Foundations of Financial Economics

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Midterm

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Total time: 2 hours and 30 minutes. Total points: 20 Instructions:

Please sit in alternate seats. This is a closed-book, closed-note exam. Please get rid of everything but pen/pencil. In your answer explain all the steps in your reasoning. Keep answers short; I don t give more credit for long answers, and I can take points o⁄ if you add things that are wrong or irrelevant.

Formulas:
If x and y are random variables then

$$E(xy) = ExEy + cov(x, y);\ \sigma^2(x) = Ex^2 - (Ex)^2$$

$$\sigma^2(ax + by) = a^2\sigma^2(x) + b^2\sigma^2(x) + 2abcov(y, x)$$

$$cov(ax, by) = abcov(x, y);\ \sigma^2(kx) = k^2\sigma^2(x)$$

$$-1 \le corr(x, y) \le 1; \quad corr(x, y) = \frac{cov(x, y)}{\sigma(x)\sigma(y)}$$

If x is normal distributed then $\exp(x)$ is lognormal, and

$$E\ exp(x) = \exp(Ex + 0.5\sigma^2(x))$$

I (12 pts.)

1. (12 pts) This group of questions concerns basic concepts.

a. (1 pt) What is a stochastic discount factor?
b. (1 pt) What is the equity premium puzzle?
c. (1 pt) What is the risk-free puzzle?
d. (1 pt) What is the CAPM?
e. (1 pt) What is the expected return, according to CAPM, on an asset that has a beta of 1:5, when the market return is 6%, and the risk-free rate is 2%?
f. (1 pt) How is the systematic risk measured in the CAPM?
g. (1 pt) What is the mean variance frontier?
h. (1 pt) What is an Arrow-Debreu security?
i. (1 pt) What is the relationship between the riskless security and the Arrow-Debreu securities?
j. (1 pt) Consider the case when all assets have independent returns. Is it true that a portfolio with more assets has always lower volatility (i.e. standard deviation of returns)? Explain.
k. (1 pt) Show that when the returns of the assets are independent and have equal volatilities then an investor that minimizes portfolio returns volatility should hold in his/her portfolio all the assets available.
l. (1 pt) Can there be arbitrage opportunities when markets are complete? Explain.

II (4 pts.)

2. Consider a representative agent economy. The consumer maximizes $E_0 \sum_{t=0}^{\infty} \beta^t u(C_t)$, where C_t is consumption of period *t*. There is a risk-free asset that pays return R_{t+1}^f, known at time *t*. There is also a risky asset that pays return R_{t+1}, unknown at time *t* and realized at time *t* + 1. The representative household chooses to invest quantity B_t in the riskless asset, and quantity S_t in the risky asset, to maximize

$$E_0 \sum_{t=0}^{\infty} \beta^t \frac{C_t^{1-\gamma}}{1-\gamma}, \qquad 0 < \beta < 1$$

subject to the budget constraint

$$B_{t+1} + S_{t+1} + C_t = R_t^f B_t + R_t S_t, \qquad \text{for all } t \geq 0$$

a. (1 pt) Write the Lagrangian of the household's problem.

b. (1 pt) What are the first order conditions of the consumer s problem?

c. (1 pt) Define $\Delta c_{t+1} \equiv lnC_{t+1} - lnC_t$. Assume that Δc_{t+1} is an i.i.d normal distribution with mean μ ad variance σ^2. Obtain the analytical solution for the riskless return.

III (2 pts.)

The problem of an investor that wants to create a portfolio with minimum variance return, using asset *A* and asset *B,* is the following:

$$\min_{\{w_a, w_b\}} \sigma^2(w_a R_a + w_b R_b)$$

s.t.

$$w_a + w_b = 1$$

a. (1 pt) Determine the w_a that solves the problem.

b. (1 pt) Assume that $cov(R_a, R_b) > 0$ and $\sigma_{R_b} < \sigma_{R_a}$. Show that w_a is higher, the higher the volatility of asset *B*.

IV (2 pts.)

Assume there are *S* states of nature in the economy and that markets are complete.

a. (1 pt) Characterize formally the stochastic discount factor.

b. (1 pt) Let the payoff of a security be $X(s) = s$ where $s = 1, 2, \ldots, S$. What is the price of this security?