviation $s = 2.36$, so $s^2 = 5.5696$. Degrees of freedom $\nu = n - 1 = 14$.

The $100(1-\alpha)\%$ CI for the variance σ^2 is

$$\left(\frac{(\nu)s^2}{\chi^2_{1-\alpha/2,\nu}}, \frac{(\nu)s^2}{\chi^2_{\alpha/2,\nu}} \right).$$

b) Would a 99% CI be wider or narrower?

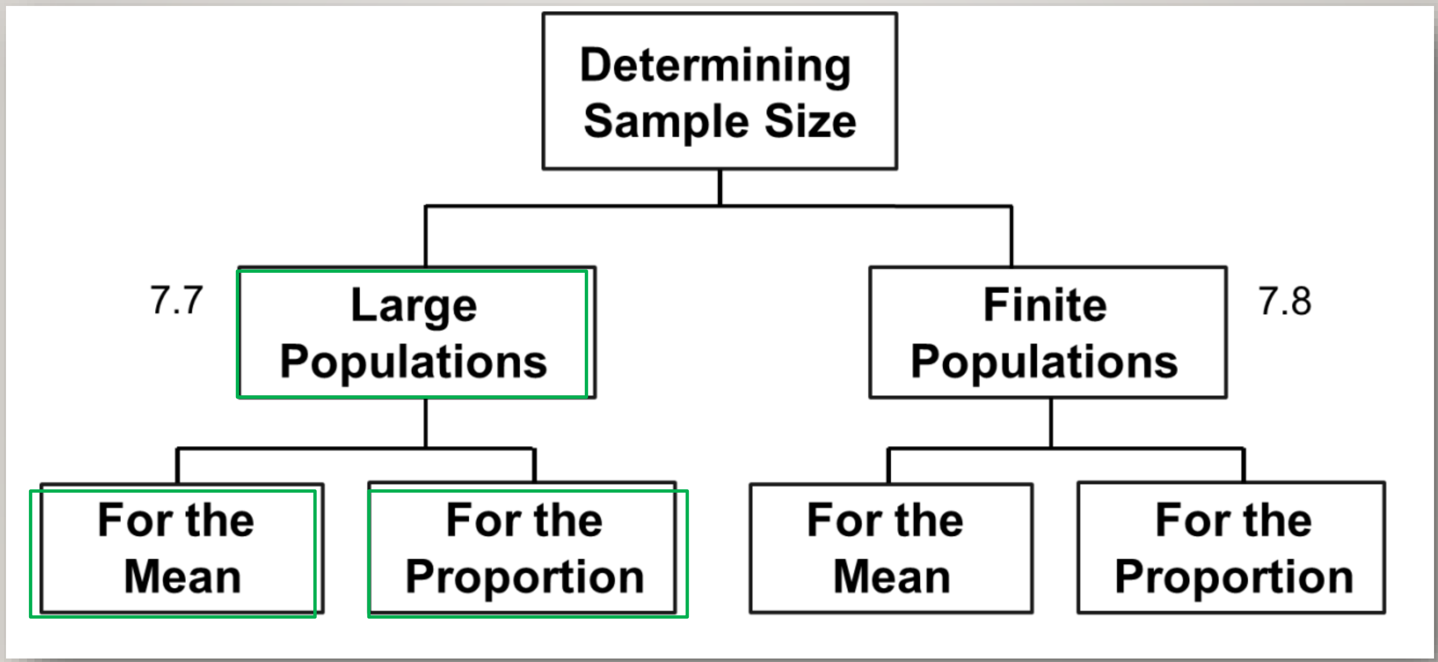
A 99% confidence interval would be **wider**. (Greater confidence requires a larger interval: the chi-square quantiles move farther into the tails, which increases the spread of the interval.)

(For reference, the 99% CI for σ^2 would be about $(2.49,\ 19.14)$, confirming it is wider than the 95% interval.)

$CI_{99\%}(\sigma^2) = (2.49, 19.14)$

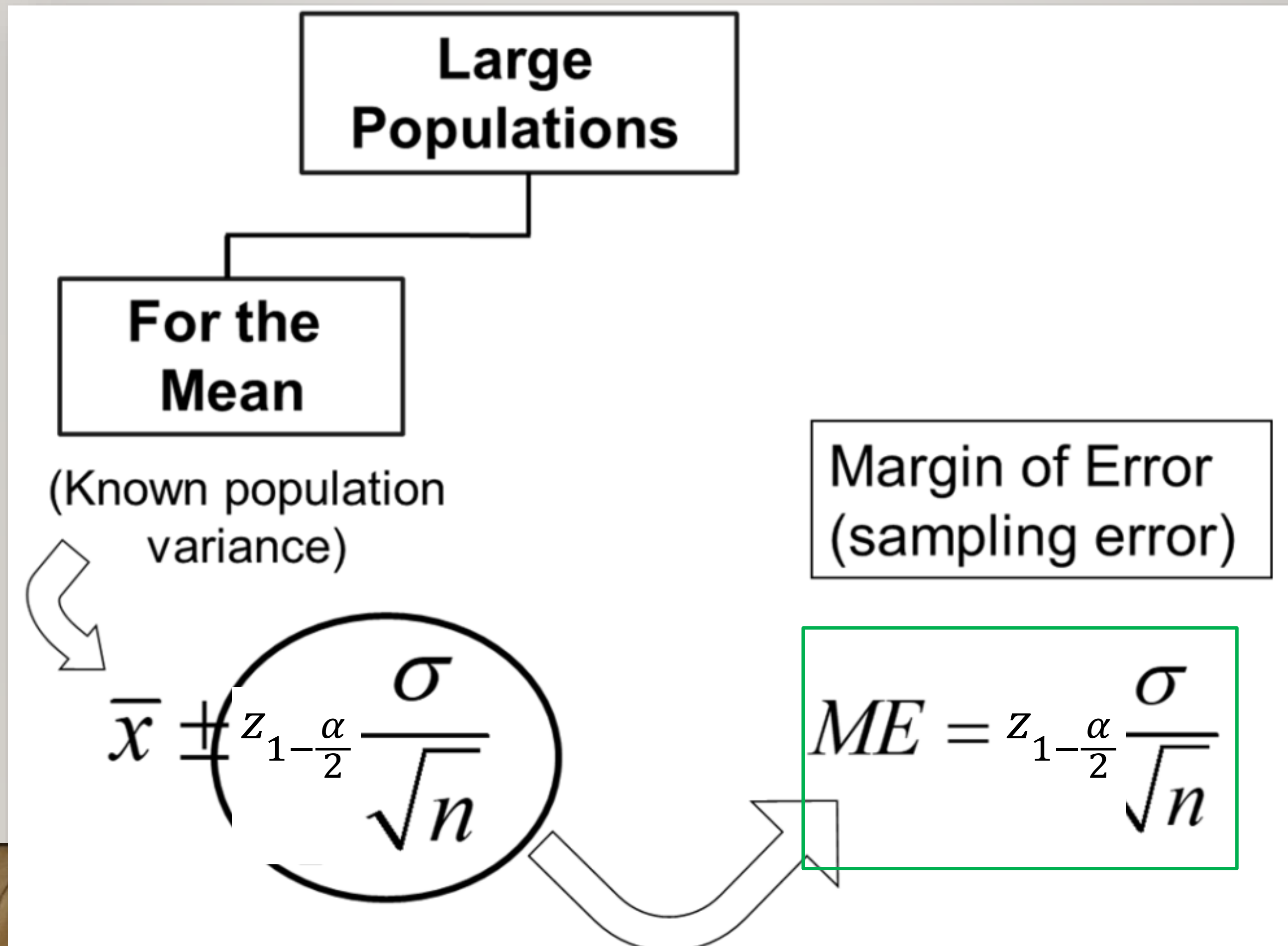
LECTURE 7: SAMPLE SIZE DETERMINATION - LARGE POPULATIONS

SAMPLE SIZE DETERMINATION

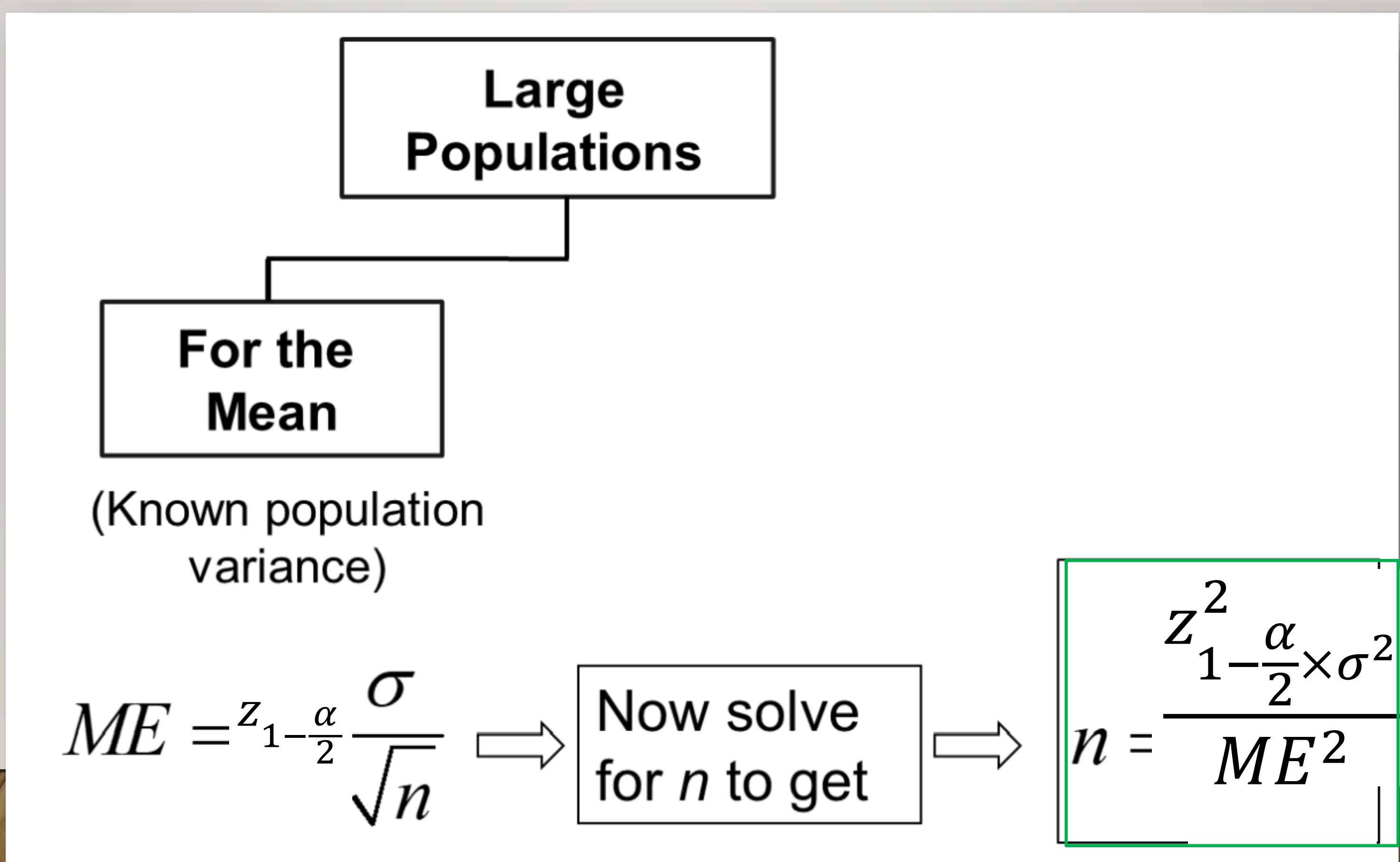


Note: Only the determination of sample size for large samples will be covered, considering either a mean or a proportion.

SAMPLE SIZE DETERMINATION: POPULATION MEAN



SAMPLE SIZE DETERMINATION: POPULATION MEAN



SAMPLE SIZE DETERMINATION: POPULATION MEAN

- To determine the required sample size for the mean, you must know:
  - The desired level of confidence $(1-\alpha)$, which determines the $z_{1-\frac{\alpha}{2}}$ value
  - The acceptable margin of error (sampling error), *ME*
  - The population standard deviation, σ

REQUIRED SAMPLE SIZE EXAMPLE: POPULATION MEAN

If $\sigma = 45$, what sample size is needed to estimate the mean within ± 5 with 90% confidence?

ME = 5

$1 - \alpha = 0.9 \Rightarrow 1 - \alpha/2 = 0.95 \Rightarrow z_{0.95} = 1.645$

$$n = \frac{z^2_{1-\frac{\alpha}{2}} \times \sigma^2}{ME^2} = \frac{(1.645)^2 (45)^2}{5^2} = 219.19$$

So the required sample size is **n = 220**

(Always round up)

Note: Since n, the sample size, must always be an integer, the result is rounded up.

EXERCISE: SAMPLE SIZE

The academic affairs committee at a university wants to estimate the average number of hours per week that students spend studying outside the classroom.

How large of a sample is required to estimate the population mean with a 90% confidence interval if the margin of error is to be at most 2 hours?

Assume that the population is normally distributed and that the population standard deviation is known to be $\sigma = 8$ hours.

EXERCISE: SOLUTION

Answer:

Given:

- Desired confidence level: $90\% \rightarrow \alpha = 0.10$
- Margin of error: $\mathrm{ME} = 2$ hours
- Population standard deviation: $\sigma = 8$ hours
- Population is normally distributed.

We want the **required sample size** n for a confidence interval for the **mean**.

Step 1: Formula

For a **known population standard deviation**, the sample size is

$$n = \left(\frac{z_{1-\alpha/2}\,\sigma}{\mathrm{ME}}\right)^2$$

Step 4: Round up

$$\boxed{n = 44}$$

Step 2: Determine $z_{1-\alpha/2}$

- Confidence level = 90% $\rightarrow 1 - \alpha/2 = 0.95$
- From the standard normal table:

$$z_{0.95} \approx 1.645$$

Answer:

The academic affairs committee should survey **at least 44 students** to estimate the mean study hours per week with a **90% confidence interval** and a **margin of error of 2 hours**.

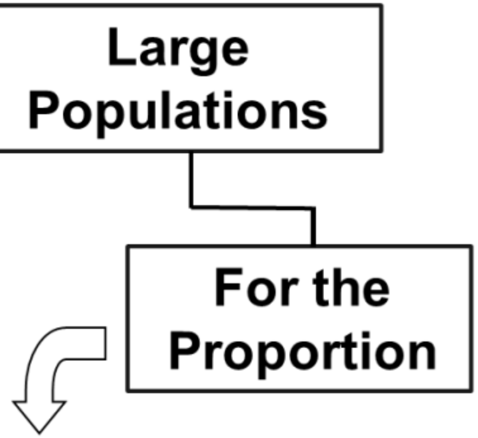
Step 3: Plug in the values

$$n = \left(\frac{1.645 \times 8}{2}\right)^2 = \left(\frac{13.16}{2}\right)^2 = (6.58)^2 \approx 43.3$$

Alternative Solution:

$\mathrm{ME} = \frac{z_{1-\frac{\alpha}{2}} \times \sigma}{\sqrt{n}} < 2 \Leftrightarrow n > (1.645 \times 8/2)^2 = 43.3 \sim 44$

SAMPLE SIZE DETERMINATION: POPULATION PROPORTION



$$\hat{p} \pm z_{1-\frac{\alpha}{2}} \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

$$ME = z_{1-\frac{\alpha}{2}} \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

Margin of Error (sampling error)

SAMPLE SIZE DETERMINATION: POPULATION PROPORTION

Large Populations

For the Proportion

$$ME = z_{1-\frac{\alpha}{2}} \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

$\hat{p}(1-\hat{p})$ cannot be larger than 0.25, when $\hat{p} = 0.5$

→ Substitute 0.25 for $\hat{p}(1-\hat{p})$ and solve for n to get

→ $$n = \frac{z^2_{1-\frac{\alpha}{2}} \times 0.25}{ME^2}$$

$$n = \left(\frac{z_{1-\alpha/2}}{\mathrm{ME}}\right)^2 p(1-p)$$

When p is unknown, we use the **conservative value**

$$p = 0.5$$

because it maximizes $p(1-p)$ and ensures a sufficiently large sample size.

SAMPLE SIZE DETERMINATION: POPULATION PROPORTION

- The sample and population proportions, $\hat{p}$ and *P*, are generally not known (since no sample has been taken yet)
- $P(1-P)=0.25$ generates the largest possible margin of error (so guarantees that the resulting sample size will meet the desired level of confidence)
- To determine the required sample size for the proportion, you must know:
  - The desired level of confidence $(1-\alpha)$, which determines the critical $z_{1-\frac{\alpha}{2}}$ value
  - The acceptable sampling error (margin of error), *ME*
  - Estimate $P(1-P)=0.25$

REQUIRED SAMPLE SIZE EXAMPLE: POPULATION PROPORTION

How large a sample would be necessary to estimate the true proportion defective in a large population within $\pm 3\%$, with 95% confidence?

ME = 0.03

$1 - \alpha = 0.95 \Rightarrow 1 - \alpha/2 = 0.975 \Rightarrow z_{0.975} = 1.96$

REQUIRED SAMPLE SIZE EXAMPLE: POPULATION PROPORTION

Solution:

For 95% confidence, use $z_{0.975} = 1.96$

$ME = 0.03$

Estimate $P(1-P) = 0.25$

$$n = \frac{z^2_{1-\frac{\alpha}{2}} \times 0.25}{ME^2} = \frac{(0.25)(1.96)^2}{(0.03)^2} = 1067.11$$

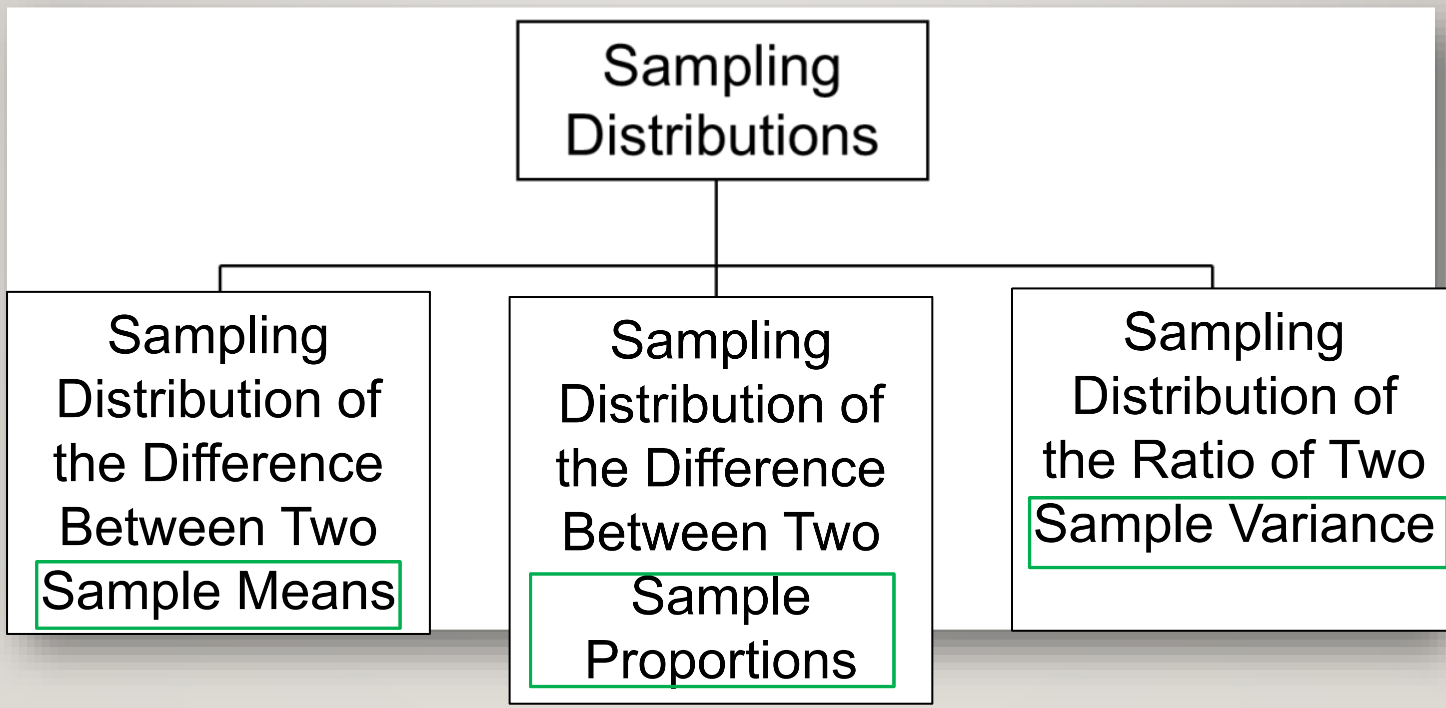
So use $n = 1068$

EXERCISE 7.70

7.70 The student government association at a university wants to estimate the percentage of the student body that supports a change being considered in the academic calendar of the university for the next academic year. How many students should be surveyed if a 90% confidence interval is desired and the margin of error is to be only 3%?

Newbold et al (2013)

SAMPLING DISTRIBUTIONS FOR TWO SAMPLES



LECTURE 7: SAMPLING DISTRIBUTION OF THE DIFFERENCE BETWEEN TWO SAMPLE MEANS

DIFFERENCE OF TWO SAMPLE MEANS

Two Independent Samples

Let

$$X_{11}, \ldots, X_{1n_1} \quad \text{and} \quad X_{21}, \ldots, X_{2n_2}$$

be **two independent samples** of sizes n_1 and n_2, respectively.

Define the sample means as

$$\bar{X}_1 = \frac{1}{n_1} \sum_{i=1}^{n_1} X_{1i}, \qquad \bar{X}_2 = \frac{1}{n_2} \sum_{i=1}^{n_2} X_{2i}.$$

We are interested in the **difference between the two sample means**:

$$\bar{X}_1 - \bar{X}_2.$$

SAMPLING DISTRIBUTION OF $\bar{X}_1 - \bar{X}_2$ (KNOWN VARIANCE)

If the populations are Normal, with $X_1 \sim N(\mu_1, \sigma_1)$ and $X_2 \sim N(\mu_2, \sigma_2)$, and σ_1 and σ_2 are known, then,

by the theorem of additivity of the Normal distribution, we have:

$$Z = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} \sim N(0, 1).$$

If the populations are not Normal, but the samples are large, then by the corollary of the Central Limit Theorem (CLT), we have $Z \sim N(0, 1)$, considering the previously discussed analogous situation.

ProbabilidadesEstatistica2019.pdf

Note:

- If we have two normal populations with known variances, the random variable $\bar{X}_1 - \bar{X}_2$ has a normal distribution.

- If the population distributions are unknown, but the variances are known and the sample sizes are large ($n_1 \geq 25$ and $n_2 \geq 25$), then the random variable $\bar{X}_1 - \bar{X}_2$ has an approximately normal distribution.

SAMPLING DISTRIBUTION OF $\bar{X}_1 - \bar{X}_2$ (UNKNOWN BUT EQUAL VARIANCES)

If the populations are Normal, with $X_1 \sim N(\mu_1, \sigma_1)$ and $X_2 \sim N(\mu_2, \sigma_2)$, and σ_1 and σ_2 are unknown but equal ($\sigma_1 = \sigma_2$), then it can be shown that:

$$T = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{(n_1-1)S_1^2+(n_2-1)S_2^2}{n_1+n_2-2}\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} \sim t_{n_1+n_2-2}.$$

If the populations are not Normal, but the samples are large, then by extension of the Central Limit Theorem (CLT), the above expression is approximately $N(0,1)$.

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Note:

- If we have two normal populations with unknown but assumed equal variances, the random variable *T* has a Student's *t* distribution with $n_1 + n_2 - 2$ degrees of freedom.

- If the population distributions are unknown, but the variances are unknown and assumed equal, and the sample sizes are large ($n_1 \geq 25$ and $n_2 \geq 25$), then the random variable $\bar{X}_1 - \bar{X}_2$ has an approximately normal distribution.

SAMPLING DISTRIBUTION OF $\bar{X}_1 - \bar{X}_2$ (UNKNOWN BUT UNEQUAL VARIANCES)

If the populations are Normal, with $X_1 \sim N(\mu_1, \sigma_1)$ and $X_2 \sim N(\mu_2, \sigma_2)$, and σ_1 and σ_2 are unknown and unequal ($\sigma_1 \neq \sigma_2$), then, by Welch's approximation (Murteira et al., 2007), we have:

$$T = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}} \sim t_v,$$

where the degrees of freedom v are given by:

$$v = \frac{\left(\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}\right)^2}{\frac{1}{n_1-1}\left(\frac{S_1^2}{n_1}\right)^2 + \frac{1}{n_2-1}\left(\frac{S_2^2}{n_2}\right)^2},$$ rounded down to the nearest integer.

Also in this case, if the populations are not Normal, but the samples are large, then by extension of the Central Limit Theorem (CLT), the above expression is approximately $N(0,1)$, with the previous observations remaining valid.

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Note:

- If we have two normal populations with unknown but assumed unequal variances, the random variable *T* has a Student's *t* distribution with v degrees of freedom.
- If the population distributions are unknown, but the variances are unknown and assumed unequal, and the sample sizes are large ($n_1 \geq 25$ and $n_2 \geq 25$), then the random variable $\bar{X}_1 - \bar{X}_2$ has an approximately normal distribution.

EXERCISE 1

A pharmaceutical company has launched a new sleep medication that has been used in hospitals. It was observed that patients **not taking this medication** sleep on average 7.5 hours, with a standard deviation of 1.4 hours, whereas patients **taking the medication** sleep on average 8 hours, with a standard deviation of 2 hours.

In a certain hospital, 31 patients were observed **not taking the medication** and 61 patients were observed **taking the medication**.

Question: What is the probability that the patients in the first group (not taking the medication) will sleep **on average more** than the patients in the second group (taking the medication)? Assume that sleep durations are normally distributed.

EXERCISE 1: SOLUTION

Answer:

We have two independent normal populations:

- Group 1 (no medication): $\mu_1 = 7.5$, $\sigma_1 = 1.4$, $n_1 = 31$
- Group 2 (medication): $\mu_2 = 8$, $\sigma_2 = 2$, $n_2 = 61$

X1 ~ Normal(7.5, 1.4^2)

X2 ~ Normal(8, 2^2)

We want $P(\bar{X}_1 > \bar{X}_2)$, the probability that the sample mean of group 1 exceeds the sample mean of group 2.

Step 1: Define the random variable

$$D = \bar{X}_1 - \bar{X}_2$$

Since $\bar{X}_1$ and $\bar{X}_2$ are independent and normally distributed, D is also normal:

$$D \sim N\left(\mu_D, \sigma_D^2\right)$$

where

$$\mu_D = \mu_1 - \mu_2 = 7.5 - 8 = -0.5$$

$$\sigma_D = \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} = \sqrt{\frac{1.4^2}{31} + \frac{2^2}{61}}$$

$$Z = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} \sim N(0,1).$$

EXERCISE 1: SOLUTION

Answer:

Step 2: Compute the standard deviation

$$\sigma_D = \sqrt{\frac{1.96}{31} + \frac{4}{61}} \approx \sqrt{0.0632 + 0.0656} = \sqrt{0.1288} \approx 0.359$$

Step 3: Standardize and compute probability

We want:

$$P(\bar{X}_1 > \bar{X}_2) = P(D > 0)$$

Standardize D using the Z-score:

$$Z = \frac{D - \mu_D}{\sigma_D} = \frac{0 - (-0.5)}{0.359} = \frac{0.5}{0.359} \approx 1.39$$

$$P(D > 0) = P(Z > 1.39)$$

$$Z = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} \sim N(0,1).$$

Step 4: Find the probability

From standard normal tables:

$$P(Z > 1.39) \approx 0.082$$

$$\boxed{P(\bar{X}_1 > \bar{X}_2) \approx 0.082 \text{ (or } 8.2\%)}$$

Interpretation: There is a low probability (about 8%) that patients **not taking the medication** will sleep more on average than patients **taking the medication**.

EXERCISE 2

A pharmaceutical company has launched a new sleep medication that has been used in hospitals. It was observed that patients **not taking this medication** sleep on average 7.5 hours, while patients **taking the medication** sleep on average 8 hours.

In a certain hospital, n_1 patients **not taking the medication** and n_2 patients **taking the medication** were observed. The observed standard deviations were 1.4 hours and 2 hours, respectively.

Question: Determine the probability that patients in the first group sleep **on average less** than patients in the second group, for $n_1 = 20$ and $n_2 = 27$, assuming normality of the distributions and considering:

a) Equal population variances.

b) Unequal population variances.

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EXERCISE 2 A): SOLUTION

✅ Answer:

We have:

- Group 1 (no medication): $\mu_1 = 7.5$, $S_1 = 1.4$, $n_1 = 20$
- Group 2 (medication): $\mu_2 = 8$, $S_2 = 2$, $n_2 = 27$

We want $P(\bar{X}_1 < \bar{X}_2)$.

X1 ~ Normal(7.5, 1.4^2)

X2 ~ Normal(8, 2^2)

$$T = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{(n_1-1)S_1^2+(n_2-1)S_2^2}{n_1+n_2-2}\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}} \sim t_{n_1+n_2-2}.$$

a) Equal population variances ($\sigma_1^2 = \sigma_2^2$)

1. **Compute pooled standard deviation**

$$S_p^2 = \frac{(n_1-1)S_1^2 + (n_2-1)S_2^2}{n_1+n_2-2} = \frac{(20-1)1.4^2 + (27-1)2^2}{20+27-2}$$

$$S_p^2 = \frac{19 \cdot 1.96 + 26 \cdot 4}{45} = \frac{37.24 + 104}{45} = \frac{141.24}{45} \approx 3.14$$

$$S_p \approx \sqrt{3.14} \approx 1.772$$

2. **Compute standard error of the difference**

$$SE = S_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}} = 1.772\sqrt{\frac{1}{20} + \frac{1}{27}}$$

$$\frac{1}{20} + \frac{1}{27} = 0.05 + 0.03704 \approx 0.08704$$

$$SE \approx 1.772 \cdot \sqrt{0.08704} \approx 1.772 \cdot 0.295 \approx 0.522$$

EXERCISE 2 A): SOLUTION

Answer:

We have:

- Group 1 (no medication): $\mu_1 = 7.5$, $S_1 = 1.4$, $n_1 = 20$
- Group 2 (medication): $\mu_2 = 8$, $S_2 = 2$, $n_2 = 27$

We want $P(\bar{X}_1 < \bar{X}_2)$.

$X1 \sim \text{Normal}(7.5, 1.4^2)$

$X2 \sim \text{Normal}(8, 2^2)$

$$T = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{(n_1-1)S_1^2+(n_2-1)S_2^2}{n_1+n_2-2}\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}} \sim t_{n_1+n_2-2}.$$

3. **Compute t-statistic**

$$t = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{SE} = \frac{0-(7.5-8)}{0.522} = \frac{0.5}{0.522} \approx 0.958$$

4. **Degrees of freedom**

$$df = n_1 + n_2 - 2 = 20 + 27 - 2 = 45$$

5. **Find probability**

$$P(\bar{X}_1 < \bar{X}_2) = P(t > -0.958) \approx P(t < 0.958) \approx 0.83$$

✅ Answer (equal variances): $\mathbf{P} \approx \mathbf{0.83}$

EXERCISE 2 B): SOLUTION

✅ Answer:

We have:

- Group 1 (no medication): $\mu_1 = 7.5$, $S_1 = 1.4$, $n_1 = 20$
- Group 2 (medication): $\mu_2 = 8$, $S_2 = 2$, $n_2 = 27$

We want $P(\bar{X}_1 < \bar{X}_2)$.

$X1 \sim$ Normal$(7.5, 1.4^2)$

$X2 \sim$ Normal$(8, 2^2)$

b) Unequal population variances ($\sigma_1^2 \neq \sigma_2^2$, Welch's t-test)

1. Standard error

$$SE = \sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}} = \sqrt{\frac{1.96}{20} + \frac{4}{27}}$$

$$SE = \sqrt{0.098 + 0.1481} = \sqrt{0.2461} \approx 0.496$$

2. t-statistic

$$t = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{SE} = \frac{0 - (-0.5)}{0.496} \approx 1.008$$

$$T = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}} \sim t_v,$$

EXERCISE 2 B): SOLUTION

✅ Answer:

We have:

- Group 1 (no medication): $\mu_1 = 7.5$, $S_1 = 1.4$, $n_1 = 20$
- Group 2 (medication): $\mu_2 = 8$, $S_2 = 2$, $n_2 = 27$

We want $P(\bar{X}_1 < \bar{X}_2)$.

$X1 \sim$ Normal(7.5, 1.4^2)

$X2 \sim$ Normal(8, 2^2)

b) Unequal population variances ($\sigma_1^2 \neq \sigma_2^2$, Welch's t-test)

1. **Standard error**

$$SE = \sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}} = \sqrt{\frac{1.96}{20} + \frac{4}{27}}$$

$$SE = \sqrt{0.098 + 0.1481} = \sqrt{0.2461} \approx 0.496$$

2. **t-statistic**

$$t = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{SE} = \frac{0 - (-0.5)}{0.496} \approx 1.008$$

EXERCISE 2 B): SOLUTION

✅ Answer:

We have:

- Group 1 (no medication): $\mu_1 = 7.5$, $S_1 = 1.4$, $n_1 = 20$
- Group 2 (medication): $\mu_2 = 8$, $S_2 = 2$, $n_2 = 27$

We want $P(\bar{X}_1 < \bar{X}_2)$.

$X1 \sim$ Normal$(7.5, 1.4^2)$

$X2 \sim$ Normal$(8, 2^2)$

3. Degrees of freedom (Welch's approximation)

$$v = \frac{\left(\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}\right)^2}{\frac{1}{n_1-1}\left(\frac{S_1^2}{n_1}\right)^2 + \frac{1}{n_2-1}\left(\frac{S_2^2}{n_2}\right)^2}$$

$$v = \frac{(0.098 + 0.1481)^2}{\frac{1}{19}(0.098)^2 + \frac{1}{26}(0.1481)^2} = \frac{0.2461^2}{0.000505 + 0.000843} = \frac{0.06055}{0.001348} \approx 44.9 \approx 44 \text{ (rounded down)}$$

4. Probability

$$P(\bar{X}_1 < \bar{X}_2) = P(t_{44} > -1.008) \approx P(t_{44} < 1.008) \approx 0.84$$

✅ Answer (unequal variances): $\mathbf{P} \approx \mathbf{0.84}$

HOMEWORK OF LECTURE 7: QUESTIONS

EXERCISE 41

A security card issuing center has **two personalization machines**, operating independently. The processing time (in seconds) for each machine is normally distributed with the **same mean**, with a standard deviation of 10 seconds for the first machine and 15 seconds for the second.

Samples of 16 cards are taken from each machine.

Questions:

a) Calculate the probability that the **absolute difference between the sample means** of the two machines exceeds 5 seconds.

b) What is the probability that the **sample standard deviation** of the first machine is greater than that of the second machine?

Murteira et al (2015), Chapter 6

Note: Only complete part (a) of Exercise 41, as part (b) involves topics not yet covered.

THANKS!

Questions?